Photon Antibunching and Possible Ways to Observe It

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I discuss the photon antibunching effect and point out that there exist many states which exhibit the effect and demonstrate a simple procedure for generating them mathematically. I also suggest here a possible approach to the experimental observation of the effect.

The Hanbury Brown–Twiss effect, or photon bunching, was detected in 1956.[1] The tendency for photons produced by natural sources to cluster has prompted much discussion in the literature[2] and I shall not consider the matter any further here. The effect may be understood quantum mechanically in terms of photon clustering or classically in terms of the stochastic character of the radiation processes in natural sources.[3]

The role of the quantized character of the electromagnetic field in elucidating the structure of optical phenomena has a large literature in which one may find opinions covering a wide spectrum as to the necessity of a quantized description. There is no doubt a good measure of validity to some of the arguments advocating a semiclassical treatment.

Here I discuss a topic in which the quantized character of the electromagnetic field is indispensible to a correct interpretation, namely, the negative Hanbury Brown–Twiss effect, also called the photon-antibunching effect or the anticorrelation effect. That this effect can occur and that it may not be comprehended in terms of an unquantized electromagnetic field has been pointed out on numerous occasions by Glauber.[4] Why has this anticorrelation effect (ACE) not been observed? There is certainly an abundance of quantum states of the field which display the effect. It is well known[5] that a state containing a definite number of photons, an n-quantum state, will display the ACE, but such states offer little hope of experimental realization.

It is clear how one may mathematically generate states which have the ACE from ones which do not. A state (of a single mode for convenience) which displays the ACE is characterized by the fact that the variance of the photon number $(\Delta N)^2$ is less than the average photon number $\langle N\rangle$, i.e.,

$$\Delta \equiv (\Delta N)^2 - \langle N\rangle < 0. \qquad (1)$$

Given a state in which Δ is positive or zero we look for some operator which when acting on that state increases the value of $\langle N\rangle$ while leaving the variance unchanged. An operator which will perform this service for an arbitrary state $|\varphi\rangle$ is the phase operator[6] $E_+ = a^\dagger(a^\dagger a+1)^{-1/2}$. If we define $|\psi\rangle = E_+|\varphi\rangle$ for an arbitrary state $|\varphi\rangle$ we have

$$\langle N\rangle_\psi = \langle N\rangle_\varphi + 1, \qquad (\Delta N)_\psi{}^2 = (\Delta N)_\varphi{}^2. \qquad (2)$$

It is clear that using this sort of procedure one may generate a large class of states possessing the ACE. I shall discuss the detailed structure of the ACE in a future paper.

I now suggest one possible way to observe ACE and, in so doing, I provide a partial explanation for the absence of previous observations. The measurement I consider here suggests that the ACE is likely to be found in transient processes, i.e., in the early stages of certain processes and lasting for very short periods of time. These are not the conditions under which photon correlation measurements are usually made.

Let us now consider the problem of generating states which have a negative value of Δ. It is clear that the process which may generate such states will begin with the field in a state having a nonnegative value of Δ since these are readily available from existing sources. So we need some device that is capable of driving Δ negative at least for some interval of time.

One such device is the degenerate parametric amplifier, i.e., a parametric amplifier for which the signal and idler are identical. The degenerate parametric amplifier has quite a different statistical behavior from the usual nondegenerate one and it can generate states having the sort of correlations required to produce the ACE during a portion of its operation. The basic formal strucutre of optical parametric amplifiers may be found in many papers and books.[7] Here we follow the treatment by Louisell[7] in which the pump is treated classically and losses are neglected. Adapting those results to the degenerate case of a single mode of frequency ω_0 we find the Hamiltonian of the system to be $H=\omega_0 aa^\dagger - k[a^{\dagger 2}e^{-i(\omega t+\varphi)}+\text{H.c.}]$. In this expression $\hbar$ is taken to be unity and k is the (positive, real) coupling constant, while $-\varphi$ is the initial phase of the pump. The pump frequency ω is equal to $2\omega_0$. The equation of motion for the (Heisenberg picture) annihilation operator $a(t)$ corresponding to the above Hamiltonian is $i\dot{a}(t)=\omega_0 a(t)-k\exp\{-i \times(\omega t+\varphi)\}a^\dagger(t)$. The solution to this equation is

$$a(t)=\exp(-i\omega_0 t)[a\cosh kt + a^\dagger i e^{-\varphi}\sinh kt], \quad (3)$$

where $a=a(0)$ is the annihilation operator at $t=0$.

Using Eq. (3) we may calculate $\Delta(t)$ for any chosen initial state. Let us take the initial state to be a coherent state $|\alpha\rangle$.

In some recent work on states of minimum uncertainty product[8] the present author studied a class of states $|z,\alpha\rangle$ generated from the coherent states $|\alpha\rangle$ by means of the unitary operator $U_z=\exp[\frac{1}{2}(za^2-z^*a^{\dagger 2})]$. In general z is an arbitrary complex number. In Ref. 8 I showed that the above states for *real* z constitute all of the minimum-uncertainty states, i.e., states which minimize the uncertainty produce of position and momentum. For complex z the $|z,\alpha\rangle$ are *not* minimal. The effect of the operator U_z on an annihilation operator a is given by $U_z a U_z^\dagger = a\cosh r + a^\dagger \sinh r e^{-i\theta}$, where $z=re^{i\theta}$. Using these results and some of the techniques employed in Ref. 8 we can show that the expectation value of the photon number is given by

$$\langle N(t)\rangle = \langle\alpha|a^\dagger(t)a(t)|\alpha\rangle$$
$$=\langle z,\alpha|a^\dagger a|z,\alpha\rangle, \quad (4)$$

where $z=kt\exp[i(\varphi+\frac{1}{2}\pi)]$. From this result we see that the states $|z,\alpha\rangle$ can be produced by the degenerate parametric amplifier. This may be stated most succinctly for the Schrödinger-picture state vector. If at $t=0$ the mode is in a coherent state $|\alpha\rangle$, then at time t the mode is in the state $|z(t),\alpha(t)\rangle$, where $z(t)=kt\exp[i(\varphi+\frac{1}{2}\pi+2\omega_0 t)]$ and $\alpha(t)=\alpha\exp(-i\omega_0 t)$.

When we calculate $\Delta(t)$ for the state of the field generated by the degenerate parametric amplifier we see that under certain conditions it will be *negative* for some finite interval. If the photons emitted during this interval are counted they will display the suppression of coincident pairs characteristic of the ACE. Using Eq. (3) we can calculate $\Delta(t)$ in a straightforward manner and get (letting $\alpha=\rho e^{i\theta}$ and $-\varphi=\varphi_p$)

$$\Delta(t)=A(t)+\rho^2[B(t)+C(t)\sin(2\theta-\varphi_p)], \quad (5)$$

where

$$A(t)=\tfrac{1}{4}(\cosh 4kt - 2\cosh 2kt + 1),$$
$$B(t)=\cosh 4kt - \cosh 2kt,$$
$$C(t)=\sinh 4kt - \sinh 2kt.$$

The functions $A(t)$, $B(t)$, and $C(t)$ are all nonnegative so that the only way $\Delta(t)$ can be negative is for $\sin(2\theta-\varphi_p)$ to be negative. The optimum choice for the anticorrelation effect to be most pronounced is for $\sin(2\theta-\varphi_p)$ to be equal to -1. This will be the case for $2\theta-\varphi_p=-\pi/2$. For this choice $\Delta(t)$ becomes

$$\Delta(t)=A(t)+\rho^2[B(t)-C(t)]. \quad (6)$$

This function is zero at $t=0$, and goes negative as t increases from zero. It is maximally negative at $t_1\approx \ln 2/2k$ and it becomes zero again at $t_2\approx\ln(2\rho^{2/3}+\frac{1}{3})/2k$ after which time it remains positive. These estimates of t_1 and t_2 are accurate provided that ρ^2 is no smaller than, say, 10.

An experimental realization of the process discussed here might proceed as follows. A strong cw laser beam is allowed to enter a nonlinear crystal and produce some second-harmonic light. Both the fundamental and the second harmonic are then allowed to enter a second crystal of the same material. The second harmonic serves as the pump and the fundamental serves as the signal and idler. The second crystal can be tuned so as to make the signal and idler modes identical. The phase combination $2\theta-\varphi_p$ may be set at the proper value by adjusting the distance between the two crystals. The second crystal will then serve as the amplifier and the emerging light at the fundamental frequency should display the ACE. The magnitude of the effect will be determined by the value of Δ which characterizes the emerging radiation. This is in turn dependent on the amount of time the light spends in the second crystal. The optimum amount of time would be t_1 at which Δ is most negative.

I have carried out my computation in the time domain, which is most natural in quantum mechanical problems of this sort. However, in order to discuss the performance of an actual parametric amplifier using nonlinear crystals we must translate the results so that they apply to a steady-state spatially dependent oscillation. This may be done by replacing kt by γz in the hyperbolic functions in Eq. (3) where z refers to the direction of wave propagation in the crystal. The constant γ may then be evaluated using conventional coupled-mode theory for optical oscillators.[9] The maximization of the ACE then occurs when the amplifier crystal has length l_1 such that $\gamma l_1 = kt_1$ which makes Δ as negative as possible.

To examine the feasibility of observing the ACE this way, let us consider a specific case and put in some typical numbers.

A suitable crystal to use here might be barium sodium niobate. This material has large phase-matchable nonlinear coefficients and good resistance to optical damage, and can be grown with good optical quality and low loss. The value of γl_1 which makes Δ most negative is $\ln 2/2 \approx 0.35$. This corresponds to a single-pass gain of about 12%. Taking the pump wavelength to be 5300 Å, and the parametric amplifier material to be barium sodium niobate, we find the value of γ for degenerate operation to be approximately

$$\gamma = 9\times10^{-4} I_p^{1/2}\ \mathrm{cm}^{-1}, \qquad (7)$$

where $I_p = 10^4\ \mathrm{W/cm^2}$.

Taking $I_p = 10^4\ \mathrm{W/cm^2}$, we find that l_1 is about 3.7 cm. This might be a bit large but is not unreasonable. In fact, the exact length of the crystal is not critical since, in the vicinity of its minimum value, Δ is a relatively weak function of l. The minimum value of Δ is approximately $-0.25\rho^2$. If the length of the crystal is reduced to one tenth of the optimum value then Δ has its magnitude reduced by a factor of 0.25. Since ρ^2 can be quite large, say 10^{10} or more, a tenfold reduction in crystal length leaves Δ still sizably negative.

The observability of the ACE in this experiment is crucially dependent on the relative phase $2\theta - \varphi_p$. The optimum value of this quantity is $-\pi/2$. The effect of fluctuations away from this value must be considered. For an amplifier crystal of optimum length the ACE will persist for values of the relative phase as much as 20° above or below the optimum value of $-\pi/2$. Of course, the value of Δ becomes less negative as the relative phase departs from its optimum value. There are several sources of fluctuation of the relative phase. The signal and pump emerge from the first (second-harmonic generation) crystal with a definite value of the relative phase. If the laser source is operating in a single mode and if the second-harmonic generation takes place with perfect phase matching, then the value of the relative phase is in fact $-\pi/2$. If it could be held at that value then one could achieve optimum relative phase at the entrant surface of the amplifier crystal by adjusting the distance between the two crystals. There will, however, be fluctuations away from $-\pi/2$ due to departures from perfect phase matching in the second-harmonic-generation crystal and from variations in the frequency and phase of the laser source. The fluctuations due to variations in the laser frequency and departures from perfect phase matching can probably be reduced sufficiently so that they will cause only small changes in the relative phase. The phase of the laser, however, undergoes a relatively slow diffusion over the full 2π range. Since this is an uncontrollable factor the detection of the ACE will require that the relative phase be monitored. Fortunately the output of the degenerate parametric amplifier is highly phase dependent and such monitoring appears to be possible.

It could be done, for example, by making use of the fact that the intensity of the amplifier output signal is *lower* than that of the input signal when the ACE is present. This is an indication that the device produces the ACE by soaking up coincident pairs of signal photons and converting them to pump photons. Under optimum conditions of relative phase and crystal lengths the output signal intensity is about *half* the input intensity. So the detection circuitry could be arranged to count photons only when the output signal intensity was lower than the input. This would enable one to exclude from the detection process the effects of large excursions of the relative phase away from its optimum value.

The role of parametric fluorescence in other modes as a source of noise will be negligible here. To calculate the contribution to Δ we consider a nondegenerate oscillator. We find that we get a positive contribution to Δ of $(\sinh\gamma l_1)^4 \approx 0.016$. This is negligible compared to the contributions from the degenerate-amplifier output.

The effect of losses should not materially change the results as long as the operation is well above the amplification threshold. We have done the calculation here using a pure initial state of the signal mode. A mixed state described by some density matrix is more realistic. The ACE will also be present in such cases too, providing there is the right sort of phase correlation between the signal and the pump analogous to $\sin(2\theta - \varphi_p)$ being negative in the pure state case. This phase correlation aspect is really the crucial thing for the effect to be generated at least in the type of process we consider here.

The state generated by the degenerate parametric amplifier, $|z(t), \alpha(t)\rangle$, is not a state of minimum uncertainty product in general. But the uncertainty product does take on its minimum value periodically every time $z(t)$ becomes real. This happens at a rate equal to the pump frequency. It does not appear that minimality is in any way crucial to the ACE especially since there are density matrices displaying the ACE but, as I have shown in a recent paper,[10] there are no minimum-uncertainty density matrices.

I have suggested here a possible specific way to detect the photon anticorrelation effect and also provided some ideas which might lead to other ways. The effect is of interest in several contexts and it seems worthwhile to pursue its detection.

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Note added.—The principal ideas contained in this paper were discussed by the author at the February 1973, New York Meeting of the American Physical Society.[11]

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Decay Width of the Neutral π Meson*

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The cross section for the photoproduction of π^0 mesons from complex nuclei at small angles has been measured at incident bremsstrahlung energies of 4.4 and 6.6 GeV. The data are fitted by a cross section calculated from a sum of Primakoff and nuclear-production amplitudes. A total decay width for the π^0 meson of 8.02 ± 0.42 eV is obtained from the magnitude of the Primakoff amplitude.

The $Y=0$, neutral pseudoscalar mesons decay into photon pairs. The partial width of this decay mode can be determined by measuring the cross section for the photoproduction of the meson in the Coulomb field.[1] Recently we reported the width of the η^0 meson measured in this way.[2] In this paper, the results of a similar experiment carried out for the π^0 meson are given.

A measurement of the π^0 photoproduction cross section at small angles was made for photon energies near 4.4 and 6.6 GeV. Data were recorded for targets of beryllium, aluminum, copper, silver, and uranium. At each machine energy a set of runs on the five targets was taken with photon hodoscope counters located directly above and below the beam line. At the lower photon energy an extra set of runs was made with the counters displaced by 15 mrad from the beam line in or-