can readily be explained either by the configuration of the doorway state leading to different intermediate structures in different channels or by the occurrence of interference between overlapping doorway states and possibly direct reaction contributions.¹² These two effects might upset correlation studies and cast some doubt on the common procedures for identifying intermediate structure by total cross-section measurements and cross correlations.

In conclusion, we emphasize that the nonstatistical cross-section behavior observed in the interaction of two "non- α -particle" nuclei at very high level densities suggests the existence of rather simple doorway-state configurations in the nuclear continuum with a small spreading width.

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Isospin-Conservation Violation in Two-Nucleon Transfer Reactions

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The analog $(p, {}^{3}\text{He})$ and (p, t) reactions on ${}^{16}\text{O}$ have been studied by a second-order distorted-wave Born-approximation (DWBA) theory. Nucleon transfer is assumed to occur here only one nucleon at a time. The contribution of the isospin-1 intermediate-state projectile was made small at forward angles by increasing the imaginary part of the corresponding optical potential. This increase can be associated with the preferential decay of an isospin-1 projectile. Convergence of the DWBA is studied by comparing one- and two-step calculations,

Isospin conservation predicts the differential cross sections for $(p, {}^{3}\text{He})$ and (p, t) reactions to have the ratio $k_{3 \text{ He}}/2k_{t}$ for protons on an isospin-zero target. Ingalls¹ has studied this prediction for 27-MeV protons on an oxygen target. He has

measured the ${}^{16}O(p, {}^{3}He){}^{14}N*(2.31 \text{ MeV}, T=1)$ and the ${}^{16}O(p, t){}^{14}O(0.00 \text{ MeV}, T=1)$ reaction cross sections for scattering angles less than 40° . For small scattering angles, less than 22.5° , the isospin-conservation prediction is satisfied, but it is violated for larger scattering angles. Distorted-wave Born-approximation (DWBA) calculations² using a one-step, direct, two-nucleon pair pickup mechanism have been able to reproduce the general trend of the experimental crosssection ratio.

I utilize a two-step reaction model where nucleons are transferred one at a time from the target, but only after interacting via a two-body potential with part or all of the projectile. Such a model requires a two-step reaction mechanism for (p, t) or $(p, {}^{3}\text{He})$ reactions. Such a two-step reaction mechanism has been $previously^{3-7}$ in (p, t)-reaction calculations and also for other reactions. Here it is applied to seek insight in explaining the isospin nonconservation in two-nucleon transfer reactions. With this model one must include and describe intermediate states of the system—states which do not occur in a direct one-step reaction model. I use a model of the target that is as simplified as possible. The target is treated as a spin-zero residual nucleus which is infinitely massive plus the two nucleons bound in simple shell-model orbits which will be picked up to form the final projectile. The twostep amplitude which will be calculated approximately has been given before.⁸ We have, for a (t, p) reaction (correct to second order in V),

$$T = \langle \chi_{\flat} \psi_{\flat} | V_{\flat} G_{d} \psi_{d} | \chi_{t} \psi_{t} \rangle,$$

where $\chi_{t(p)}^{\pm}$ is the incoming triton (outgoing proton) relative-motion distorted wave. $\psi_{p(t)}$ are the wave functions for the proton (triton) including spin. Temporarily treating the nucleons as distinguishable, and labeling them m, n, and p, after making the usual distorted-wave approximations of canceling the optical potentials with the corresponding nucleon-nucleus potential, we have

$$V_{p} = V_{pn} + V_{pm},$$
$$V_{d} = V_{pn} + V_{nm}.$$

 G_d^+ stands for the Green's function (propagator) of the system where *n* and *p* propagate as a deuteronlike system on the intermediate nucleus. This nucleus is the nucleon *m* bound to the residual nucleus. The initial and final states are now all to be antisymmetrized separately for protons and for neutrons. We write

$$G_{d}^{+} = \iint d^{3}r \, d^{3}r' \sum_{i} |\psi_{i}\rangle g_{i}(\mathbf{\bar{r}}, \mathbf{\bar{r}}') \langle \Psi_{i}|,$$

where Ψ_i represents the wave function of the two clusters; one cluster is the nucleon bound to the

nucleus, the other the relative motion of the two nucleons in the intermediate-state projectile. $g_i(\mathbf{r}, \mathbf{r}')$ describes the propagation of the intermediate-projectile center of mass with respect to the (residual plus one nucleon) intermediatestate nucleus. The index i labels the various states of the intermediate nucleus and of the projectile (including spin). These states are restricted as follows: The nucleon in the intermediate nucleus is assumed to be in the same state as in the target nucleus. The states describing the projectile internal motion are all assumed to be bound (l=0) s waves. This follows three-nucleon calculations⁹ using separable potentials which describe neutron-proton states as completely dominated by the bound deuteron and a low-lying $({}^{1}S_{0})$ resonance which is also treated as a bound state. Consistent with these calculations, internal states of intermediate-projectile orbital angular momentum with l greater than zero are ignored. The triplet and singlet deuteron and di-proton binding energies are assumed here to be equal. The energy of the center of mass of the intermediate-state projectile is then fixed by the relation

 $\hbar^2 k^2 / 2M = E_p - B_n + B_d.$

 B_n represents the binding energy for the initially bound nucleon and B_d is the binding energy of the intermediate-state projectile. A complex (absorptive) optical-model potential is assumed to describe the propagation of the intermediate projectile such that, after expanding in partial waves,

$$g_{l}(r, r') = -(2mk/\hbar^{2})U_{l}^{\circ}(r_{s})U_{l}^{i}(r_{s}).$$

 $U_l^{\text{r,o}}(r)$ is the solution to the radial Schrödinger equation with appropriate regular-at-origin or outgoing boundary conditions.¹⁰ The di-proton intermediate-state projectile possible in the (*p*, ³He) reaction is also assumed to be bound with the binding energy of the deuteron for purposes of determining *k*. These binding energies can be varied but calculations (not shown) resulted in no noticeable dependence on their variation, so they were set equal.

The optical-model potentials used to describe the propagation of the center of mass of the intermediate-state projectile can differ. This difference is due in part to the energy dependence of the optical-model potentials, and if nonequivalent nucleons are present in the assumed model of the target, the energy of the intermediatestate projectile can differ by an amount reflecting the q-value difference of the nonequivalent

	V (MeV)	r₀ ≡r₅₀ (fm)	$a = a_{so}$ (fm)	<i>W</i> (MeV)	$W^1 = 4W_D$ (MeV)	γ ₀ ¹ (fm)	a ¹ (fm)	V _{so} (MeV)	γ _{0C} (fm)	D _i
þ	53.2	1.125	0.57	0	32.3	1.125	0.50	5.5	1.4	
t (³ не)	177	1.138	0.724	12	0	1.602	0.769	5	1.4	
Bound states		1.26	0.6					$\lambda = 25$	1.26	
d(T=0)	107.2	1.125	0.57	0	32.3	1.125	0.5	5.5	1.4	1
d(T=1)	107.2	1.125	0.57	10	32.3	1.125	0.5	5.5	1.4	0.77
$^{2}\mathrm{He}(T=1)$	107.2	1.125	0.57	10	32.3	1.125	0.5	5.5	1.4	0.889

orbits. Also, the projectiles may not be equally stable; presumably (because of Coulomb forces) the di-proton would break up more readily than the singlet deuteron, which would be less stable than the triplet deuteron. This instability difference is approximately included by increasing the imaginary volume absorptive part of the appropriate projectile optical potential. For calculations reported here, the singlet-deuteron (φ) and di-proton optical potentials have been kept equal, except for the Coulomb potential. This two-step model has isospin-conservation-violation contributions from Coulomb effects, q-value differences, varying stability of intermediate-state projectiles, and the intermediate states permitted. For a (p, t) reaction or $(n, {}^{3}\text{He})$ reaction, the intermediate projectile must be a singlet or triplet deuteron in this model. For a $(p, {}^{3}\text{He})$ reaction [or a (n, t) reaction], a di-proton [dineutron] intermediate projectile is possible as well. δ function zero-ranged potentials are used for the perturbative nucleon-nucleon potentials. For the two-step reactions I write the potential overlap as $\langle \varphi_i | V | \varphi_i \rangle \langle \varphi_i | V | \varphi_p \rangle = D_i$ times two δ functions, in analogy to DWUCK¹¹ which takes $\langle \varphi_{a} | V | \varphi_{p} \rangle$ $\approx D_0 \delta(\mathbf{\tilde{r}}_{np})$. The subscript *i* denotes the intermediate-state-projectile internal wave function. The optical-potential parameters which best fit the experiment in the one-step DWBA analysis² are used here for the p, t, and ³He projectiles. The two-nucleon potential parameters used are listed in Table I. When Coulomb forces are neglected as well as contributions from the di-proton intermediate state for the $(p, {}^{3}\text{He})$ reaction, the shapes of the (p, t) and $(p, {}^{3}\text{He})$ reaction cross sections are the same. Figure 1 shows the cross sections that would result if only a single mechanism were contributing to the reaction. The two-step contributions depend on the optical potentials used in a generally standard way. For instance, increasing the imaginary potential lowers the cross section. Note that the one-step amplitudes

calculated are smaller than the two-step amplitudes. The differences between the one-step and two-step cross sections indicate that the DWBA series has important contributions after the first nonvanishing terms for the optical potentials used. See also below.

These amplitudes have used a standard (taken



FIG. 1. Cross sections due to various mechanisms acting alone. Curve A, $(p, d, {}^{3}\text{He})$, cross section with d intermediate state; curve B, (p, d, t); curve C, $(p, {}^{2}\text{He}, {}^{3}\text{He})$; curve D, (p, t), one-step mechanism; curve E, $(p, \varphi, {}^{3}\text{He})$, two-step; curve F, (p, φ, t) , twostep; curve G, $(p, {}^{3}\text{He})$, one-step. Normalizations match DWUCK. Units are mb/sr. Proton energy is 27 MeV, on an ¹⁶O target.

from DWUCK) potential overlap D_i that was independent of the intermediate state. Because of the differing overlap of the deuteron internal wave function or of the T=1 internal wave function with a triton wave function, one expects the D_i to be larger for a deuteron intermediate state than for the T=1 projectile intermediate states. A χ^2 analysis was performed for the (p, ³He) reaction allowing the D_i to be independently varied. The results are shown in Table I. Minimizing χ^2 with $V_d = V_{\varphi} = V_{^2\text{He}}$ resulted in D_i of 1, 0.54, and 2.5 for d, φ , and ²He, respectively, contrary to this expectation as the di-proton amplitude dominated the final amplitude. The absorption of the T=1 two-nucleon channel was then increased and the analysis repeated. χ^2 found a slightly lower minimum than before and the D_i producing this minimum are in the ratios expected (see Table I). The $(p, {}^{3}\text{He})$ cross section calculated for the minimum χ^2 is shown in Fig. 2. Keeping these D_i unchanged, I calculated the (p, t) cross section also shown in Fig. 2.

The magnitudes of the cross sections found in this manner very nearly equal the one-step calculated cross sections for scattering angles less than 15° . We see that the two-step amplitudes are large compared to the one-step amplitudes. but cancelations occur at forward angles such that the first-order DWBA result approximates that of the first- plus second-order DWBA. Independently varying the D_i for the (p, t) reaction slightly reduces χ^2 to 7.7 from 8.0. The shape of the cross section differs in these two cases by less than the width of the plotted line. Such a result was not obtained when $V_d = V_{\varphi}$ (then $D_{\varphi} > D_d$ and $\chi^2 = 60$). The ratio of cross sections $\sigma(p, {}^{3}\text{He})/$ $\sigma(p, t)$ is also shown in Fig. 2. The ratio calculated is in somewhat poorer agreement with experiment than when using the one-step DWBA with the same optical potentials. Note, however, that the two-step DWBA requires and is sensitive to two-nucleon optical potentials in addition to those required in the one-step DWBA. The cross sections past 40° differ in the two calculations. the one-step calculation predicting a large peak at about 80° that is absent in the two-step calculation.

Here it has been shown that the two-step model of two-nucleon transfer reactions is able to explain the isospin conservation seen at forward angles in the cross-section ratio of (p, t) and $(p, {}^{3}\text{He})$ reactions by separate treatment of the intermediate-state projectiles. The fragility of the T = 1 intermediate-state projectile is incor-



FIG. 2. ¹⁶O(p, t) and ¹⁶O(p, ³He) cross sections for 27-MeV protons in arbitrary units. (p, t), experimental data; (p, ³He), experimental data; experimental ratio: dashed curve, calculated one-step ratio by Ref. 2; curve R, calculated two-step ratio.

porated here by increasing the corresponding optical-model imaginary potential and reducing the potential overlap D_i . Such considerations are not possible in a one-step reaction model. The calculations reported here are sensitive to the choice of optical-model parameters. The T=1intermediate-state contribution would spoil the agreement between calculation and experiment except for the fact that it was made somewhat smaller. Thus we can explain isospin conservation at forward angles (less than 20°) in that the T=1 contribution has been made small there. This model predicts isospin nonconservation at larger angles but the calculated ratio of cross sections differs from the experimental ratio. Thus the model, as parametrized here, does not completely explain the experimentally observed isospin nonconservation. This might be remedied by searching on intermediate-state opticalmodel parameters or by treating the intermediate projectile as in part two scattering nucleons.

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rather than as a loosely bound two-body system. The first-order DWBA is seen to nearly agree in magnitude with the first- plus second-order DWBA for forward scattering angles, but the shapes differ for larger angles.

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