Continuum Random-Phase–Approximation Calculations of Higher Multipole Giant Resonances in ¹⁶O and ⁴⁰Ca

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The positions and widths of E0, E1, E2, E3, and E4 giant resonances are calculated with a density-dependent force.

Within the last two years, giant resonances of a higher multipolarity than the well-known dipole resonance have been detected experimentally by inelastic electron and proton scattering.¹⁻³ These states and their widths have been calculated within the random-phase approximation (RPA) by taking into account the coupling of the particles to the continuum. This is an extension of recent calculations⁴ which discretized the continuum and therefore could not get the widths.

Our techniques differ from previous approaches⁵ as they use the energy-separation method.⁶ The continuum shell-model equations⁷ have been generalized to a RPA wave function,

$$\Psi(E) = \sum_{\alpha} a_{E}(\alpha) |\alpha\rangle + \sum_{c} \int_{0}^{\infty} d\epsilon \, a_{E}(\epsilon, c) |\epsilon, c\rangle,$$

where $|\alpha\rangle$ denotes a bound particle-hole configuration and $|\epsilon, c\rangle$ a particle-hole configuration with the particle in the continuum, both acting on a RPA-correlated ground state. Thus the ground state contains also correlations with particles in the continuum. The inclusion of the RPA corre-

TABLE I. The Woods-Saxon parameters of ¹⁶O and ⁴⁰Ca. We used $r_C \approx 1.25$ fm for the reduced Coulomb radius, a = 0.53 fm for the diffuseness, and the oscillator constants b = 1.639 fm in ¹⁶O and b = 1.8 fm in ⁴⁰Ca.

Nucleus	τ	V (MeV)	<i>r</i> (fm)	V _{SL} (MeV)
¹⁶ O	Þ	52.8789	3.0828	5.5327
	n	53.1384	3.0828	5.1859
⁴⁰ Ca	Þ	55.8056	4.1	6.6748
	n	55.4979	4.1	7.1539

lations shifts the maximum of the giant dipole (giant quadrupole) peak of 16 O to lower energies by 0.8 MeV (2.8 MeV) and reduces the width by



FIG. 1. The giant dipole $(\gamma, p) + (\gamma, n)$ cross section in ¹⁶O (upper part) and ⁴⁰Ca (lower part). The experiment (Ref. 10) in ¹⁶O is drawn in arbitrary units. The experiment in ⁴⁰Ca is taken from Ref. 11.

TABLE II. Comparison of the isovector (E1) and isovector plus isoscalar (E2, E3, E4) sum rules (Ref. 12) with the present calculation, integrated up to 45 MeV for E1, up to 52 MeV for E2, and up to 80 MeV for E3 and E4.

	σ ₀ (1 ⁻) (mb MeV)	σ ₋₂ (2 ⁺) (μb/MeV)	σ ₋₄ (3 ⁻) (nb/MeV ³)	$\sigma_{-6}(4^+)$ (pb/MeV ⁵)
¹⁶ O calc	253.8	13.65	0.059	446
sum rule	240.0	15.25	0.190	611
⁴⁰ Ca calc	620.5	54.04	0.350	3348
sum rule	600.0	62.39	1.240	6420

about 40% (50%) compared with a Tamm-Dancoffapproximation calculation. The sum rules are also affected. The most outstanding example is the giant dipole resonance in ¹⁶O. Here we get a reduction of 38% of the Tamm-Dancoff-approximation result. More details will be given in a forthcoming publication. The width of a resonance can be obtained directly from the crosssection plot. In the present stage of the calculations (RPA), this width corresponds to the "decay width" caused by the escape of a nucleon. The inclusion of more complicated configurations yields an additional shift and a fine structure which will further spread out the resonance.

We used the density-dependent δ -function force proposed by Migdal with parameters⁸ fitted in the ²⁰⁸Pb region to electromagnetic properties. The interpolation radius and the total strength were readjusted to reproduce the low-lying collective 3⁻ state and its B(E3) value. The singleparticle energies of the bound configurations were taken from a Hartree-Fock calculation.⁹ The wave functions were obtained from a Woods-Saxon potential, the parameters of which are given in Table I.

The giant-dipole-resonance calculations are presented in Fig. 1. The agreement with previous calculations⁵ provides an excellent check of the method. The total cross sections are overestimated, as expected in a one-particle, onehole approximation. The integrated cross sections, however, agree with the dipole sum rule without exchange correction as is shown in Table II.

The giant-quadrupole $(\gamma, p) + (\gamma, n)$ cross-section calculations for ¹⁶O and ⁴⁰Ca are presented in Fig. 2. The giant quadrupole resonance in ¹⁶O is found near 22 MeV. This is in agreement with the analysis of Geramb, Sprickmann, and Strobel² and with the experimental results of Hanna *et al.*³ who found quadrupole strength in the 20–28-MeV region with the maximum at 24 MeV. The coupling to more complicated configurations will yield a larger width because of the fine structure and will give additional strength below 20 MeV. This might explain a recent (α, γ_0) experiment¹³ which excites predominantly more complicated 2^+ states. The calculated peak at 22 MeV is mainly isoscalar, whereas the isovector strength is spread out above 30 MeV. In ⁴⁰Ca, an isoscalar peak is obtained at 17 MeV. Experimentally,¹⁴ it is found at about 18.25 MeV. The high-lying structures seen in ¹⁶O and ⁴⁰Ca are due to bound particle-hole configurations smeared out by the coupling to the continuum.

The E0 resonances ("breathing modes"), which



FIG. 2. The giant quadrupole $(\gamma, p) + (\gamma, n)$ cross sections of ¹⁶O and ⁴⁰Ca in the lower part of the figure, and the 0⁺ resonance characterized by the transition strength r^2 to the ground state in the upper part.



FIG. 3. The $(\gamma, p) + (\gamma, n)$ cross sections of the 3⁻ and 4⁺ resonances in ¹⁶O and ⁴⁰Ca.

are also displayed in Fig. 2, have not yet been observed experimentally. The E3 and E4 resonances (Fig. 3) appear as very broad structures above 30 MeV. They can be qualitatively compared with Ref. 2. In both nuclei, all resonances have the same gross structure and show a mass dependence of the peaks which follows qualitatively an $A^{-1/3}$ law. The contribution of the resonances in the continuum to the photonuclear sum rules¹² is shown in Table II. The main restriction of the present approach is the description of the residual nucleus in terms of single-hole shell-model states. Taking into account correlations within the residual nucleus will yield a more quantitative description of the spreading width.

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