Hot-Ion Distribution Function in the Oak Ridge Tokamak*

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Observed Doppler shifts of neutral-hydrogen H_α radiation in the poloidal and toroidal directions in the Oak Ridge tokamak experiment are shown to be consistent with neoclassical theory. The inferred shift and distortion of the proton distribution function in the banana-plateau regime is caused by the radial pressure gradient and by the ambipolar electric field, whose steady-state value is determined by nonambipolar diffusion due to charge exchange.

The distribution of hot ions in the Oak Ridge tokamak (ORMAK) does not appear to be Maxwellian. Observed Doppler shifts in the H_α (6563 Å) spectral line originating from neutrals within the discharge reflect a shift and a distortion of the proton distribution function since the neutrals are produced by charge-exchange reactions with energetic protons. The ORMAK plasma parameters are such that $\nu_{*p} \gtrsim 1$, $\omega_{\rm tr} \gg \nu_{pp} \gg \nu_{\rm cx}$, where ν_{*p} is the proton collisionality parameter,[1] $\omega_{\rm tr}$ is the toroidal transit frequency, ν_{pp} is the proton-proton collision frequency, and $\nu_{\rm cx} = n_0\langle\sigma_{\rm cx}v\rangle$ is the charge-exchange frequency. In this Letter, (1) we outline the theory of toroidal plasma flow and ambipolar potential in the presence of charge-exchange friction; (2) we give the expression for the parallel velocity distribution of the hot protons in the banana and plateau regimes; and (3) we present experimental data on the Doppler shifts observed in ORMAK which are consistent with this theory, with respect to magnitude, direction, and scaling with temperature.

In the following, we develop a theoretical expression for the plasma drifts expected in tokamak plasmas in order to compare with the experimental observations on ORMAK. Since plasma drifts are in part driven by the radial electric field, we first consider the mechanisms for the production and relaxation of this field.

In the short-mean-free-path (Pfirsch-Schlüter) regime, the plasma exhibits a transient poloidal flow velocity with nonzero flux surface average, driven by the pressure gradient and radial electric field. The transient buildup originates from nonambipolar processes early in the discharge[2,3] and has been calculated by Stringer.[4] In the long-mean-free-path regime, Stix[5] has related the decay of poloidal rotation to magnetic pumping on a time scale slower than ν_{pp}^{-1}. The theory also predicts a toroidal flow velocity $u_\parallel$ in this regime. Its relaxation due to ion viscosity has been calculated by Hazeltine[6] and Tsang.[7] However, the dominant relaxation process determining the ambipolar electric field and $u_\parallel$ is charge exchange.[8] In the Pfirsch-Schlüter regime it has been shown[6,7] that $u_\parallel$ vanishes. By contrast, in the long-mean-free-path regime, characteristic of hot tokamak plasmas, $u_\parallel$ remains at a finite value[8] for $t \gg \nu_{\rm cx}^{-1}$. Since the ORMAK plasma is in this long-mean-free-path (banana-plateau) regime we will obtain an expression for the nonzero diamagnetic drift velocity in the toroidal direction to compare with the experimental observations.

To zero order in $(r/R)^{1/2}$, its parallel component is[1,9]

$$u_{\parallel 0} = u_E + u_g, \tag{1}$$

with the neoclassical form of the electric drift

$$u_E = E_r/B_\theta, \tag{2}$$

and the gradient-driven drift

$$u_g = -\frac{T_i}{e_iB_\theta}\left[\frac{n_i'}{n_i} + \left(y - \frac{3}{2}\right)\frac{T_i'}{T_i}\right]; \tag{3}$$

E_r is the radial electric field, B_θ is the poloidal magnetic field, i stands for ion (proton), n' and T' denote radial gradients of density and temperature, and the parameter y has been taken from Hazeltine and Hinton[10]:

$$y = 1.31(1 + 1.65\nu_{*i}^{1/2})/(1 + 0.86\nu_{*i}^{1/2}), \tag{4}$$

with ν_{*i} the ion collisionality parameter defined in Ref. 1. Higher-order terms to $u_\parallel$ are driven solely by the ion-temperature gradient and are $O((r/R)^{1/2})$ in the banana regime and $O(r/R)$ in the plateau regime.

Whereas u_g and B_θ evolve smoothly on the diffusion time scale, the time-asymptotic value of E_r, and thereby u_E, depends on the nature of the fastest nonambipolar diffusion mechanism. Anomalous diffusion due to low-frequency turbulence can be shown to be ambipolar.[11] In the banana regime, lack of toroidal symmetry due to magnetic field ripples has been considered by Rosenbluth[12] and Tsang,[7] and nonambipolar diffusion due to charge exchange in Ref. 8. For the highly symmetric ORMAK plasma, with central neutral-hydrogen densities[13] of 2×10^8 to $2\times10^9/\mathrm{cm}^3$ and surface densities of $10^{10}/\mathrm{cm}^2$, charge exchange appears to be the dominant mechanism. The loss of toroidal momentum of the protons due to charge-exchange reactions, $R_{\parallel \mathrm{cx}}$, corresponds to a parallel friction force producing a radial drift of the ions. Since the plasma electrons do not experience this frictional force, this radial ion drift is nonambipolar and changes E_r. We have calculated $R_{\parallel \mathrm{cx}}$ to order $(r/R)^{1/2}$ and find that the charge flow due to charge-exchange friction relaxes E_r on a time scale of ν_{cx}^{-1} to an asymptotic value such that for $t \gg \nu_{\mathrm{cx}}^{-1}$,

$$R_{\parallel \mathrm{cx}} = -m_i n_i \nu_1 (E_r/B_\theta + u_g) + 1.46 m_i n_i \nu_2 (r/R)^{1/2} T_i'/e_i B_\theta = 0. \tag{5}$$

Here, ν_1 and ν_2 are constants proportional to ν_{cx}. Equation (5) implies an $(r/R)^{1/2}$ expansion of E_r. The results of the expansion are, for the zero-order term,

$$E_r^0/B_\theta + u_g = 0, \tag{6a}$$

and for the first-order term in $(r/R)^{1/2}$,

$$E_r^1/B_\theta = (\nu_2/\nu_1) 1.46 (r/R)^{1/2} T_i'/e_i B_\theta. \tag{6b}$$

In the limit of small ratio of neutral temperature T_n to proton temperature T_i, one finds[8]

$$\nu_2/\nu_1 = 1.67 - 0.51 T_n/T_p + O(T_n^2/T_p^2). \tag{7}$$

As will become apparent, in the plateau regime, the expansion of $R_{\parallel \mathrm{cx}}$ and E_r is in powers of r/R rather than $(r/R)^{1/2}$. To zeroth order in that expansion, E_r^0 is again determined from Eq. (6a). The $O(r/R)$ term E_r^1 has not been worked out so far; however, we will not require this expression.

The shape of the H_α line of charge-exchange-produced neutral atoms observed tangentially to the toroidal magnetic field can be related to the proton distribution function of parallel velocities $F_p(v_\parallel) = \int 2\pi v_\perp \, dv_\perp f(v_\perp, v_\parallel)$. Since our major concern is with the radiation emitted by the superthermal neutrals, we assume that the wings of the H_α spectrum characterize the proton distribution function in a small region near the peak temperature in the observed volume shown in Fig. 1. Delayed cascading of the excited neutrals sets an upper limit of about 10 cm to the diameter of this volume. Because of the finite spectrometer slit width, the low-velocity portion of F_p is unresolvable and one observes essentially the thermal and superthermal parallel velocities, corresponding to circulating protons.

Rosenbluth and co-workers[1,9] have given an expression for the ion distribution function in the banana and plateau regimes. In the limit $v \to v_\parallel$ describing well-circulating particles, their result can be written as

$$F_p(v_\parallel) = F_0 + F_1, \tag{8}$$

where F_0 is Maxwellian (even in $v_\parallel$), and the odd piece in $v_\parallel$ is given by

$$\frac{F_1}{F_0} = \frac{v_\parallel m_i}{T_i}\left[\left(\frac{E_r}{B_\theta} + u_g\right) - \frac{T_i'}{e_i B_\theta}\left(\frac{v_\parallel^2}{\alpha_i^2} - y\right)(\sqrt{\epsilon} \text{ or } \epsilon/2)\right]. \tag{9}$$

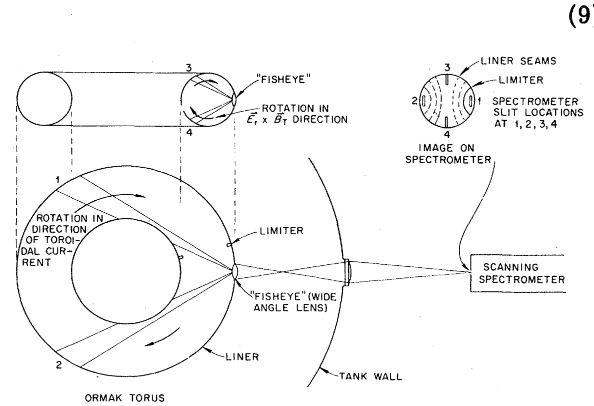

FIG. 1. Schema for observing plasma rotation. A wide-angle lens in the liner is located about 20° from the limiter. Another lens at the tank wall focuses a real image on the spectrometer slit. The spectrometer is adjusted to sweep over H_α (6563 Å) every 8 msec, giving six to eight useful H_α line profiles every ORMAK shot. A distorted H_α profile is seen when the entrance slit of the spectrometer is placed at any one of the four positions. The spatial distribution of H_α radiation shows an intensity maximum at $r = 15$–18 cm. The observation angle is about 45° with respect to direction of motion.

Here, $\alpha_i^2 = 2T_i/m_i$, $\epsilon = r/R$; $\sqrt{\epsilon}$ applies in the extreme banana regime, and $\epsilon/2$ applies in the extreme plateau regime. (There is, of course, a continuous transition between the two regimes.) Inserting the charge-exchange-relaxed value for $E_r = E_r^0 + E_r^1$ with E_r^0 and E_r^1 given by Eqs. (6a) and (6b) into Eq. (8), one obtains for the distribution function for well-circulating protons in the banana regime

$$\frac{F_1}{F_0} \cong \frac{m_i}{T_i}\frac{T_i'}{e_iB_\theta}\left(y + 1.46\frac{\nu_2}{\nu_1}\right)(r/R)^{1/2}v_\parallel - \frac{m_i}{T_i}\frac{T_i'}{e_iB_\theta}\alpha_i^{-2}(r/R)^{1/2}v_\parallel^3, \tag{10}$$

with y given by Eq. (4) and ν_2/ν_1 by Eq. (7). This form of the distribution function should describe the superthermal ions in the experimental plasma. In the plateau regime, where the expression for E_r^1 has not yet been determined, one has

$$\frac{F_1}{F_0} \cong \frac{m_i}{T_i}\left(\frac{E_r^1}{B_\theta} + \frac{1}{2}\frac{r}{R}\frac{T_i'}{e_iB_\theta}y\right)v_\parallel - \frac{m_i}{2T_i}\frac{T_i'}{e_iB_\theta}\alpha_i^{-2}(r/R)v_\parallel^3. \tag{11}$$

Since E_r^1 is $O(r/R)$, F_1/F_0 in the plateau regime is down from the banana-regime value by a factor $O((r/R)^{1/2})$.

We conclude that the perturbed proton distribution F_1 for the ions of interest in the experimental measurement consists of a shifted Maxwellian ($\sim v_\parallel F_0$), and a distortion ($\sim v_\parallel^3 F_0$) influenced primarily by the superthermal particles. The shift depends on the collisionality parameter ν_{*i} through y, and both the shift and the distortion depend on the collisionality regime as indicated by the factor $(r/R)^{1/2}$ or r/R in Eqs. (10) and (11).

Experimental observations.—Observing in the poloidal direction (Fig. 1) we note a shift in the centroid of the H_α (6563 Å) profile, implying a poloidal drift velocity. Within a period of 10 msec after breakdown, the shift appears to be proportional to the loss of charge in escaping runaway electrons. The data are in Fig. 2. This charge loss can be inferred from the intensity of hard x-ray bursts.[2,3] The poloidal rotation is seen to decay on a 10-msec time scale consistent with the charge-exchange friction relaxing the plasma charge.

For observation in the toroidal direction (Fig. 1) at later times, the spectral line has a central peak due to radiation from cold charge-exchange neutrals on the plasma perimeter with a half-width approximately equal to the slit width ($\Delta\lambda_{\text{cold}} \sim 1.5$ Å). In addition, the contribution of radiation from hot charge-exchange neutrals in the plasma interior extends the line profile to $\Delta\lambda_{\max} \sim 8$ Å, approximately to 2–3 times thermal velocity, depending on the hot-ion temperature. The center of the cold peak is assumed to indicate the location $v_\parallel = 0$ of the hot-proton distribution. The total spectral line is then decomposed into an even piece corresponding to $F_0(v_\parallel)$ yielding the hot-proton temperature, and an odd piece corresponding to $F_1(v_\parallel)$. Experimentally the λ-centroid displacement due to a distorted distribution function is indistinguishable from that due to a displaced Gaussian distribution function; hence, the experiment yields an equivalent drift velocity which can be due to either a shift or a distortion of the distribution function. The data are in Fig. 3.

In order to facilitate comparison with theory

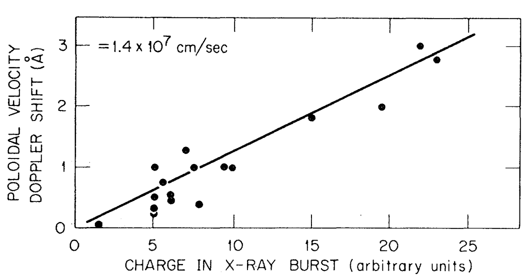

FIG. 2. Poloidal plasma flow. The straight line suggests a possible relation between the poloidal flow velocity and the plasma charge.

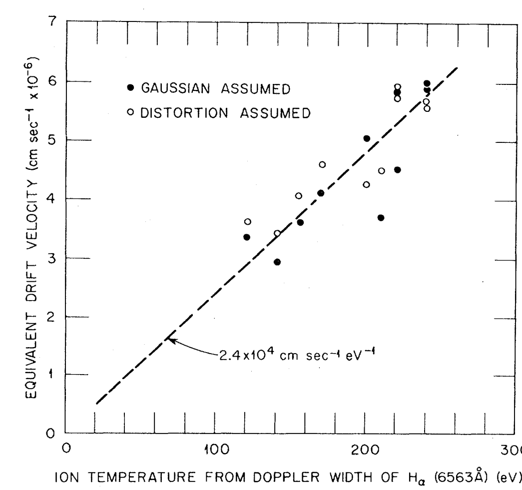

FIG. 3. Toroidal plasma flow. The straight line suggests a possible relation between the toroidal drift velocity of the ion species and its temperature.

we analyze data from quiescent periods in the experiment. These are times between magnetohydrodynamic bursts which are associated with irreproducible runaway electron dumps. (As noted above, these dumps change the plasma charge and consequently $u_{\parallel}$.) We only analyze discharges with a low-resistance anomaly permitting the protons to approach the banana regime.[14] The subthermal to thermal protons are always in the plateau regime, and are described by the shift piece of F_1, Eq. (11), which is proportional to r/R. The superthermal protons are in the banana regime and are described by the distortion piece of F_1, Eq. (10), which is proportional to $(r/R)^{1/2}$. The direction and magnitude of the experimental drift velocity agrees with that predicted from the distortion term in Eq. (10), and it changes sign when the plasma current is reversed, as it should.

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[1]F. L. Hinton and M. N. Rosenbluth, Phys. Fluids 16, 836 (1973).

[2]C. F. Barnett *et al.*, in *Proceedings of the Sixth European Conference on Controlled Fusion and Plasma Physics, Moscow, 1973* (U.S.S.R. Academy of Science, Moscow, 1973), Vol. I, p. 67.

[3]P. Chrisman, J. F. Clarke, and J. A. Rome, ORNL Report No. ORNL-TM-4501 (unpublished).

[4]T. E. Stringer, J. Plasma Phys. 10, 433 (1973).

[5]T. H. Stix, Phys. Fluids 16, 1260 (1973).

[6]R. D. Hazeltine, Phys. Fluids 17, 961 (1974).

[7]K. T. Tsang, Ph.D. thesis, Princeton University, 1974 (unpublished).

[8]D. J. Sigmar and J. F. Clarke, ORNL Report No. ORNL TM 4606 (to be published).

[9]M. N. Rosenbluth, R. D. Hazeltine, and F. L. Hinton, Phys. Fluids 15, 116 (1972). There should be a minus in front of the step function in Eq. (180).

[10]R. D. Hazeltine and F. L. Hinton, Phys. Fluids 16, 1883 (1973).

[11]R. J. Bickerton, Nucl. Fusion 13, 290 (1973).

[12]M. N. Rosenbluth, Bull. Amer. Phys. Soc. 18, 1337 (1973).

[13]L. A. Berry, J. F. Clarke, and J. T. Hogan, Phys. Rev. Lett. 32, 362 (1974).

[14]E. C. Crume, ORNL Report No. ORMAK-TM-148 (to be published).

Dielectric Response of a Superionic Conductor

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A microscopic theory of ionic transport in superionic conductors is presented. Based on a liquidlike description of the ionic carriers, it describes the frequency and temperature dependences of the conductivity. A sum rule is obtained and the theory predicts the existence of a conductivity peak which scales with the inverse of the ionic carrier mass. The results are in good agreement with recent data on Na and Ag β-aluminas.

Superionic conductors, exemplified by systems like AgI, Na-β-Al_2O_3, and PbF_2, display a variety of properties that make them potentially useful for energy storage schemes.[1] With diffusion coefficients close to those of liquids, and ionic carrier concentrations of the order of 10^{23} ions/cm^3, they also pose interesting questions regarding the nature of the extremely high conductivities they display, which in some cases are connected with very well defined phase transitions.[2]

Recently there have been some theoretical attempts at incorporating detailed correlations into the usual random-walk theories in order to deal with diffusion in very dense systems.[3] Although the assumptions underlying these models seem reasonable and their limiting behavior in the absence of interactions yields the usual Arrhenius result, it is difficult to check their validity experimentally by using static probes. A more suitable approach would be to measure their dynamic properties, as superionic conductors are