higher-spin states in the high-Ω $i_{13/2}$ bands provide an essential constraint on the wave functions obtained in the band-fitting procedure.

Finally, an important question to be answered is just how sensitive the wave functions obtained from the Coriolos matrix diagonalizations of various groups are to the details of the fitting parameters. This is very important for the reliability of the curves in Fig. 1, since only the wave functions are used in our calculations. Fortunately, it is our experience and apparently that of other groups as well[10] that the wave functions are not in fact very sensitive to the quality of the fit to the energy data. In the case of ^{187}Os, for example, we performed numerous fits at various times as the analysis of our data progressed. Though the eigenvectors for the higher-lying (unseen) bands showed considerable variation in these fits, those of the two bands for which experimental data were available ($\frac{11}{2}^+[615]$ and $\frac{9}{2}^+[624]$) varied but little with relatively large changes in the input parameters and the quality of the energy fits.

In summary, a simple, quantitative criterion for the tendency of high-j particles to decouple from the nuclear core rotation has been presented. If decoupling of $i_{13/2}$ neutrons from the core is indeed the explanation for "backbending" in the ground rotational band of neutron-deficient even-even deformed rare-earth nuclei where the effect was first reported, then that explanation can apply equally well to the W and Os isotopes which have also been found to exhibit strong backbending behavior.[5] While the possible role of $h_{9/2}$ protons in this region should not be ignored,[4] it appears that an understanding of the ground-band behavior in ^{180}W and $^{182-186}$Os does not require the involvement of protons. The case of ^{170}Yb is difficult to explain in this picture, however, and may offer the first instance of a departure from the straightforward Stephens-Simon model.

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Threshold Photoproduction of Pions on ^{6}Li

F. Cannata,* C. W. Lucas, Jr.,† and C. W. Werntz†
The Catholic University of America, Washington, D.C. 20017
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We examine the dependence of the ^{6}Li(γ,π^+)^{6}He cross section on the pion optical potential and on the nuclear size parameters within the context of the distorted-wave impulse approximation. Other factors affecting the cross section are also considered and it is shown that the theoretical prediction remains about 60% higher than the observed cross section. Comparison to radiative pion capture in ^{6}Li is made.

In a recent experiment Deutsch *et al.*[1] have measured near threshold the ratio of photoproduction of positive pions on ^{6}Li and the proton. Using the absolute experimental proton cross section[2] near threshold, they have obtained a value for the ^{6}Li cross section as a function of photon energy above the threshold. Their measured value is significantly lower than that calculated by Koch and Donnelly.[3]

Near threshold the momentum-dependent terms[4] in the photoproduction amplitude should be unimportant so the distorted-wave impulse approximation (DWIA) which was used in Ref. 3 is effectively

$$\sigma_N=\frac{(1+\omega/M)^2}{2(2J_i+1)}\frac{1}{1+\epsilon_\pi/M_f}\frac{q}{\omega}\tfrac{1}{2}A^2\int d\Omega q\sum_{\substack{M_i,M_f,\\ \lambda}}|\langle J_f M_f|\sum_\alpha \varphi_\pi^{(-)*}(\vec{r}_\alpha)\tau_\alpha^{(-)}\vec{\sigma}_\alpha\cdot\hat{\epsilon}_\lambda \exp(i\vec{k}\cdot\vec{r}_\alpha)|J_i M_i\rangle|^2. \tag{1}$$

In the above ω is the photon energy, ϵ_π and q are the total energy and momentum of the pion, M is the nucleon mass, and M_f is the mass of the final nucleus. The constant A is related to threshold π^+ photoproduction from the proton by

$$\sigma=4\pi(q/\omega)A^2. \tag{2}$$

Sufficiently near threshold the matrix element in Eq. (2) should be accurately given by using only the S-wave distorted pion wave, $\varphi_\pi^{(-)}(\vec{r})\cong\varphi_0(r)$.

The disagreement of the calculation with experiment is intriguing because the same theory when applied to radiative pion capture[5,6] in ^{6}Li tends to overestimate[7] the branching ratio for radiative capture as compared to absorption. In this note we examine four different questions connected with the calculation and come to the conclusion that the discrepancy cannot be removed as long as Eq. (1) is the basis of the calculation.

(1) The optical potential used by Koch and Donnelly[3] for ^{6}Li was of the Krell-Ericson[8] form,

$$V_{\text{opt}}(\vec{r})=-(4\pi/2\mu)\{\tilde{b}_0\rho(\vec{r})+\tilde{b}_1[\rho_p(\vec{r})-\rho_n(\vec{r})]+i\,\mathrm{Im}B_0\rho^2(\vec{r})+\nabla\cdot\alpha(\vec{r})\nabla\}, \tag{3}$$

$$\alpha(\vec{r})=\tilde{c}_0\rho(\vec{r})+\tilde{c}_1[\rho_p(\vec{r})-\rho_n(\vec{r})]+i\,\mathrm{Im}C_0\rho^2(\vec{r}),$$

where the effective scattering lengths and scattering volumes give the correct energy shifts and widths for a large number of pionic atom levels including the 1S state in ^{6}Li. The question can be asked whether this potential can be radically different for $\pi^+ + {}^6$He.

The constants multiplying the term linear in the densities are effective parameters and differ from their free pion-nucleon values. This difference can be semiquantitatively understood by evaluating higher-order contributions to the optical potential using multiple-scattering theory. The $\rho^2(\vec{r})$ terms, which are real, are then taken account of by modifying the linear terms. Goldberger and Watson[9] define the second-order potential by an expectation value in the nuclear ground state:

$$V_{\text{opt}}^{(2)}(\vec{x}',\vec{x})=-\frac{4\pi}{2\epsilon_\pi}\langle\Psi_0|\sum_{\beta\neq\alpha}t_\beta^{\pi\pi}\delta(\vec{x}'-\vec{x}_\beta)\frac{1}{d}(1-\Lambda_0)t_\alpha^{\pi\pi}\delta(\vec{x}-\vec{x}_\alpha)|\Psi_0\rangle. \tag{4}$$

Here $1-\Lambda_0$ eliminates the ground state in the intermediate nuclear states excited by pion scattering.

For low-energy pions the second-order part of the S-wave potential is dominated by long-range Pauli correlations; and the P-wave scattering, by short-range hard-core correlations.[10] With use of the methods of Johnston and Watson[11] the second-order correction to the optical potential in ^{6}He is

$$V_{\text{opt}}^{(2)}(\vec{x}',\vec{x})=-\frac{4\pi}{2\epsilon_\pi}\bigg(\frac{\rho(\vec{x}')\rho(\vec{x})}{|\vec{x}-\vec{x}'|}[-\tfrac{7}{27}b_1^2-G_F(\vec{x}',\vec{x})\tfrac{1}{9}(b_0^2-2b_0b_1+2b_1^2)]$$
$$+\tfrac{4}{3}\pi\delta(\vec{x}'-\vec{x})\nabla\cdot[c_0^2\rho^2(\vec{x})+2c_0c_1\rho(\vec{x})(\rho_p(\vec{x})-\rho_n(\vec{x}))+c_1^2(\rho_p(\vec{x})-\rho_n(\vec{x}))^2]\nabla\bigg). \tag{5}$$

Here $\rho(\vec{x})$ is the nucleon density, b_0 and b_1 are the free pion-nucleon scattering lengths, and $G_F(\vec{x},\vec{x}')$ is a Fermi correlation function. (Note the presence of a term not proportional to the correlation function that is quite large in light nuclei.[11,12]) The correction is repulsive and, for example, accounts for the value of $\tilde{b}_0=-0.03\mu^{-1}$ in Eq. (3) compared to the free $b_0=-0.008\mu^{-1}$.

If we calculate the *difference* between the second-order potentials in ^{6}Li and ^{6}He, we find

$$\Delta V_{\text{opt}}^{(2)}=-\frac{4\pi}{2\epsilon_\pi}\bigg(\frac{\rho(\vec{x}')\rho(\vec{x})}{|\vec{x}'-\vec{x}|}[\tfrac{2}{27}b_1^2+\tfrac{2}{9}b_0b_1G_F(\vec{x},\vec{x}')]$$
$$+\tfrac{4}{3}\pi\delta(\vec{x}-\vec{x}')\nabla\cdot[2c_0c_1\rho(\vec{x})(\rho_p(\vec{x})-\rho_n(\vec{x}))+c_1^2(\rho_p(\vec{x})-\rho_n(\vec{x}))^2]\nabla\bigg); \tag{6}$$

$$b_0=-0.008\mu^{-1},\quad b_1=-0.09\mu^{-1},\quad c_0=+0.21\mu^{-3},\quad c_1=+0.18\mu^{-3}. \tag{7}$$

One sees that the potential should be slightly more attractive for ^{6}He than for ^{6}Li, but that the change in the *correction* is small for the non-gradient terms.

Since $\varphi_0(\vec{x})$ does not vary too much over the nuclear volume, the nonlocal part of Eqs. (5) and (6) can be evaluated approximately by simple integration over $\vec{x}'$. We solved the wave equation for both the effective potential in Eq. (3) and a potential that was a sum of the first order [Eq. (3)] and second order [Eq. (5)] but in which free nucleon-nucleon parameters were used. The difference is too small to plot and a common curve is shown in Fig. 1. We used for the $^6\mathrm{Li}\to{}^6\mathrm{He}$ nuclear transition the same model as in Ref. 5 but with the oscillator parameters different for the 1s and 1p nucleons in ^{6}He: $b_s = 1.63$ F, $b_p = 1.98$ F. This form fits the elastic-electron-scattering form factor[13] in ^{6}Li out to $q^2 = 3$ F^{-1} and still allows one to deduce ρ_n and ρ_p for ^{6}He. The values of B_0 and B_1 were unchanged although the work of Dover[14] indicates that the absorption of pions occurs more readily in a $^3S_1$ np pair than in a $^1S_0$ pair, implying a greater absorption in ^{6}Li.

To illustrate the lack of sensitivity to the optical potential, we have made gross changes in the potential. Setting $\tilde{c}_0 = \tilde{c}_1 = 0$, we find only a very small change, as one would expect for the gradient terms. With $\tilde{b}_1 = 0$, the optical potential in Eq. (3) becomes repulsive and one would expect a drop in the cross section. These cases are shown in Fig. 1. With both $\tilde{b}_0$ and $\tilde{b}_1$ present, the local part of V_{opt} is very small for ^{6}He since $\tilde{b}_0 \cong \frac{1}{3}\tilde{b}_1$. For this reason the pure Coulomb result ($V_{\mathrm{opt}} \equiv 0$) is close to the cross section calculated with the full potential of Eq. (3). We conclude

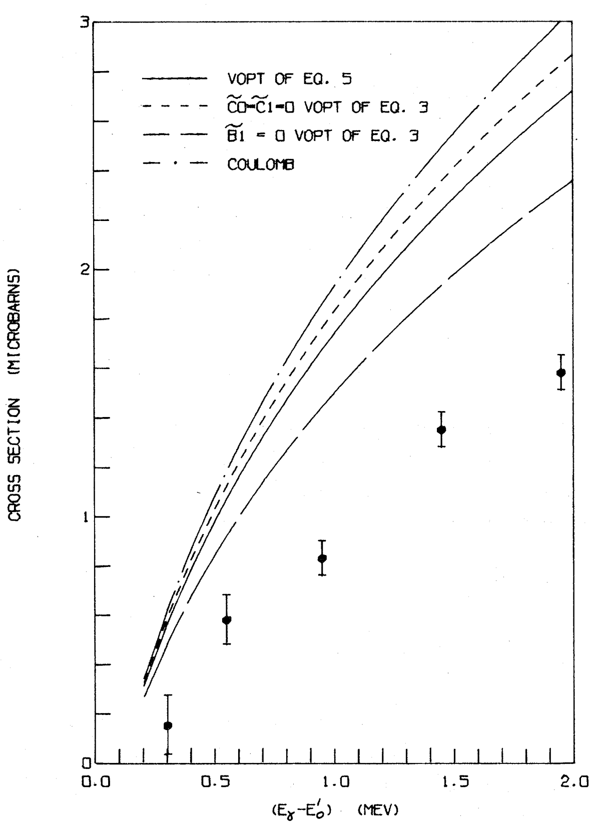

FIG. 1. Theoretical $^6\mathrm{Li}(\gamma,\pi^+)^6\mathrm{He}$ total cross sections as a function of photon energies above threshold, compared to the data of Ref. 1. The solid curve was calculated with a potential whose real parts were a sum of Eqs. (3) and (5) using free pion-nucleon parameters, and whose imaginary part came from Eq. (3). The curves representing grossly modified potentials were calculated using the simpler potential of Eq. (3). The 1p oscillator parameter was taken to be $b_p = 1.98$ F.

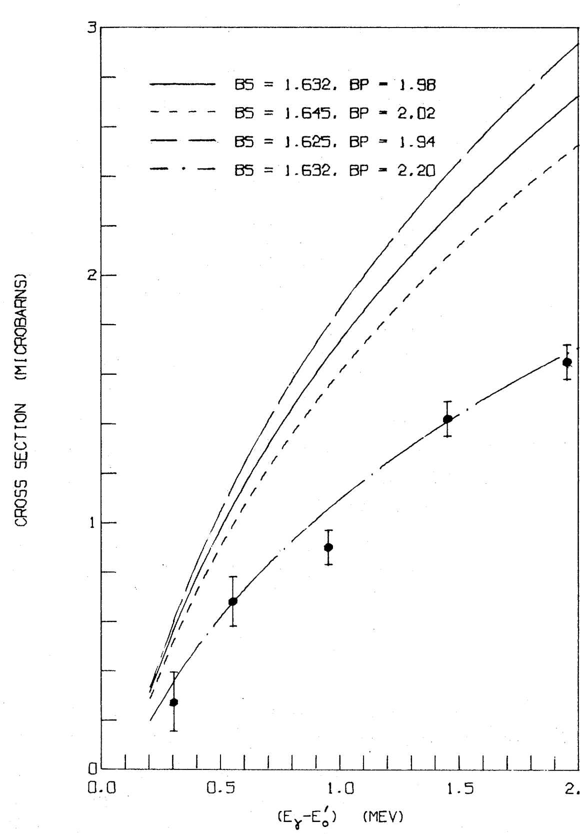

FIG. 2. Theoretical cross sections for indicated values of the oscillator parameters. All curves were calculated with the full potential which can be described as a sum of Eqs. (3) and (5). The upper three curves represent the range of value of the oscillator parameter consistent with elastic electron scattering from ^{6}Li. The lower curve is a fit to the experimental data of Ref. 1.

that the optical potential plays a small role in determining the cross section.

(2) What is the effect of nuclear size on the cross section? The cross section depends strongly on the spatial extent of the outer nucleons in $^6\mathrm{Li} \to {}^6\mathrm{He}$, chiefly through the matrix element $\langle J_f M_f | \sum_\alpha \vec{\sigma}_\alpha \cdot \hat{\epsilon}_\lambda \exp(i\vec{k}\cdot\vec{x}_\alpha) | J_i M_i\rangle$ since $\varphi_0(x)$ is almost constant. In Fig. 2 we show the curves for $b_p = 1.94$, 1.98, and 2.02 F. This range reflects the uncertainty in the $^6\mathrm{Li}(e,e')$ work. Koch and Donnelly[3] chose $b_p = 2.03 \pm 0.02$ F for ^{6}Li from considering $^6\mathrm{Li}(e,e')^6\mathrm{Li}^*(3.56)$, but we prefer to compare states with nearly equal binding energies for the last nucleon. We find that $b_p = 2.20$ F would fit the data but this is completely unreasonable in light of the μ-capture rate and the cross section[15] for $^6\mathrm{Li}(e,e')^6\mathrm{Li}^*(3.56)$.

(3) Does the Fermi motion of the nucleons, which causes the relative momentum between a zero-energy pion and an internal nucleon to be nonzero, have an effect? The relative kinetic energy, $(\mu/M)(p_N^2/2M)$, can be as high as 10 MeV.

From Berends's amplitudes[16] one finds that, retaining only S waves in the pion-nucleon rescattering corrections, A decreases by ~10% from its threshold value to its value at $T_\pi = 10$ MeV. Again the effect is small.

(4) The second-order potential in Eq. (4) arises from excited intermediate nuclear states and includes pion charge exchange. For consistency, one must allow for photoproduction of either a π^+ or π^0 pion with the intermediate nucleus in an excited state. This is equivalent to using the "impinging pion wave" in Eq. (1) instead of the average wave.[17] The matrix element in Eq. (1) is to be corrected by[18]

$$\Delta M^{(2)} = \langle f | \sum_{\beta \neq \alpha} \varphi_\pi^{(-)*}(\vec{r}_\beta) t_\beta^{\pi\pi} d^{-1}(1-\Lambda_0) t_\alpha^{\pi\gamma} \exp(i\vec{k}\cdot\vec{r}_\alpha) | i\rangle. \qquad (8)$$

We estimate a +5% correction to the matrix element which roughly cancels the reduction in A due to Fermi motion.

In Fig. 2 we see that the cross section calculated with an unrealistically large value of the $1p$ oscillator parameter, $b_p = 2.20$ F, fits the experimental data. If we take the ratio of our calculated σ for the reasonable value $b_p = 1.98$ F we find that, independent of energy, $\sigma_{\mathrm{IA}}/\sigma_{\mathrm{expt}} = 1.59 \pm 0.18$. Here σ_{IA} is the DWIA result and σ_{expt} is the experimental value. The radiative pion-capture calculation is more uncertain and in the past we have been generous with the error limits. If we use $b_p = 1.98$ F and the new value[19] of the width of the $1s$ level in ^{6}Li, we get that the ratio of theoretical to experimental branching ratios is

$$\frac{R_{\mathrm{IA}}}{R_{\mathrm{expt}}} = \frac{0.38 \pm 0.09}{0.306 \pm 0.035} = 1.24 \pm 0.43, \qquad (9)$$

and the two values are probably consistent. Unexplained would be the success of the impulse approximation in predicting the Panofsky ratio[20] in ^{3}He.

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