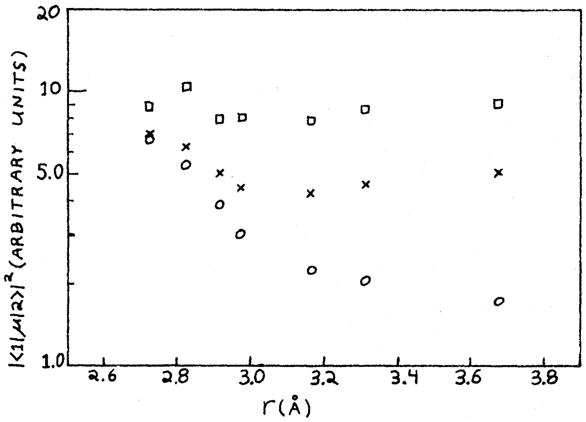

FIG. 2. The square of the transition moment as a function of the internuclear separation. The analysis of Ref. 2 is repeated and indicated by circles. The modified analysis of case *a* is denoted by crosses and the modified analysis of case *b* by squares.

the fluorescence signal.

Finally in Fig. 2 we present a reanalysis of the sodium induced–atomic-fluorescence data based on the present theory. The variation of the dipole moment is not as drastic as in our previous analysis but still does exhibit an interesting variation as a function of internuclear separation.

After having completed this manuscript we received a preprint of the work by Hessel, Smith, and Drullinger[7] on the dependence of the transition moment on internuclear separation in sodium deduced purely from molecular-fluorescence measurements. Their data and the results presented here are in reasonable agreement.

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Decoupled $i_{13/2}$ Neutrons and Backbending in W and Os Isotopes

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Analysis of rotational band structure in odd-N deformed rare-earth nuclei shows that if rotation-particle decoupling is the correct explanation for backbending in the region $N = 92$–96, then it can also explain the phenomenon in ^{180}W and ^{182,184,186}Os.

The discovery in 1972 of the so-called "backbending" effect[1] in the *yrast* band structure of even-even rare-earth nuclei prompted an initial interpretation that the data confirmed the breakdown of nuclear superfluidity at high spins, an effect proposed ten years earlier by Mottelson and Valatin.[2] More recently, however, Stephens and Simon[3] (SS) proposed that the observed effect could be explained in a much simpler, albeit related, way as arising from the recoupling of a single pair of $i_{13/2}$ neutrons in a "rotation-aligned" coupling scheme.

Subsequent experimental data, especially those obtained in the odd-A neighbors of the "backbending" nuclei, have for the most part tended to support the SS model. If indeed the $i_{13/2}$ neutrons nearest the Fermi surface were primarily responsible for the backbending effect, it seemed clear that the high-spin behavior of these neutrons in odd-N nuclei should provide a qualitative picture of what to expect in the neighboring even-even nuclei.

Stephens *et al.*[4] recently proposed a very simple criterion for gauging the adiabaticity of the single-particle motion in the presence of a rotating core: The spin $\frac{17}{2}\to\frac{13}{2}$ energy-level spacing of the lowest $i_{13/2}$ band is compared with the average $2\to 0$ spacing in neighboring even-even nuclei, the assumption being that the spacing should be identical in the decoupled limit. The results of such an analysis are less than conclusive, however, for the W and Os nuclei in precisely that region of the nuclear chart where the recent observation of striking "backbending" phenomena[5] seemed most difficult to understand in terms of the original SS model.

In this paper we show that a quantitative analysis of the particle-core decoupling in odd-neutron $i_{13/2}$ rotational bands places the backbending phenomena in the even-even Os nuclei squarely within the domain of the SS model, and in fact can account for all such effects so far observed in the "rare-earth" deformed region with the notable exception of the case of ^{170}Yb.[6]

The experimental data pertinent to the calculations described consist of even-parity rotational and intrinsic states known to be associated with the $i_{13/2}$ neutron orbitals. We have not made an exhaustive study of the data in this regard, but instead have made use of the available published wave functions obtained from Coriolis-coupling calculations on even-parity band structure in several odd-N rare-earth nuclei. All of the published calculations involve a similar procedure, more or less that described by Lindblad, Ryde, and Barnéoud,[7] for example. Briefly, the expansion coefficients in Ω for the single-particle motion are obtained by diagonalizing the Coriolis secular determinant, where the single-particle energies are determined in a Nilsson-model calculation and corrected for the effects of pairing.

The Hamiltonian appropriate to the description of the various rotational and intrinsic states is taken as the sum of the quasiparticle energy and the rotational contribution, and is diagonalized with a consistent parameter set for each spin state. This yields the eigenvectors consisting of linear combinations in Ω of the conventional wave functions which describe a nuclear rotation superimposed on an intrinsic state with projection Ω:

$$|IM\rangle=\left(\frac{2I+1}{16\pi^2}\right)^{1/2}\sum_{\Omega>0} f_\Omega{}^I\left[\mathfrak{D}_{M\Omega}{}^I\chi_\Omega+(-1)^{I-\Omega}\mathfrak{D}_{M-\Omega}\chi_{-\Omega}\right].$$

Here, the $f_\Omega{}^I$ are the expansion coefficients in Ω obtained from the diagonalization for each spin.

With the availability of the coefficients $f_\Omega{}^I$ for numerous even-parity rotational bands in the rare-earth region, it becomes a particularly simple matter to solve for the rotational energy, $\langle\vec{R}^2\rangle\hbar^2/2\mathscr{I}$, if one first transforms the conventional wave functions to the $|IRj\rangle$ representation suggested by Vogel,[8]

$$|IRj\rangle=\left(\frac{2I+1}{8\pi^2}\right)^{1/2}\sum_{\Omega=-j}^{+j}(-1)^{j-\Omega}(Ij\Omega-\Omega|R0)\,\mathfrak{D}_{M\Omega}{}^I|j\,\Omega\rangle.$$

The expectation value for the operator $\vec{R}^2$ is then easily evaluated,

$$\langle\vec{R}^2\rangle=\sum_{R,\,j}\langle IRj|IM\rangle^2R(R+1),$$

where R is restricted to even values, and

$$\langle IRj|IM\rangle=\sqrt{2}\sum_{\Omega=1/2}^{j}(Ij\Omega-\Omega|R0)f_\Omega{}^IC_{jl}{}^\Omega.$$

The $C_{jl}{}^\Omega$ are the usual coefficients of the Nilsson wave functions; in this case only the $i_{13/2}$ and $g_{9/2}$ (~10%) components are significant.

One is thus provided with a simple criterion for estimating the degree of particle-core decoupling in rotational bands. We have obtained values of $\langle\vec{R}^2\rangle$ for the even-parity bands in eighteen deformed odd-N nuclei from $^{155}_{64}$Gd$_{91}$ to $^{187}_{76}$Os$_{111}$. Fortunately, most of the available data span the region of greatest interest, i.e., those nuclei where a transition between backbending and normal behavior seems to occur. In Fig. 1 we show $[\langle\vec{R}^2\rangle-\langle\vec{R}_{\rm dec}{}^2\rangle]$ plotted as a function of spin for selected cases of interest. Here, $\vec{R}_{\rm dec}{}^2$ is just the decoupled limit; in that limit, the quantity plotted should approach zero. In the strong-coupling limit it is an increasing function of spin (cf. ^{179}Hf). On the left of Fig. 1 we show schematically just the $I=\frac{13}{2}$ values for other nuclei for which reliable wave functions were available, and we indicate (boxes) those odd-A nuclides where one or more

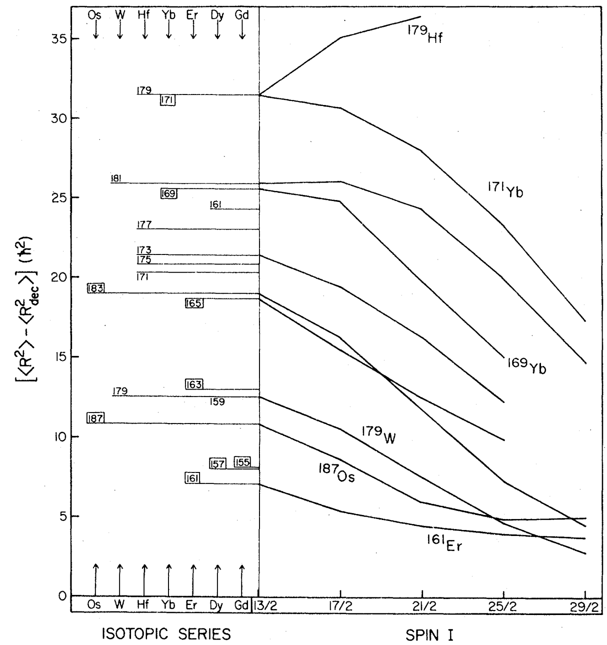

FIG. 1. The approach to rotation-particle decoupling for even-parity (predominant $i_{13/2}$) rotational bands in several odd-A rare-earth nuclei. Not all spins are plotted for all nuclei, but data for spin $\frac{13}{2}$ are indicated on the left portion in each case. The ^{179,181}W and ^{183,187}Os data are from our laboratory; wave functions for all other nuclei are taken from references cited in Ref. 4.

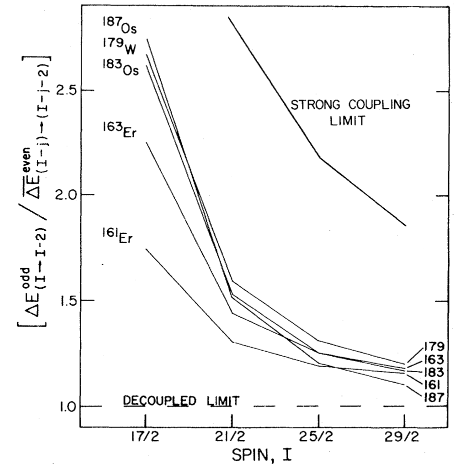

FIG. 2. Qualitative estimate of particle-core decoupling deduced from ratio of transition energies in odd-A and neighboring even-even nuclei (cf. Ref. 4). The coupled limit for spin $\frac{17}{2}$ is 5.33.

of the even-A neighbors display backbending[4] in the strictest sense (decreasing rotational frequency with increasing spin).

The picture emerges rather clearly from Fig. 1 that the regions of backbending nuclei are in general characterized by rotation-particle decoupling in the neighboring odd-A $i_{13/2}$ band structure. The principal difference between the conclusions from our calculations and the data presented in Ref. 4 is the clear inclusion in Fig. 1 of ^{187}Os and ^{179}W in the region of essential particle-core decoupling. Though the case for ^{183}Os (bounded by backbending ^{182}Os and ^{184}Os) is not so clear, it too becomes quite decoupled at relatively low spin. One also notes that ^{179}W and ^{159}Dy adjoin ^{180}W and ^{158,160}Dy, all three of which fail to meet the strict "backbending" requirement by only a few keV for at least one high-spin state; the ground rotational band behavior of these three even-even nuclei is certainly anomalous.[4]

The single difficulty in drawing a broad conclusion from Fig. 1 is the case of ^{170}Yb. The appearance of "backbending" in the ground band of this nucleus is not so easily understood in the SS model. Neither ^{169}Yb nor ^{171}Yb approaches the degree of decoupling displayed by all the other odd-A nuclei in the vicinity of "backbenders." Some authors[6,9] have attempted to explain the ^{170}Yb data in the pairing-collapse theory as a partial consequence of the variation in level density near the Fermi surface for the series ^{166,168,170}Yb, and in view of the modest decoupling exhibited by the $i_{13/2}$ particles in ^{169}Yb and ^{171}Yb, it may well be that the ^{170}Yb ground rotational band singularity arises from a more concerted breakdown of pairing at high spins.

An obvious question to be asked with regard to the results for ^{179}W and ^{183,187}Os in Fig. 1 is why the comparison of rotational energy spacings in the $\frac{9}{2}^+[624]$ and $\frac{11}{2}^+[615]$ bands in these nuclei fails to indicate the highly decoupled character of those bands. Although the band fitting which we have carried out for those nuclei makes use of much more experimental information than does the analysis in Ref. 4, it is also true that the even-even moment of inertia is implicit in the band-fitting calculations. One might suspect that the $\frac{17}{2}\to\frac{13}{2}$ spacing of the $i_{13/2}$ bands in ^{179}W and ^{187}Os is somewhat misleading, and there is some evidence that this is indeed the case, as the data in Fig. 2 show. The $\frac{17}{2}\to\frac{13}{2}$ energy spacing in the higher-Ω bands appears to be anomalously large when compared with ^{161}Er and ^{163}Er, both relatively well-decoupled cases. For higher-spin states, however, the spacings of the ^{183,187}Os and ^{179}W "favored" spin states ($I=j+2, 4, 6, \ldots$) fall rather quickly into much closer agreement with the ^{161,163}Er spacings. It thus seems likely that the higher-spin states in the high-Ω $i_{13/2}$ bands provide an essential constraint on the wave functions obtained in the band-fitting procedure.

Finally, an important question to be answered is just how sensitive the wave functions obtained from the Coriolos matrix diagonalizations of various groups are to the details of the fitting parameters. This is very important for the reliability of the curves in Fig. 1, since only the wave functions are used in our calculations. Fortunately, it is our experience and apparently that of other groups as well[10] that the wave functions are not in fact very sensitive to the quality of the fit to the energy data. In the case of ^{187}Os, for example, we performed numerous fits at various times as the analysis of our data progressed. Though the eigenvectors for the higher-lying (unseen) bands showed considerable variation in these fits, those of the two bands for which experimental data were available ($\frac{11}{2}^+$[615] and $\frac{9}{2}^+$[624]) varied but little with relatively large changes in the input parameters and the quality of the energy fits.

In summary, a simple, quantitative criterion for the tendency of high-j particles to decouple from the nuclear core rotation has been presented. If decoupling of $i_{13/2}$ neutrons from the core is indeed the explanation for "backbending" in the ground rotational band of neutron-deficient even-even deformed rare-earth nuclei where the effect was first reported, then that explanation can apply equally well to the W and Os isotopes which have also been found to exhibit strong backbending behavior.[5] While the possible role of $h_{9/2}$ protons in this region should not be ignored,[4] it appears that an understanding of the ground-band behavior in ^{180}W and $^{182-186}$Os does not require the involvement of protons. The case of ^{170}Yb is difficult to explain in this picture, however, and may offer the first instance of a departure from the straightforward Stephens-Simon model.

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Threshold Photoproduction of Pions on ^{6}Li

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We examine the dependence of the ^{6}Li(γ,π^+)^{6}He cross section on the pion optical potential and on the nuclear size parameters within the context of the distorted-wave impulse approximation. Other factors affecting the cross section are also considered and it is shown that the theoretical prediction remains about 60% higher than the observed cross section. Comparison to radiative pion capture in ^{6}Li is made.

In a recent experiment Deutsch *et al.*[1] have measured near threshold the ratio of photoproduction of positive pions on ^{6}Li and the proton. Using the absolute experimental proton cross section[2] near threshold, they have obtained a value for the ^{6}Li cross section as a function of photon energy above the threshold. Their measured value is significantly lower than that calculated by Koch and Donnelly.[3]

Near threshold the momentum-dependent terms[4] in the photoproduction amplitude should be unimpor-