cal.

[6]Unit integrated intensity is defined to be 100% of the incident beam arriving at the detector throughout an angular rotation of the crystal of 1 rad.

[7]We emphasize that Eq. (8) applies to an interferometer for which the beam splitter is very thin. Rays are refracted as they enter and leave the beam splitter (near B and E). Phase differences caused by this phenomenon could not arise were the angle $\epsilon - \delta$, given by Eqs. (4) and (6), equal to zero. Since $\epsilon - \delta \sim 10^{-8}$, a small correction to Eq. (8) may result for a beam splitter of finite thickness, depending on design.

[8]Equation (8) can also be derived when no gravitational field is present provided the nuclear reactor, beam port, etc., and the interferometer have a uniform acceleration g.

Multiplicity Distributions and Multiplicity Scaling in $p + p \to p$ + MM at 28.5 GeV/c*

T. S. Clifford,† A. R. Erwin, J. R. Ficenec, G. P. Larson, E. Lazarus, K. M. Moy, W. N. Schreiner,† P. Schübelin, W. P. Trower, and F. Turkot

Brookhaven National Laboratory, Upton, New York 11973, and Virginia Polytechnic Institute and State University, Blacksburg, Virginia 24061, and University of Wisconsin, Madison, Wisconsin 53706

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The charged particles associated with a fast forward trigger proton of four-momentum transfer t in the reaction $p+p \to p$ + MM at 28.5 GeV/c were measured using the Multiparticle Argo Spectrometer System. Charged-particle multiplicity distributions show multiplicity scaling with respect to the missing mass, MM, to ~10% within each of two t regions, $0<|t|<2$ (GeV/c)^2 and $3<|t|<5$ (GeV/c)^2. The width of the scaling curve shrinks between regions.

We previously reported the behavior of the average number of charged particles, $\langle n_c\rangle$, produced in the reaction

$$p_1 + p_2 \to p_3 + \mathrm{MM} \tag{1}$$

at 28.5 GeV/c.[1] We observed a sharp rise in $\langle n_c\rangle$ with increasing t, the four-momentum transfer from p_1 to p_3, for fixed mass MM recoiling against p_3. Here we describe certain regularities in the shapes of the charge multiplicity distributions, $P_{n_c}(\mathrm{MM}, t)$, and examine the scaling properties in MM and t and their correlation with the rise in $\langle n_c\rangle$. Such observations may be helpful in understanding the dynamical mechanism of particle production in high-energy pp collisions.

Our data base consists of ~ 200 000 events of Reaction (1). The data were taken at Brookhaven National Laboratory by the Multiparticle Argo Spectrometer System (MASS).[2] The high-momentum spectrometer (HMS) momentum analyzed and identified p_3, and the vertex spectrometer (VS)[3] momentum analyzed the remaining charged particles. The VS tracks were reconstructed by the computer code PITRACK[4] and fitted to a common vertex. Corrections were applied[1] to the multiplicity distributions to compensate for loss of charged particles due to limited solid angle (~ 10%), low-momentum cutoff (~ 1%), and software inefficiencies (~ 6%).

The probability for a given charged multiplicity associated with a trigger proton of given MM and t is

$$P_{n_c}(\mathrm{MM}, t) \equiv \frac{d^2\sigma_{n_c}/d(\mathrm{MM})dt}{\sum_i d^2\sigma_i/d(\mathrm{MM})dt}. \tag{2}$$

The data, which cover the range $1.2<\mathrm{MM}<6.5$ GeV and $0.2<|t|<6.5$ (GeV/c)^2, have been divided into 31 bins of MM and t by taking six intervals in MM and seven in t. For each bin we calculated the average charged multiplicity in the final state, $\langle n_c\rangle$, the dispersion, $D^2 \equiv \langle n_c^2\rangle - \langle n_c\rangle^2$, and two higher normalized moments, $C_i \equiv \langle n_c^i\rangle/\langle n_c\rangle^i$, $i = 3, 4$, of $P_{n_c}(\mathrm{MM}, t)$; the results are given in Table I. In analyzing the data with finer binning[5] one sees that for fixed MM > 2 GeV, the average charge multiplicity is approximately constant for $|t|<2$ (GeV/c)^2, then rises abruptly by $\Delta_{n_c} \sim 0.6$ charged particles and again becomes approximately constant for $|t|>3$ (GeV/c)^2. This constancy of $\langle n_c\rangle$ for fixed MM in the two t regions is also observed[5] in the individual P_{n_c}. The magnitude of the increase and the t value at which it is centered are approximately constant for all intervals of MM > 2 GeV. We found that the rise is centered at $|t| \sim 2.5$ (GeV/c)^2 and occurs in a characteristic interval $\Delta t \sim 1$ (GeV/c)^2.

Figure 1 is a plot of $P_{n_c}(\mathrm{MM}, t)$ versus MM averaged over two t intervals on either side of the rise in $\langle n_c\rangle$: Region I has $1<|t|<2$ (GeV/c)^2 and region II has $3<|t|<5$ (GeV/c)^2. Compari-

TABLE I. Differential moments of the multiplicity distributions in Reaction (1). The errors are statistical only.

$\langle\lvert t\rvert\rangle$	MM[a]	Events	$\langle n_c\rangle$	D	C_3	C_4
0.326	1.651	4957	2.70 ± .03	1.12 ± .02	1.63 ± .06	2.65 ± .15
0.424	2.551	8203	3.75 ± .03	1.43 ± .02	1.48 ± .04	2.15 ± .09
0.540	3.522	11581	4.35 ± .03	1.67 ± .02	1.49 ± .03	2.15 ± .06
0.637	4.451	10728	4.92 ± .03	1.86 ± .02	1.46 ± .03	2.05 ± .05
0.799	5.225	2785	5.39 ± .04	2.00 ± .03	1.44 ± .05	1.98 ± .10
1.319	1.673	1266	2.89 ± .05	1.22 ± .05	1.67 ± .15	2.93 ± .46
1.389	2.579	3746	3.68 ± .03	1.43 ± .02	1.49 ± .05	2.14 ± .10
1.430	3.608	12803	4.40 ± .03	1.74 ± .02	1.51 ± .03	2.22 ± .06
1.496	4.570	40019	5.01 ± .02	1.93 ± .01	1.48 ± .02	2.09 ± .03
1.567	5.241	19294	5.48 ± .03	2.11 ± .01	1.47 ± .02	2.08 ± .04
1.898	6.043	84	6.26 ± .27	2.26 ± .20	1.41 ± .29	1.90 ± .48
2.322	1.692	126	2.75 ± .15	1.00 ± .06	1.43 ± .30	1.97 ± .52
2.348	2.585	589	3.75 ± .08	1.41 ± .05	1.45 ± .13	2.03 ± .24
2.348	3.623	2779	4.61 ± .04	1.72 ± .03	1.44 ± .06	2.02 ± .11
2.272	4.546	9145	5.16 ± .03	1.91 ± .02	1.44 ± .03	2.00 ± .06
2.432	5.504	11599	5.85 ± .03	2.11 ± .02	1.41 ± .03	1.91 ± .04
2.614	6.088	604	6.25 ± .10	2.20 ± .07	1.38 ± .10	1.83 ± .17
3.283	2.600	68	4.09 ± .25	1.59 ± .16	1.48 ± .38	2.07 ± .68
3.475	3.623	390	4.73 ± .11	1.75 ± .07	1.43 ± .15	1.96 ± .25
3.603	4.686	3208	5.61 ± .04	2.03 ± .03	1.42 ± .05	1.94 ± .09
3.651	5.561	19291	6.26 ± .03	2.19 ± .01	1.38 ± .02	1.85 ± .03
3.462	6.105	2273	6.34 ± .05	2.16 ± .04	1.36 ± .05	1.80 ± .09
4.414	3.657	268	4.72 ± .13	1.80 ± .12	1.49 ± .20	2.17 ± .40
4.418	4.678	2186	5.78 ± .05	2.07 ± .04	1.41 ± .06	1.91 ± .10
4.355	5.643	15621	6.37 ± .03	2.23 ± .02	1.38 ± .02	1.84 ± .04
4.628	6.090	4830	6.70 ± .04	2.36 ± .03	1.39 ± .04	1.87 ± .07
5.415	3.661	87	4.92 ± .24	1.42 ± .13	1.24 ± .25	1.50 ± .37
5.254	4.593	418	5.60 ± .12	2.01 ± .10	1.41 ± .14	1.95 ± .26
5.201	5.688	1216	6.33 ± .07	2.18 ± .05	1.36 ± .07	1.79 ± .12
5.475	6.193	4472	6.67 ± .04	2.30 ± .03	1.37 ± .04	1.80 ± .06
6.300	6.269	2235	6.63 ± .05	2.30 ± .04	1.37 ± .05	1.83 ± .09

[a] These values of MM represent the average value for each bin.

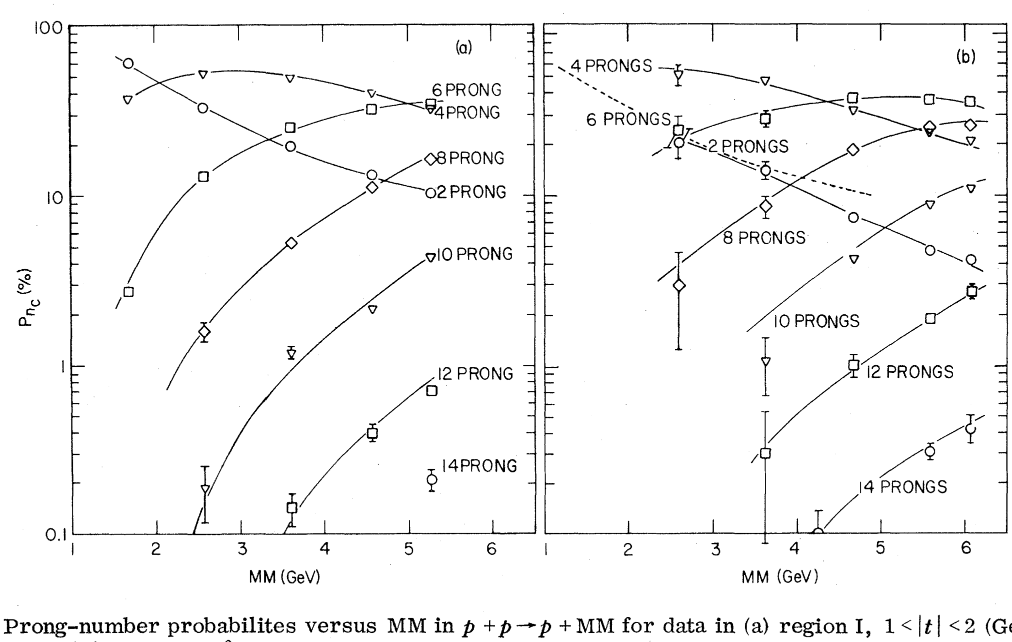

FIG. 1. Prong-number probabilites versus MM in $p+p\rightarrow p+\mathrm{MM}$ for data in (a) region I, $1<\lvert t\rvert<2$ (GeV/c)^2, and (b) region II, $3<\lvert t\rvert<5$ (GeV/c)^2. The hand-drawn curves (excepting the two-prong case) were obtained with (b) superimposed on (a) and displaced by +0.75 GeV. The dotted line is the two-prong curve from (a).

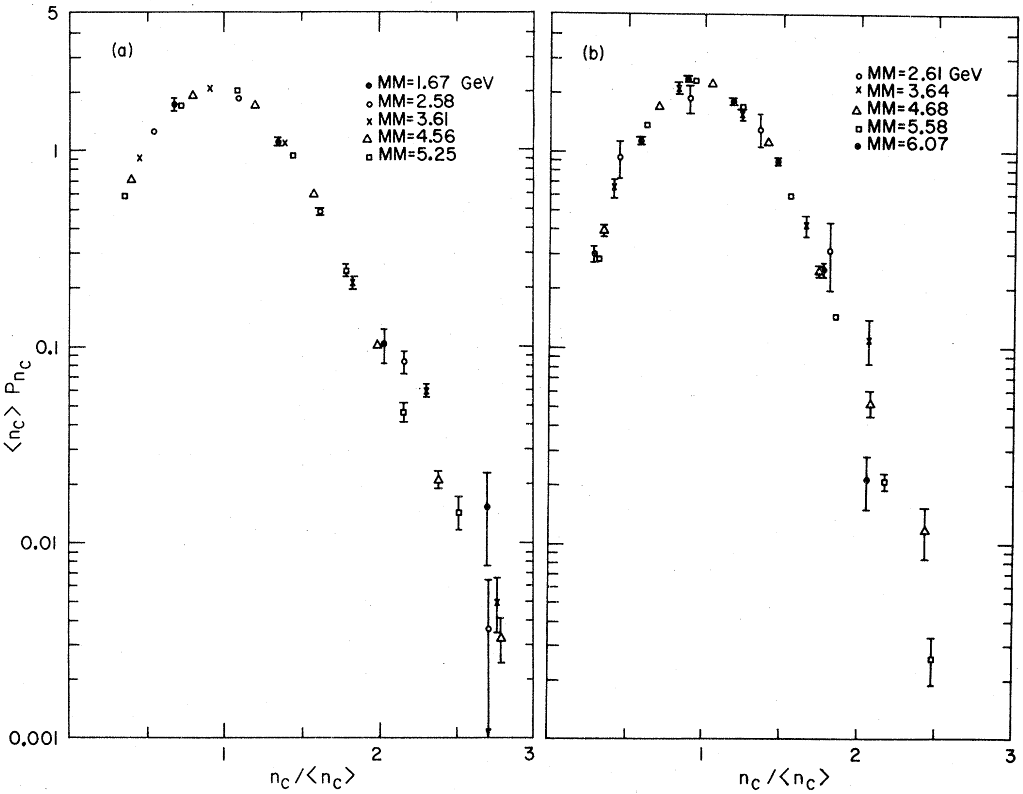

FIG. 2. $\langle n(\mathrm{MM})\rangle P_{n_c}(\mathrm{MM})$ versus $n_c/\langle n_c(\mathrm{MM})\rangle$ averaged over t in (a) region I and (b) region II.

son of the P_{n_c} in these regions indicates that Δn_c is distributed in prong numbers as if MM were effectively ~0.7 GeV larger in region II; the behavior of the two-prongs is the sole exception, showing a reduction corresponding to an even larger equivalent shift in MM.

Recently, Barshay and Yamaguchi[6] suggested that the associated charge multiplicity, $n_a \equiv n_c - 1$ [the multiplicity within MM in Reaction (1)], may exhibit a scaling behavior with respect to MM:

$$\langle n_a(\mathrm{MM}, t)\rangle P_{n_a}(\mathrm{MM}, t) = \varphi(n_a(\mathrm{MM}, t)/\langle n_a(\mathrm{MM}, t)\rangle, t), \qquad (3)$$

where the scaling function φ can also depend on incident energy. If such behavior exists it would be an empirical result independent of the well-known Koba-Nielsen-Olesen (KNO) multiplicity scaling[7] with the total energy, since Reaction (1) is semi-inclusive and for KNO scaling one integrates over MM. Barshay *et al.*[8] have observed evidence in experimental data for scaling in MM when integrating over the low-t region, $0.0 \leq |t| \leq 0.5$ (GeV/c)^2, accessible to the bubble chamber.

To examine our data for scaling behavior with respect to MM we plot in Fig. 2 $\langle n_c(\mathrm{MM}, t)\rangle P_{n_c}(\mathrm{MM}, t)$ versus $Z \equiv n_c(\mathrm{MM}, t)/\langle n_c(\mathrm{MM}, t)\rangle$ for regions I and II. Approximate scaling within each region is observed[9]; however, the scaling function in II is narrower than in I. This effect is primarily due to the reduced percentage of two-prongs in region II. Since the P_{n_g} are essentially constant within regions I and II,[5] one has the trivial result of doubly differential multiplicity scaling within those regions.

In order to make a more quantitative test of the goodness of scaling in MM, we plot in Fig. 3(a) the ratio $\langle n_c\rangle/D$ versus MM. Some variation is observed within each region and this may be used as a measure of the minimum deviation from scaling in MM over the range $2.0 < \mathrm{MM} < 6.5$ GeV. $\langle n_c\rangle/D$ being 10% larger in region II than in region I corresponds to the narrowing of the scaling curve when $|t|$ goes from 2 to 3 (GeV/c)^2. The normalized moments C_i versus MM in region I are shown in Fig. 3(b); there is a slight indication from C_3 and C_4 that scaling in MM is better for higher MM. From Fig. 3 it is apparent that $\langle n\rangle/D$ is a more sensitive variable than C_2

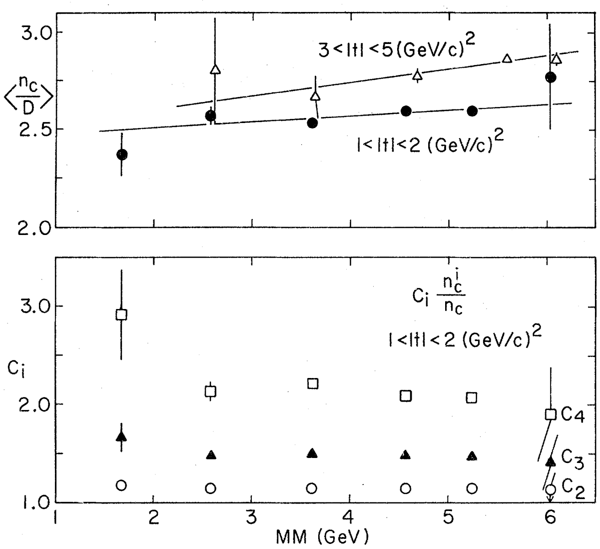

FIG. 3. Deviations from exact multiplicity scaling in MM. (a) $\langle n_c\rangle/D$ versus MM in regions I and II. The lines fitted to the data are $\langle n_c\rangle/D = (0.03 \pm 0.04)\,\mathrm{MM} + (2.45 \pm 0.06)$ in region I and $\langle n_c\rangle/D = (0.07 \pm 0.03)\,\mathrm{MM} + (2.46 \pm 0.16)$ in region II. (b) The normalized higher moments (see text) versus MM.

to test scaling.[10]

Initially one may be tempted to look for scaling of the charged multiplicity within MM as expressed by Eq. (3). However, since the width of the associated charged multiplicity distribution, D_a, equals the width of the total charged multiplicity distribution, D, at fixed MM, a plot similar to Fig. 3(a) of $\langle n_a\rangle/D_a = \langle n_c - 1\rangle/D$ versus MM would show a more pronounced percentage variation with MM. In fact the data of Table I indicate that the best scaling in MM can be obtained with the variable $n' = n_c + 0.7 = n_a + 1.7$ as $\langle n'\rangle/D$ is flat.

Attempts have been made in the case of KNO scaling to find a scaling variable that improved the regularity.[11,12] In particular Buras, Dias de Deus, and Møller[12] used the variable $Z' = (n_c - \alpha)/\langle n_c - \alpha\rangle$ and suggested that the quantity α represents the average number of unfragmented leading particles, $0 \leq \alpha \leq 2$. They plotted the Nth root of the Nth central moment, μ_N, versus $\langle n_c\rangle$ and found that an abscissa intercept of $\alpha = +0.9$ works well over the incident-momentum range from 5 to 300 GeV/c. If their interpretation of α is correct, scaling would improve in our differential data with that variable. This is inconsistent with our "best scaling value" $\alpha \approx -0.7$.

The higher normalized moments of the associated multiplicity n_a, calculated from Table I for region I, join smoothly with those of Barshay *et al.*[8] at MM ~ 5.5 GeV/c. This indicates that over the incident momentum range from 28.5 to 400 GeV/c, differential multiplicity scaling for all values of MM may be observed if the modified scaling variable $(n_c + \alpha)/\langle n_c + \alpha\rangle$, with $\alpha \sim 0.7$, is chosen.

In summary, we have shown for Reaction (1) at 28.5 GeV/c that $P_{n_c}(\mathrm{MM}, t)$ exhibits approximate scaling for $1.2 < \mathrm{MM} < 6.5$ GeV in the $|t|$ intervals 1–2 (GeV/c)^2 and 3–5 (GeV/c)^2, the scaling function φ is narrower at large t than at low t, and the narrowing of φ occurs in the same t region where $\langle n_c\rangle$ is observed to have a sharp rise.

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