have deduced from the three-nucleon problem specific information about the two-nucleon interaction that has not yet been attainable.

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[1]J. C. Aarons and I. H. Sloan, Nucl. Phys. A182, 369 (1972); I. H. Sloan and J. C. Aarons, *ibid.* A198, 321 (1972).

[2]S. C. Pieper, Nucl. Phys. A193, 529 (1972).

[3]P. Doleschall, Phys. Lett. 40B, 443 (1972), and Nucl. Phys. A201, 264 (1973).

[4]S. C. Pieper, Phys. Rev. C 6, 1157 (1972).

[5]P. Doleschall, Nucl. Phys. A220, 491 (1974).

[6]H. E. Conzett, G. Igo, and W. J. Knox, Phys. Rev. Lett. 12, 222 (1964).

[7]H. E. Conzett, H. S. Goldberg, E. Shield, R. J. Slobodrian, and S. Yamabe, Phys. Lett. 11, 68 (1964).

[8]T. B. Clegg and W. Haeberli, Nucl. Phys. A90, 593 (1967).

[9]S. N. Bunker, J. M. Cameron, R. F. Carlson, J. R. Richardson, P. Tomas, W. T. H. van Oers, and J. W. Verba, Nucl. Phys. A113, 461 (1968).

[10]J. C. Faivre, D. Garreta, J. Jungerman, A. Papineau, J. Sura, and A. Tarrats, Nucl. Phys. A124, 169 (1969).

[11]R. E. White, W. Grüebler, V. König, R. Risler, A. Ruch, P. A. Schmelzbach, and P. Marmier, Nucl. Phys. A180, 593 (1972).

[12]J. S. C. McKee, H. E. Conzett, R. M. Larimer, and Ch. Leemann, Phys. Rev. Lett. 29, 1613 (1972).

[13]A. Fiore, J. Arvieux, N. V. Sen, G. Perrin, F. Merchez, J. C. Gondrand, C. Perrin, J. L. Durand, and R. Darves-Blanc, Phys. Rev. C 8, 2019 (1973).

[14]R. Aaron, R. D. Amado, and Y. Y. Yam, Phys. Rev. 140, B1291 (1965).

[15]I. H. Sloan, Phys. Rev. 185, 1361 (1969), and Nucl. Phys. A168, 211 (1971).

[16]We use the nomenclature and notation of the Madison Convention; see *Polarization Phenomena in Nuclear Reactions*, edited by H. H. Barschall and W. Haeberli (Univ. of Wisconsin Press, Madison, Wis., 1971), p. XXV.

[17]F. N. Rad, J. Birchall, H. E. Conzett, S. Chintalapudi, R. M. Larimer, and R. Roy, Bull. Amer. Phys. Soc. 19, 476 (1974).

[18]S. C. Pieper, Phys. Rev. C 8, 1702 (1973).

[19]M. H. MacGregor, R. A. Arndt, and R. M. Wright, Phys. Rev. 182, 1714 (1969); R. A. Arndt, J. Binstock, and R. Bryan, Phys. Rev. D 8, 1397 (1973); J. Binstock and R. Bryan, Phys. Rev. D 9, 2528 (1974).

[20]Ch. Leemann, H. E. Conzett, W. Dahme, J. A. MacDonald, and J. P. Meulders, Bull. Amer. Phys. Soc. 17, 562 (1972), and to be published.

[21]The constrained-solution values of ϵ_1 and $\delta(^1P_1)$ as given in Table VII of MacGregor, Arndt, and Wright, Ref. 19, were used.

[22]J. J. Malanify, J. E. Simmons, R. B. Perkins, and R. L. Walters, Phys. Rev. 146, 532 (1966); C. L. Morris, R. Rotter, W. Dean, and S. T. Thorton, Phys. Rev. C 9, 1687 (1974).

[23]S. C. Pieper, Phys. Rev. C 9, 883 (1974).

What Can We Learn from Three-Body Reactions?

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Deuteron-breakup calculations are performed by using phase-equivalent potentials that differ off shell with and without the constraint of a fixed triton binding energy. This approach is compared with the previous work of Brayshaw. We examine various regions of phase space and energy between 14 and 65 MeV. Our results show that new off-shell information can be obtained from deuteron-breakup studies in the final-state region of phase space.

Nuclear reactions involving three nucleons have been extensively studied in the last few years. One motivation has been the possibility of learning new information about short-range two-particle interactions, in particular, the off-shell behavior of the nuclear force. Several authors have suggested regions of phase space in which this information may be readily obtainable. They generally agree that the $L=0$ part of the M_{D2} amplitude is the one sensitive to off-shell effects and that the regions of phase space that should be investigated are the ones in which these dominate. Jain, Rogers, and Saylor[1] have suggested an energy-sharing kinematic locus, while Kloet and Tjon[2] have suggested certain regions of phase space where breakup results are sensitive to the presence or absence of a repulsive core in the potential. Brayshaw[3] has examined these regions at 14 MeV using a three-body boundary-condition model in which two-body on-shell properties are fixed, but short-range correlations are varied. The variations in short-range correlations correspond to some (unspecified) combinations of off-shell effects and three-body forces. Brayshaw found that while breakup results exhibit some sensitivity to short-range correlations, this sensitivity virtually disappears (to a few percent) if a fixed value of the n-d doublet scattering-length prediction (^2a) is used as a constraint. Furthermore, the cross section is small and observation is very difficult in the regions investigated. Brayshaw concluded that no off-shell information could be obtained which was not already implicit in the value of the n-d doublet scattering length.

In view of the importance of Brayshaw's conclusion, we thought it valuable to investigate the situation using a different approach. There are three points at which his conclusion can be questioned: (a) Would other models lead to different conclusions? (b) Are there other regions of phase space in which observable off-shell differences would show up? (c) Is the insensitivity suggested by Brayshaw energy independent?

This paper discusses deuteron-breakup results obtained with three separable S-wave potentials by using the Ebenhöh code.[4] The potentials differ from the Yamaguchi potentials employed heretofore in their off-shell momentum dependence; i.e., they have different form factors. We employ an energy-dependent modification of the two-nucleon (N-N) T matrix[5] which enables us to obtain phase-shift equivalence of the three potentials and to adjust the triton binding energy ($E_T = 8$ or 11 MeV).

For separable potentials, the momentum-space matrix elements have the form

$$\langle k|V|k'\rangle = -\lambda g(k)g(k'), \tag{1a}$$

and the T matrix,

$$\langle k|T(E)|k'\rangle = g(k)g(k')\tau(E), \tag{1b}$$

where $\tau(E) = -[\lambda^{-1} + 4\pi\int_0^\infty q^2\,dq\,g^2(q)/(E - q^2 + i\epsilon)]^{-1}$. (We have suppressed angular momentum, spin, and isospin labels.) Most previous calculations have used the Yamaguchi form factor $g(k) \sim (k^2 + \beta^2)^{-1}$, where λ and β are chosen to fit the N-N effective-range parameters. Here, we consider an alternate form factor, $g(k) \sim (k_c^2 - k^2)/(k^2+\beta^2)^2$. Different values of k_c^2 give different off-shell behavior, while λ and β (for a fixed k_c^2) are such as to give the experimental effective-range parameters. We consider two form factors differing in values of k_c^2. The form factor HA ($k_c^2 = +4\ \mathrm{fm}^{-2}$) simulates a repulsive core (like the Reid soft-core potential[6]), while HB ($k_c^2 = -2.89\ \mathrm{fm}^{-2}$) is similar to the "softer" Yamaguchi form factor.

In general, separable potentials can differ in both their off-shell and on-shell behaviors. In order to obtain the same on-shell behavior (phase shifts), we employ the method of Ref. 5 by replacing $\tau(E)$ in Eq. (1b) by

$$\tau'(E) = \rho(E)\tau(E)\{1 - 2i\pi^2\sqrt{E}\,g^2(\sqrt{E})\tau(E)[1-\rho(E)]\}^{-1} \tag{2}$$

(where energy E has units of momentum squared). The presence of the denominator means that the replacement $\tau(E)\to\tau'(E)$ corresponds to multiplying the on-shell K matrix element, $K(E) = -[\tan\delta(E)]/2\pi^2\sqrt{E}$, by $\rho(E)$. Here $\delta(E)$ is the phase shift predicted by $g(k)$. A real $\rho(E)$ guarantees a unitary two-body amplitude. In fact, the replacement of $\tau(E)\to\tau'(E)$ merely corresponds to an energy-dependent λ $[\lambda\to\lambda(E)]$ in the potential [see Eq. (1a)].

We modify the T matrix of the HB form factor to gain the phase shifts of HA. Explicitly,

$$\rho(E) = \begin{cases} 1 + [\tan\delta_{\mathrm{HA}}(E)/\tan\delta_{\mathrm{HB}}(E) - 1]\{1 + \exp[-\gamma(E - E_0)]\}^{-1} & \text{for } E > E_1, \\ 1 & \text{for } E \leq E_1. \end{cases} \tag{3}$$

The form factors, triton binding energy, and $\rho(E)$ parameters for each potential appear in Table I. With the parameters in Table I for γ and E_0, we see that for $E>0$, $\rho(E)$ is very closely equal to $\tan\delta_{\mathrm{HA}}(E)/\tan\delta_{\mathrm{HB}}(E)$; i.e., potentials HB2-8.0 and HB2-11.0 are virtually phase equivalent to HA2-8. In the nomenclature of Table I, HA and HB refer to the form factors, 2 refers to this particular set of phase shifts (i.e., those predicted by the

TABLE I. Potential parameters, effective-range parameters, and triton binding energy.

	Potential		
	HA2-8	HB2-8	HB2-11
k_c^2 (fm^{-2})	4.0	-2.89	-2.89
β_t (fm^{-1})	3.318	1.541	1.541
a_t (fm)	5.395	5.395	5.395
r_{0t} (fm)	1.750	1.750	1.750
β_s^{np} (fm^{-1}) [a]	2.246	1.374	1.374
a_s^{np} (fm) [a]	-23.68	-23.68	-23.68
r_{0s}^{np} (fm) [a]	2.670	2.670	2.670
β_s^{pp} (fm^{-1})	2.286	1.390	1.390
a_s^{pp} (fm)	-7.766	-7.766	-7.766
r_{0s}^{pp} (fm)	2.860	2.860	2.860
γ (MeV^{-1})	b	0.500	0.500
E_0 (MeV)	b	-45.50	-40.00
E_1 (MeV)	b	-75.00	0.00
E_T (MeV)	8.04	8.08	10.90

[a]The n-n interaction is taken to be the same as the n-p interaction.

[b]Not applicable as $\rho(E)=1$ for the HA form factor.

form factor HA), and 8 or 11 refers to the approximate triton binding energy. Note that by altering the parameters E_0 and E_1, which essentially affect only the $E<0$ T matrix, we can change the triton binding-energy predictions [here $\delta(E)$ is the analytic continuation of the phase-shift function]. Work by Phillips[7] has indicated that E_T and ^2a are strongly correlated; we list E_T rather than ^2a. The potential HB2-11 corresponds closely to the potential used by Ebenhöh. A comparison using the potentials HB2-11 and HA2-8 corresponds to a change in the off-shell momentum dependence, i.e., the form factor, with no constraint on the triton binding energy. A comparison using HA2-8 and HB2-8 corresponds to an off-shell change with the constraint of a fixed triton binding energy. The potentials HB2-11 and HB2-8 predict a change in triton binding energy with no change in the off-shell momentum dependence.

In Fig. 1, we plot predictions of the ^{2}H$(n,nn)p$ cross section at 14 MeV, following Kloet and Tjon and Brayshaw. In this region it is very important to compare phase-equivalent potentials, as it is a region of minimum spectator energy and is thus very sensitive to the on-shell part of the interaction as well as to the off-shell part. The predictions of our phase-equivalent potentials HA2-8 and HB2-8 differ by a larger amount than do the predictions obtained by Brayshaw using his boundary-condition approach. The results

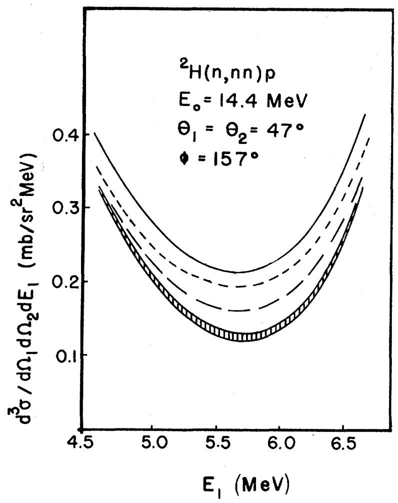

FIG. 1. Differential cross section for n-d breakup versus the energy of an outgoing neutron. The solid curve is the Kloet-Tjon calculation using the Malfiet-Tjon I-III potential. The shaded area represents the maximum variation produced by Brayshaw. The long-dashed and short-dashed curves indicate our results with the HA2-8 and HB2-8 potentials which predict the same phase shifts, N-N scattering length, and triton binding energy but differ in their off-shell characteristics.

are qualitatively consistent but do indicate that conclusions drawn from Brayshaw's approach are somewhat model dependent.

We have also made calculations for the ^{2}H$(p, pp)n$ cross section at 23 MeV for the energy-sharing locus of Jain, Rogers, and Saylor.[1] In this case, as in Fig. 1, there exist regions where some relatively large (up to about 30%) sensitivity occurs, several times that obtained by Brayshaw at 14 MeV. However, where the sensitivity is relatively large, the cross sections are small and even a 30% difference would be difficult to distinguish experimentally. In fact, for the $L=0$ amplitude to dominate, the cross section is usually small since one is ruling out the mechanism that accounts for the quasifree-scattering enhancement (i.e., the coherent contributions of many partial waves). Furthermore, off-shell changes seem primarily to renormalize the cross sections making it more difficult to distinguish between potentials experimentally.

Is it possible to find regions of phase space where the $L=0$, $S=\frac{1}{2}$ M_{D2} amplitude is important yet the cross section is not very small? In the final-state-interaction (FSI) region, the enhancement is due to the $^1S_0$ resonance in the N-N scat-

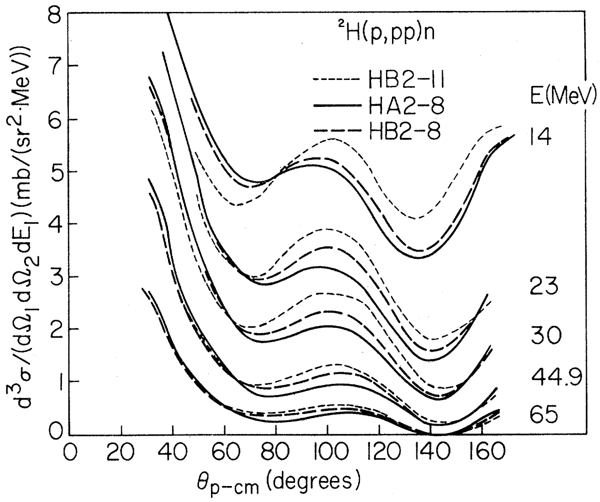

FIG. 2. Peak differential cross sections at final-state angles for the various beam energies indicated on the right. A comparison using HB2-11 and HA2-8 corresponds to an off-shell change with no constraint on triton binding energy, while a comparison using HA2-8 and HB2-8 corresponds to an off-shell change with fixed triton binding energy. A comparison using HB2-11 and HB2-8 corresponds to changing triton binding energy with a fixed off-shell momentum dependence.

tering amplitude at zero energy; therefore, unlike quasifree scattering, the $L=0$ contribution could be important yet the cross section large.

Since the relative importance of the $L=0$ amplitude varies with the center-of-mass angle of the FSI pair, both the magnitude and shape of the FSI angular distribution should be sensitive to off-shell changes. Figure 2 illustrates FSI distributions at a number of energies. At 14 MeV the effect of a change of form factor is greatly reduced by a constraint on E_T. At higher energies, however, even the constraint on E_T still allows changes in magnitude and shape under off-shell changes. The maximum sensitivity occurs between 90° and 130° and is large enough, even for fixed E_T, that the difference in cross section is easily experimentally observable. The change in shape as well as magnitude improves the experimental distinguishability of the potentials.

Our calculations indicate the following: (1) Off-shell variations in cross-section calculations at 14 MeV are slightly model dependent. (2) The final-state region of phase space is the most practical region in which to search for off-shell effects. The other regions sensitive to off-shell effects have small cross sections and off-shell differences would be very difficult to distinguish experimentally. (3) The practicability of observing off-shell effects is a function of energy. In the region between 20 and 45 MeV cross sections for final-state interactions are large and are sensitive to the off-shell character of the nuclear potential. Therefore, careful measurements of the FSI angular distribution can lead to off-shell information beyond that obtainable from the low-energy three-nucleon properties E_T and ^2a. The question of distinguishability of off-shell effects and three-body forces remains an open question.

[1] M. Jain, J. G. Rogers, and D. P. Saylor, Phys. Rev. Lett. 31, 838 (1973).

[2] W. M. Kloet and J. A. Tjon, Nucl. Phys. A210, 380 (1973).

[3] D. D. Brayshaw, Phys. Rev. Lett. 32, 382 (1974).

[4] W. Ebenhöh, Nucl. Phys. A191, 97 (1972).

[5] E. L. Petersen *et al.*, Phys. Rev. C 9, 508 (1974).

[6] R. V. Reid, Ann. Phys. (New York) 50, 411 (1968).

[7] A. C. Phillips, Nucl. Phys. A107, 209 (1968). The values of ^2a were 1.29 fm for HA2-8, 1.42 for HB2-8, and -0.50 for HB2-11. These values are consistent with the variation about the Phillips line observed in Brayshaw's work. We want to thank Jon Wallace for providing the values of ^2a.