Parametric Instabilities in Turbulent, Inhomogeneous Plasma*

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(Received 19 August 1974)

It is known that parametric instabilities involving two coupled modes, with oppositely directed group velocities, saturate convectively if the medium is inhomogeneous. This work considers the modification of that result when weak long-wave turbulence is present, in addition to the background inhomogeneity. We find that the convective saturation disappears when the turbulence exceeds a certain level, absolute growth occurring instead.

The behavior of parametric instabilities in *homogeneous* media is now fairly well understood.[1-3] Study of the *inhomogeneous* situation is progressing rapidly,[4-10] motivated by the problems of laser fusion,[11] heating of magnetically confined plasma, and ionospheric modification.

Most theories of coupled-mode effects assume that the pump (or driver), which is responsible for coupling the two modes, has a definite phase relationship to them. In practice, there will be a finite bandwidth[12] in the driver; furthermore, there may be spatial and temporal fluctuations in the background medium which would affect the coupling and propagation of the modes.[13]

When all inhomogeneities are stationary in time, and slowly varying in space with respect to the wavelengths of the three waves, the coupled-mode equations can be cast into the form[5,8]

$$\begin{aligned}\left(\frac{\partial}{\partial t}+V_1\frac{\partial}{\partial x}\right)a_1(x,t)&=\gamma_0 a_2(x,t)\exp\left[i\int_0^x\kappa(x')\,dx'\right],\\ \left(\frac{\partial}{\partial t}+V_2\frac{\partial}{\partial x}\right)a_2(x,t)&=\gamma_0 a_1(x,t)\exp\left[-i\int_0^x\kappa(x')\,dx'\right],\end{aligned} \tag{1}$$

where $a_1(x,t)$ and $a_2(x,t)$ are the slowly varying (in space and time) amplitudes of the waves; V_1 and V_2 are the group velocities of the waves in the absence of coupling; γ_0 is the coupling constant, usually proportional to the amplitude of the third wave, and here assumed constant in space and time; and $\kappa(x)$ is the wave number mismatch between the three waves in a WKBJ sense: $\kappa(x)=k_0(x)-k_1(x)-k_2(x)$. The origin $x=0$ is chosen as the reference point of exact wave-number matching, i.e., $\kappa(x=0)=0$, in the absence of turbulence. The parameters V_1, V_2, and γ_0 can be combined to form the fundamental length $L_0\equiv |V_1V_2|^{1/2}/\gamma_0$. If $V_1V_2>0$, L_0 is the amplification length; if $V_1V_2<0$, L_0 is the minimum length of a finite system for the existence of absolute instability.

For purposes of this paper, we separate the mismatch function $\kappa(x)$ into a nonrandom part, representing a constant gradient, and a random part with zero mean:

$$\kappa(x)=\kappa'x+\delta\kappa(x);\quad \langle\delta\kappa(x)\rangle=0. \tag{2}$$

The turbulent contribution is characterized by the rms mismatch $\Delta\equiv\langle[\delta\kappa(x)]^2\rangle^{1/2}$ and the correlation length L_T.

In the case $V_1V_2>0$, there is no possibility of absolute instability,[2] and one can assume a steady state. This case has been considered by Kaw *et al.*[14] For $\kappa'=0$, and with a constant source at $x=0$, the growth length was found to be increased by the presence of turbulence, from L_0 to $(L_0\Delta)^2\times L_T$ for $\Delta^2\gg 1/L_TL_0$. For $\kappa'\neq 0$ and no turbulence, we know[5] that the maximum amplification of a temporally constant source at $x=0$ is $\sim\exp(\pi\lambda)$, where $\lambda\equiv\gamma_0^2/V_1V_2\kappa'$. In the turbulent case a similar result was found, but with the amplification length increased.

In this paper we consider the case $V_1V_2<0$, which occurs in backscatter instabilities. If $\kappa'=0$, and there is no turbulence, an absolute instability exists. On the other hand, for $\kappa'\neq 0$ and in the absence of turbulence, the absolute instability is replaced by saturation of an initial pulse at a value $\exp(\pi\lambda)$. We interpret this saturation, at a fixed point in space, as due to destructive interference by the waves. We expect that the effectiveness of this interference would be reduced if phase incoherence were introduced by spatial turbulence; and that the saturation could be eliminated, with absolute instability again obtained, for a sufficient level of turbulence.

Spatial turbulence is characterized by an amplitude Δ and a correlation length L_T. We take the correlation function to be statistically uniform and Gaussian:

$$\langle \delta\kappa(x)\, \delta\kappa(x')\rangle = \Delta^2 \exp[-\tfrac{1}{2}(x-x')^2/L_T^{\,2}]. \tag{3}$$

Consistent with Eq. (3), we represent the random function $\delta\kappa(x)$ as

$$\delta\kappa(x) = (32\pi)^{1/4}(L_T/L)^{1/2}\Delta \sum_{j=1}^{\infty} \exp[-(k_j L_T)^2/4]\sin(k_j x + \alpha_j), \tag{4}$$

where $k_j = 2\pi j/L$; L is a length much larger than L_0, L_T, and the pulse width at all times of interest; α_j is a random phase, with probability uniformly distributed between zero and 2π; and the upper limit of summation is taken to be large,[15] such that $(k_j)_{\max} L_T \gg 1$. For a given realization of $\{\alpha_j\}$, and a particular set of parameters, the total mismatch gradient $d\kappa(x)/dx \equiv \kappa' + d[\delta\kappa(x)]/dx$ is illustrated in Fig. 1(a).

Given this model, the coupled-mode equations (1) are integrated numerically to determine the effect of the spatial turbulence on the response of the system to an initial perturbation. The main result of this study is that if Δ exceeds a threshold value (dependent on L_T), the instability no longer saturates at a value $\sim\exp(\pi\lambda)$, but grows exponentially at fixed x for all time, at a growth rate γ_a lower than that for a nonturbulent homogeneous medium. In Fig. 1(b) we show the temporal development of a typical unstable case with initial conditions $a_1(x, t=0) = \delta(x)$, $a_2(x, t=0) = 0$. Fluctuations reminiscent of Ref. 7 are observed, but with a less regular character. The most unstable part of the pulse has the behavior of a temporal normal mode, maintaining its shape while growing exponentially.

In Fig. 1(c) we show the absolute growth rate

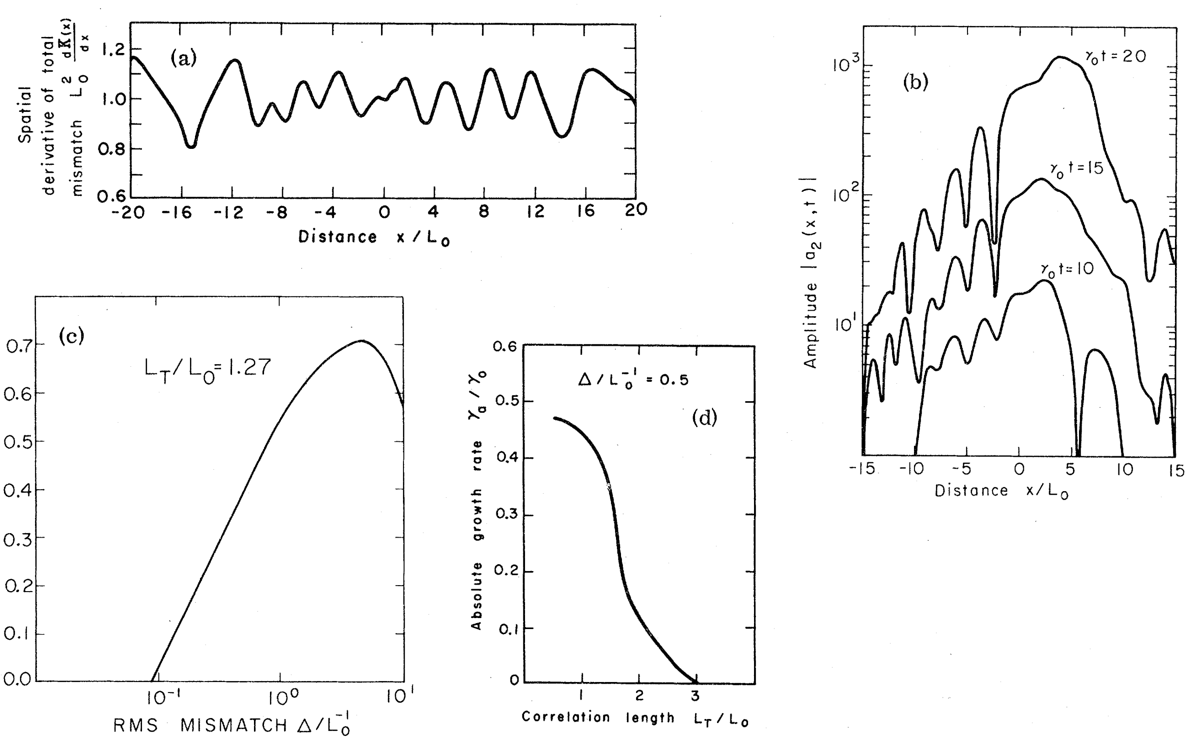

FIG. 1. $V_2/V_1 = -1$, $\lambda^{-1} \equiv \kappa' L_0^{\,2} = 1$ $(L/L_0 = 400)$. A particular realization of the set $\{\alpha_j\}$ is used. (a) The function $L_0^{\,2} d\kappa(x)/dx$ versus x/L_0 at the threshold value $\Delta/L_0^{-1} \approx 0.1$ in (c). $L_T/L_0 = 1.27$. (b) The temporal evolution of $|a_2(x,t)|$ versus x/L_0 for the initial conditions $a_1(x, t=0) = \delta(x)$, $a_2(x, t=0) = 0$. $\Delta/L_0^{-1} = 0.5$, $L_T/L_0 = 1.27$. (c) The absolute growth rate γ_a/γ_0 versus the rms mismatch function Δ/L_0^{-1}. (d) The absolute growth rate γ_a/γ_0 versus the correlation length L_T/L_0.

γ_a/γ_0 versus Δ/L_0 for $V_1/V_2 = -1$, $\lambda^{-1} \equiv \kappa' L_0^2 = 1$, $L_T/L_0 = 1.27$. The threshold turbulence level is seen to occur at $\Delta/L_0^{-1} \approx 0.1$. The maximum growth rate is $\gamma_a/\gamma_0 \approx 0.70$, which is comparable to the homogeneous growth rate γ_0.

The function $d\kappa(x)/dx$ shown in Fig. 1(a) corresponds to the threshold case of Fig. 1(c). This function is seen to lie in the range $0.80 < L_0^2 d\kappa(x)/dx < 1.20$. This shows that the coupled-mode equations can produce absolute instability even if $d\kappa(x)/dx$ vanishes nowhere in the medium, in contrast to the result of Kaw *et al.*[14]

In Fig. 1(d) we show the growth rate γ_a as a function of correlation length L_T, for fixed fluctuation level Δ. For this calculation we use the same realization of the set $\{\alpha_j\}$ in Eq. (4), varying L_T with $\Delta/L_0^{-1} = 0.5$. We see that the absolute growth rate decreases with increasing correlation length.

We have also considered parameters corresponding to Raman backscattering[16] in a laser-pellet geometry.[11] Results similar to those above were found.

It should be noted that in this work the turbulent wavelengths are quite long, the shortest being equal to the standard length $L_0 \equiv |V_1 V_2|^{1/2}/\gamma_0$. A further point is that for a given value of Δ, the absolute growth rate depends somewhat on the realization of $\{\alpha_j\}$ chosen in Eq. (4). The relative dispersion of the growth rates is of the order of 30–40%.

We interpret our results as follows. The convective saturation of the linearly inhomogeneous coupled-mode problem,[5,7] with oppositely directed group velocities, seems to be due to destructive interferences between responses originating at large positive and negative positions. This interpretation is supported by the work of White *et al.*,[17] who found that replacing the constant pump [γ_0 in Eq. (1)] by a Gaussian in x resulted in absolute instability, when the Gaussian width was larger than L_0 but smaller than $(K'L_0)^{-1}$; i.e., removing the responses at large x removed the destructive interference at $x = 0$. The analogy in our work is that the turbulence upsets the delicately balanced destructive interferences, allowing the system to grow absolutely.

We conclude that the presence of long-wave turbulence tends to destabilize the convective saturation found[5,7] for the coupled-mode equations, with oppositely directed group velocities, in an inhomogeneous medium. This destabilization occurs at relatively small turbulence levels—so small that the condition $\partial\kappa(x)/\partial x = 0$ is never satisfied.

For useful discussions we gratefully thank B. Cohen, C. N. Lashmore-Davies, G. Johnston, P. Kaw, Y. C. Lee, C. S. Liu, C. E. Max, M. Mostrom, M. N. Rosenbluth, C. Oberman, G. Smith, and R. White.

*Work supported in part by the U. S. Atomic Energy Commission.

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Influence of Intense ac Electric Fields on the Electron-Ion Collision Rate in a Plasma*

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The classical electron-ion collision rate, ν_{ei}, has been deduced in the absence of Ohmic heating from the inverse bremsstrahlung absorption rate measured on a laboratory plasma subjected to an intense microwave electric field E whose frequency ω is near the electron plasma frequency ω_p. The observations, carried out in the transition range, where $eE/(m\omega) \approx (kT_e/m)^{1/2}$, yield a simple empirical relation that describes the decrease of ν_{ei} with increasing E.

Current interest in the heating of several plasma configurations[1,2] to thermonuclear temperatures centers on efficient absorption of electromagnetic energy by plasma particles. In these cases the time-dependent absorption goes hand in hand with other important transport processes, such as heat flow and electron-ion energy equilibration. Although such particle heating is well known to influence other classical transport processes indirectly through their temperature dependence, it is generally not recognized that the application of intense electromagnetic heating fields may also directly modify classical transport parameters on a time scale that is much shorter than the heating time, without generating plasma instabilities. This effect arises when the electromagnetic field is intense enough to produce significant modifications in the hyperbolic orbits traversed during Coulomb encounters and thus occurs as soon as these fields begin to oscillate. Several theoretical treatments[3,4] predict such an orbit-modification–electric-field effect for the classical ac electrical resistivity or, equivalently, the classical inverse bremsstrahlung absorption rate.[5,6] It is the purpose of this Letter to report the first experimental observation of this effect. Our measurements of inverse bremsstrahlung absorption yield an electric-field-dependent electron-ion momentum-transfer collision rate which can also be used to estimate the dependence of other transport parameters upon the strength of applied ac fields. On the basis of these observations we conclude that the application of intense electromagnetic heating fields to a long magnetized plasma column[2] could significantly reduce absorption and equilibration rates and could significantly increase the electron-thermal conduction loss to the column ends compared to the generally accepted values that apply only in the absence of an electromagnetic field. In the case of the laser-illuminated DT pellet experiment,[1] increased heat flow could occur in the pellet's low-density plasma ablation cloud, and this would reduce the requirements for symmetric laser illumination imposed by the need for a high-compression spherical implosion.[7]

Our experiment measures microwave absorption by a magnetized 110–150-cm-long potassium plasma column contact-ionized on a single hot plate at 2250°K. The plasma[8] is fully ionized, has a nearly trapezoidal density profile $n(r)/n(0)$ with a mean diameter of 2.5 cm, and $\omega_{ce}/\omega_{p0} \approx 6-10$. Microwaves are stored in an 8.6-cm-long cylindrical TM_{010} cavity that is coaxial with the plasma column, resonates near $\omega/2\pi = 2$ GHz, and has a loaded Q of 21 000 without plasma. The measured change in Q due to plasma, expressed as $\Delta(1/Q)$, is independent of applied magnetic field B and is given by[9]

$$\Delta(1/Q) = \int\left[\frac{\omega_{p0}{}^2 n(r)}{\omega^2 n(0)}\right]\left[\frac{\nu_{ei}(E,T)n(r)}{\omega n(0)}\right]E^2\,dV \times\left[\int E^2 dV\right]^{-1}. \quad (1)$$

Here ω_{p0} is the electron-plasma frequency at $r=0$, E is the microwave electric field, and $\nu_{ei}(E, T)$ is the electric-field- and temperature-dependent electron-ion momentum-transfer colli-