is indeed observed.

The ratio of the two in-plane diffusion coefficients in smectic-C systems is finally obtained as

$$(D_\perp{}^0/D_\perp{}^{0\prime}) = \exp[-(U_\perp - U_\perp{}')/kT] \qquad (10)$$

and is expected to be relatively close to 1.

It should be mentioned that within a "free-flow" model of the smectic-A phase the ratio of the "out-of-plane" (D^0) diffusion constants is obtained as

$$\left(\frac{D_\parallel{}^0}{D_\perp{}^0}\right) = \left(\frac{l_\parallel}{l_\perp}\right)^2 \exp\{-[(U_\parallel - U_\perp)/kT]\}, \qquad (11)$$

where $l_\parallel$ and $l_\perp$ stand for the mean free path for the "out-of-plane" and "in-plane" motion. If the two-dimensional free-flow model should have any meaning $l_\perp$ should be much longer than $l_\parallel$ so that the ratio $(D_\parallel{}^0/D_\perp{}^0)$ should be much smaller than observed experimentally or predicted by Eq. (9).

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Evidence for Upward or "Anomalous" Dispersion in the Excitation Spectrum of He II

R. C. Dynes and V. Narayanamurti
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 12 June 1974)

The dispersion and attenuation of superthermal phonons ($\hbar\omega \gg kT$) in He II at 0.1 K are remeasured and reinterpreted as a function of pressure. We find evidence for the existence of the three-phonon process and an upper energy cutoff to this process implying anomalous dispersion. Our earlier conclusion on the absence of anomalous dispersion at saturated vapor pressure is shown to be incorrect.

In order to explain ultrasonic data,[1] it was first suggested by Maris and Massey[2,3] that the $\omega(k)$ relation in He II at saturated vapor pressure (SVP) had a small but finite upward curvature in the energy range up to ~8 K. Recently, we reported group-velocity measurements[4] ($v_g = d\omega/dk$) of propagating phonons generated by a superconducting Al film and it was concluded that the dispersion relation was linear below 4.5 K. This result disagreed not only with other data,[5] but also with theoretical[3,6] estimates of the mean free path for these phonons [under decay via the allowed three-phonon process (3pp) where one phonon decays into two] which should be substantially shorter than our propagation length (a few millimeters) at SVP. It remained vital, then, to show that the phonons measured were indeed of the energy range 0.5 K to 4 K. We report here new measurements and a reappraisal of these earlier measurements which imply that phonons of energy *greater* than this, generated by quasiparticle relaxation, escape from the Al film and are long lived in the He II. It is these higher-frequency phonons which were detected and measured.

By studying the response of a detector superconducting tunnel junction as a function of the bias of a generating junction separated by a medium, one can determine whether phonons of energy $\hbar\omega = 2\Delta$ (the superconducting energy gap) propagate through that medium. At generator bias voltages $V=2\Delta/e$ and $4\Delta/e$, changes in the slope of the detector signal S should be observed. The change at $2\Delta/e$ corresponds to the detection of phonons generated by the quasiparticle recombination process, and at $4\Delta/e$ to a quasiparticle relaxation to the superconducting-gap edge via phonon emission. At a bias of $V=4\Delta/e$, the maximum relaxation phonon energy is 2Δ, the minimum detectable energy of a similar tunnel junction.

In our earlier paper[4] we had attempted to look for such structure in the S-versus-I (generator current) curves at 24 bar and at SVP. The signal at SVP was interpreted this way but was sufficiently weak below $V=4\Delta/e$ to raise serious doubts. Here we alleviate this problem by measuring the derivative dS/dI as a function of the generator bias by using phase-sensitive modulation techniques. Some improvement in signal was also achieved through a reduction of the generator-detector separation to 0.7 mm. The thin-film (~1000 Å) Al-oxide-Al tunnel-junction generator and detector were evaporated on glass substrates. The normal-state resistances were ~0.5 Ω and $2\Delta_{\rm Al}\approx 0.38$ meV. The results of this experiment, for various pressures up to the solidification point and $T=0.1$ K, are shown in Fig. 1 where we plot dS/dI as a function of the generator voltage above the energy gap ($eV-2\Delta$). At 24 bar it is generally believed that the dispersion is normal and hence that the 3pp is inoperative. Thus one expects propagation and detection of phonons $\hbar\omega=2\Delta_{\rm Al}$ and this is borne out by the data. At 0 and 0.38 meV ($eV-2\Delta_{\rm Al}=2\Delta_{\rm Al}$) we see steps in dS/dI corresponding to changes in the slope of S. The peak at low energies (0.05 meV) is probably due to direct quasiparticle recombination and need not concern us here.

Reducing the pressure we see dramatic changes. At 12 bar, the step at 2Δ diminishes in strength and broadens to higher energies. The step at 0 disappears. We interpret this as the onset of the 3pp for phonons of energy 4.5 K. By reducing the pressure further, the "2Δ" step becomes much weaker and occurs at higher energies. This implies that phonons of energies greater than 2Δ escape from the Al film and that those of energy less than the step are strongly scattered.[7] The step gives us a measure of the energy at which the 3pp cuts off (E_c). As the pressure is reduced to SVP the onset increases in energy to 0.8 meV or 9.5 K. Above $T=0.3$ K the structure at 2Δ at 24 bar disappears. In this case the generated phonons decay rapidly and approach a thermal distribution.

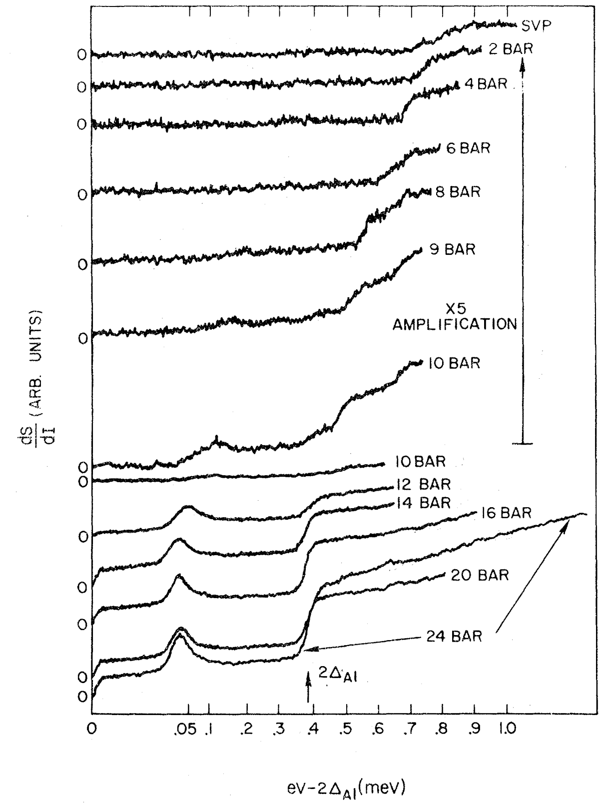

FIG. 1. dS/dI versus generator voltage for various pressures. Note that the nonlinear voltage scale is measured from $2\Delta_{\rm Al}$ and note the change of scale at 10 bar.

These results indicate that in our earlier experiments at SVP and 0.1 K, the detected phonons were not those of energy $2\Delta_{\rm Al}$ but of higher energy (0.8 meV). These were not as efficiently down-converted[8] to 2Δ as they would be in a stronger electron-phonon–coupled superconductor such as Sn.[9] Tuning 2Δ down by magnetic field had little influence on these high-frequency phonons and no change in velocity with field was observed. For phonons of energy $2\Delta_{\rm Al}$ the 3pp was indeed operative. As the pressure was increased to 12 bar, E_c decreased and the medium became transparent for phonons of energy $2\Delta_{\rm Al}$. This resulted in a sharp rise of the signal at 12 bar. These data, then, imply that the 3pp acts as a filter with a high-frequency cutoff. At SVP this cutoff E_c is measured from the data of Fig. 1 to be 9.5 K and corresponds to phonons of wave vector $k_c=0.51$ Å^{-1}. Phonons with $k>k_c$ are not subject to the 3pp and are thus detected. From detailed measurements of the type shown in Fig. 1 and magnetic tuning of the superconducting energy gap, E_c and k_c as a function of pressure can be extracted. The existence of the 3pp and a well-defined cutoff energy imply an upward curvature to the excitation spectrum. In addition, from pulsed time-of-flight measurements the group velocity of these phonons at k_c can be measured and comparisons with existing models for the excitation spectrum can be made. These detailed analyses and comparisons will be the subject of a future publication.[10]

In summary we have direct evidence for the existence of the 3pp operative in an energy range up to a cutoff E_c in the excitation spectrum of He II. This implies that upward dispersion does exist. Our previous conclusions from pulsed measuremens were erroneous as we were detecting phonons of energy $E > E_c$. From detailed measurements of this type we can directly compare results with various $E(k)$ models for He II.[10]

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Instability of a Vortex Array in He II*

William I. Glaberson, Warren W. Johnson, and Richard M. Ostermeier
Department of Physics, Rutgers University, New Brunswick, New Jersey 08903
(Received 28 August 1974)

We propose an instability of a vortex array in the presence of axial normal-fluid flow in He II. This instability may be responsible for the influence of thermal counterflow on ion-trapping cross sections reported by Cheng, Cromar, and Donnelly.

It is generally accepted that, upon rotation, He II is threaded by an array of quantized vortex lines parallel to the axis of rotation.[1] This array exhibits oscillation modes which in the long-wavelength limit are similar to classical inertial waves and in the short-wavelength limit are related to the waves of isolated vortex lines. Hall[2] predicted these modes on the basis of a set of phenomenological two-fluid equations and was able to observe the modes and, from their dispersion relation, to determine the vortex core parameter. In this paper we use the Hall[3] equations to investigate the normal modes of a vortex array in the presence of axial normal-fluid flow. We find that some of the modes become unstable when the velocity of the axial flow exceeds some (rather small) critical value. This implied disruption of the vortex array may be the explanation for the anomalous ion-trapping results reported by Cheng, Cromar, and Donnelly[4] and may be relevant for an interpretation of some results of Williams and Packard.[5]

We begin by considering an apparently unrelated situation, that of a vortex ring in counterflow.[6] Viewed in the frame of reference in which the superfluid is at rest at infinity, a ring having radius R travels with velocity

$$V = (\kappa/4\pi R)\ln(8R/a - \tfrac{1}{2}), \quad \kappa = h/m, \tag{1}$$

where h is Planck's constant, m is the mass of a helium atom, and a is the core parameter (~1 Å). If the normal fluid is at rest in this frame, dragging of the vortex ring through the normal fluid causes the ring to lose energy steadily, and hence to decrease in size. If, on the other hand, the normal fluid is moving faster than (and in the same direction as) the ring, it is clear that energy is being added to the ring and, because of the peculiar dynamics of a ring, it will continue to grow. Equation (1) then represents the critical