cm^{-1}. The line shape allows one to determine the approximate magnitude and the sign of $q_{j'} = P_{\Phi j'}/\pi V_E{}^* P_{\psi_E j'}$ for two different intermediate states, j'. We do not yet understand why the experimental shapes have higher wings than the theoretical.

Thus, we have a new technique for studying autoionizing levels. By determining q for transitions between an autoionizing level and several intermediate states (j'), one can gain information to supplement what is known about the level from absorption spectroscopy. This can lead to new classifications for the quantum numbers of the discrete-state component of the autoionizing level. An accurate relative measurement of $|\chi^{(3)}|^2$ using different j' levels determines the $q_{j'}$ and the ratios of the $P_{\Phi j'}$. To do this accurately requires a knowledge of the coherence length for the parametric mixing process,[1] the absorption coefficients at ν_1, ν_2, and ν_{vuv} as functions of frequency, and a measurement of the intensities of the light at ν_1, ν_2, and ν_{vuv}. For our measurements in Sr, the weak oscillator strength of the autoionizing transition and its broad linewidth resulted in negligible contributions to the coherence length and absorption coefficient, so that the observed line shape was characteristic of $|\chi^{(3)}(\nu)|^2$ alone, but we did not measure the relative intensities.

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Low-Temperature Studies of the Rayleigh-Bénard Instability and Turbulence

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Low-temperature techniques were applied to the study of a hydrodynamic instability. High-precision results for the Nusselt number N as a function of the Rayleigh number R for liquid and gaseous helium revealed no singularities in $N(R)$, except at the convective threshold R_c. For $R > 2.19R_c$, a new turbulent state was found and characterized by measuring the frequency spectrum and the amplitude of $N(R)$.

I wish to report on a number of quantitative experimental results relating to heat transport by thermal convection at low temperatures in liquid and gaseous He^4. The measurements were made on a horizontal layer of the fluid heated from below. They thus pertain to the Rayleigh-Bénard instability,[1] a particularly simple case of a hydrodynamic instability which has caused considerable interest[2-6] among physicists recently. The present work exploits some of the experimental advantages of low-temperature techniques[7] which permit thermal measurements of very high resolution and great accuracy. In addition to providing accurate measurements of the onset of convection and of the heat transport by the fluid under a wide range of conditions, the experiments reveal a transition to, and provide a quantitative description of, a new turbulent state. The properties of this state are described rather well by a theory developed recently by McLaughlin and Martin.[6]

The apparatus has been described in detail elsewhere.[7] Measurements were made on samples contained in the part of the apparatus referred to as the "probe" in Ref. 7. These samples had cylindrical symmetry, with a height h of 0.088 ± 0.002 cm and a diameter D of 0.927 ± 0.002 cm. Their top and bottom boundaries were provided by isothermal copper plates having thermal relaxation times of 10^{-3} sec. The walls consisted of 0.013-cm-thick stainless steel. All heat-conductivity measurements were corrected for wall conduction. One of the advantages of the low-temperature environment is the virtual absence of any other parallel heat-transport mechanisms.

The effective thermal conductivity λ_{eff} was determined by imposing a time-independent heat current Q, and measuring the temperature increase ΔT of the bottom plate while holding constant the temperature of the top plate. One can express λ_{eff} in terms of the Nusselt number $N = \lambda_{eff}/\lambda$, where λ is the thermal conductivity of the fluid at rest. It is expected[1] that N is a function of only two independent dimensionless parameters, the Rayleigh number

$$R = g\alpha_P \Delta T h^3/\nu\kappa , \tag{1}$$

and the Prandtl number $\sigma = \nu/\kappa$.[1] Here g is the gravitational acceleration, α_P the isobaric thermal expansion coefficient, ν the kinematic viscosity, and κ the thermal diffusivity. In the present experiments, I could obtain $0.6 \lesssim \sigma \lesssim 1.4$. The results exhibited no dependence upon σ.

It is expected[1] that $\lambda_{eff} = \lambda$ for all R less than some critical value R_c because in that case the fluid remains at rest. For $R > R_c$, $\lambda_{eff} > \lambda$ because there is a contribution to the heat transport from fluid flow. Experimental values of N are shown as a function of $r \equiv R/R_c$ in Fig. 1. Although the region $r < 1$ is not adequately represented in the figure, the temperature resolution of about 10^{-7} K made it possible to measure N with a precision of 0.1% or better for $r \gtrsim 0.01$. For all $r \lesssim 0.93$, N was equal to unity within experimental error.

Much of the theory of hydrodynamic stability employs an approximation due to Boussinesq,[1] in which it is assumed that ΔT is small in the sense that all the parameters which enter into R may be regarded as constant. In order to describe the effect of departures from the Boussinesq approximation, I define a parameter $B \equiv \delta R/\langle R\rangle$. Here $\delta R = |R_b - R_t|$, where R_b and R_t correspond to the fluid at the hot and cold boundaries, respectively, and $\langle R\rangle$ is an average based upon the mean values of the properties of the fluid. By

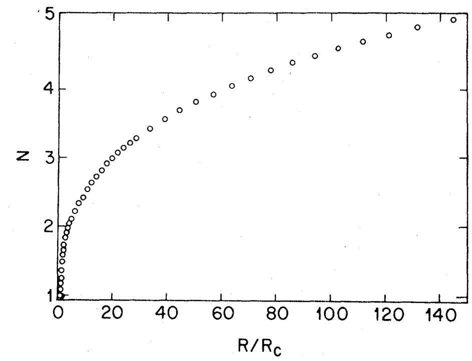

FIG. 1. The Nusselt number N as a function of the reduced Rayleigh number $r = R/R_c$.

varying the sample pressure and temperature, I could investigate the range $0.01 \lesssim B \lesssim 0.66$, and I found that $\langle R\rangle_c$ was constant within a random scatter of 1%. Systematic errors in $\langle R\rangle_c$ due to systematic errors in ν and h were large, yielding $\langle R\rangle_c = 1840 \pm 150$, consistent with the theoretical value.[1]

For the range $1.07 \leqslant r \leqslant 2.5$, the results for N in a particular sample ($B = 0.016$, $\sigma = 1.17$) could be represented within a random scatter of 0.1% by $N - 1 = f(\epsilon)$, with

$$f(\epsilon) = 1.034\epsilon + 0.981\epsilon^3 - 0.866\epsilon^5, \tag{2}$$

where $\epsilon \equiv 1 - R_c/R$. Results for all other samples differed from Eq. (2) by no more than possible systematic experimental errors, which for large B tended to be as large as 2% because of the large Q and ΔT involved in the measurement. Equation (2) can be compared with measurements for fluids with $\sigma \gtrsim 450$ by Pallas,[8] whose sample also had cylindrical symmetry.[9] His values are higher than Eq. (2) by about 9% of $N - 1$. Although the reason for the difference is not known, the large difference in σ between the two investigations should be noted. Equation (2) is consistent with the theoretical prediction that $N - 1 \sim \epsilon$,[1] but inconsistent with the analysis of recent light-scattering measurements[5] which would correspond to $N - 1 \sim \epsilon^{1.2}$.

Although one would expect Eq. (2) to be valid for $\epsilon \gtrsim 10^{-5}$,[3] I find a rather large range $0.95 \lesssim r \lesssim 1.05$ over which the data for N are "rounded." The contribution δN, equal to $N - f(\epsilon)$ for $r > 1$ and to $N - 1$ for $r \leqslant 1$, can be represented by

$$\delta N = 0.025 \exp[-(\epsilon/0.0554)^2]. \tag{3}$$

Within 0.1% of N this "rounding" is independent of B. It is possible, however, that this effect is

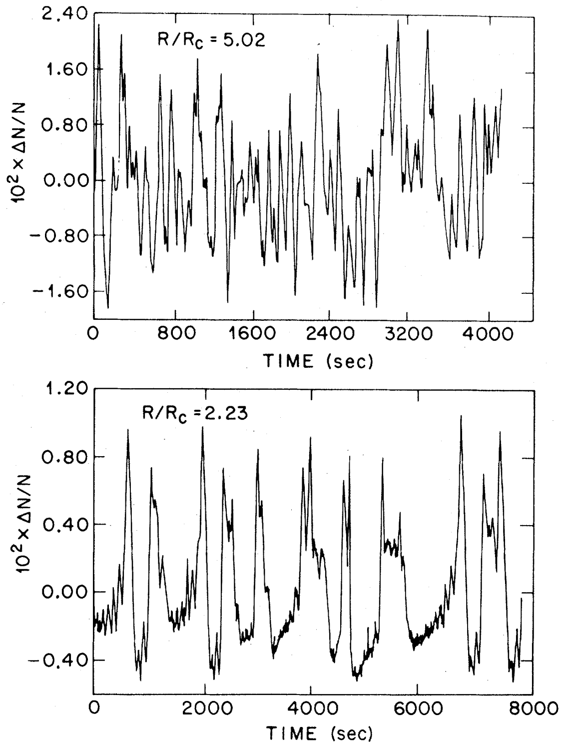

FIG. 2. Time dependence of N for two values of R/R_c.

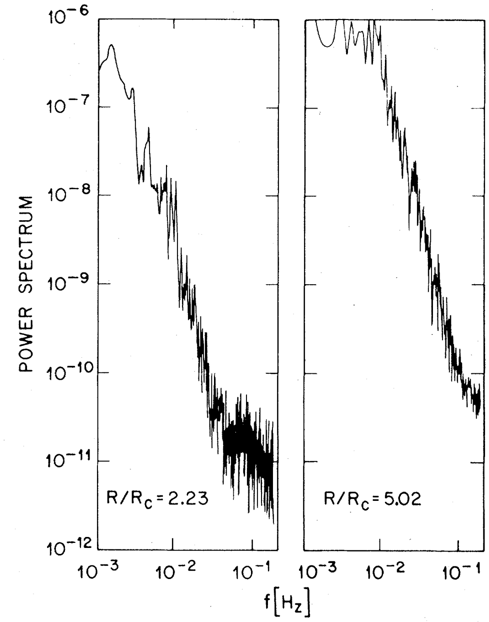

FIG. 3. The power spectrum [Fourier transform of $N(t)$] for two values of R/R_c.

peculiar to my particular samples, caused for instance by a slight variation of h in the horizontal plane.

For $30 \lesssim r \lesssim 150$, the data can be represented by

$$N - 1 = 0.77(r-1)^{0.334}. \tag{4}$$

The exponent in Eq. (4) is somewhat higher than most of those reported previously[10]; but perhaps my value of r was not sufficiently large to determine the value pertinent to the large-r limit.

There have been persistent reports[11-13] of heat-flux transitions which manifest themselves as singularities in $N(R)$. I have examined the data by plotting the deviations from Eqs. (2) and (4) on high-resolution graphs. Within a precision of 0.1%, the data can be represented by a function with a continuous derivative. Thus, there is no evidence for discrete heat-flux transitions over the range $1 < r \lesssim 150$, although with the resolution of the present experiment I should easily have seen singularities of the type reported by others.[11-13]

At $r = r_T \cong 2$, a transition occurred for all the samples from a region where N at constant heat current Q was independent of time to a region where N was time dependent. I shall refer to the time-dependent state as being turbulent. The possibility of observing the time dependence without inserting disturbing probes into the fluid is another of the advantages of the low-temperature experiment. The heat capacity of the end plates was negligible compared to that of the fluid, and the temperature of the hot plate readily followed fluctuations in the liquid. The high thermal conductivity of copper assured that the fluctuations were averaged in the horizontal plane. I investigated the turbulent state in detail at 4.515 K and at a pressure of 2.38 bar ($B = 0.01$, $\nu = 2.9\times10^{-4}$ cm^2/sec, $\kappa = 3.36\times10^{-4}$ cm^2/sec). I found $r_T = 2.19 \pm 0.03$. For $r > r_T$, I used the time average of N in Fig. 1 and for Eqs. (2) and (4). The fractional deviations from the mean values are shown in Fig. 2 for two values of r. Amplitudes were typically of the order of 0.5% of N, and fluctuations became more rapid with increasing r. For all $r < r_T$, there was no measurable time dependence of N although the experimental noise was about a factor of 20 smaller than the amplitude of $N(t)$ for r just above r_T. Since the time dependence was not periodic, I examined it in more detail by calculating the Fourier transform of $N(t)$. Each transform was based on approximately 10^3 data points, with sampling rates between 0.4 sec^{-1} at small r and 4 sec^{-1} at large r. Two such power spectra are shown in Fig. 3. In or-

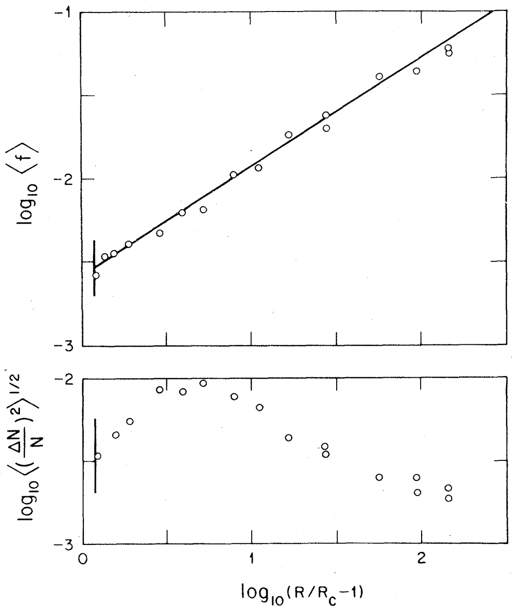

FIG. 4. The first moments $\langle f \rangle$ in hertz of the power spectra, and the rms amplitudes of $N(t)$, as a function of $R/R_c - 1$ on logarithmic scales. To the left of the vertical lines in the figures, there was no time dependence.

der to characterize them by a single frequency, I calculated their first moments $\langle f \rangle$ which are shown in Fig. 4. They could be described by

$$\langle f \rangle = 2.7\times 10^{-3}(r-1)^{0.65}\ \mathrm{Hz}. \tag{5}$$

Equation (5) extrapolates to $\langle f \rangle = 0$ for $r=1$; but the observed time dependence ceased discontinuously at r_T where $\langle f \rangle = 3.0\times 10^{-3}$ Hz. The amplitude of N, also shown in Fig. 4, is not readily described by a simple function; but it is also discontinuous at r_T and nonzero only for $r > r_T$.

The results for helium should be comparable to those obtained at higher temperatures for air, because air has the very similar Prandtl number $\sigma \cong 0.7$. For air, singularities in $N(R)$ have been reported, and on the basis of local temperature measurements in the fluid these singularities have been associated with transitions to turbulent states.[12,13] I have seen no singularities in $N(R)$, and the values of $\langle f \rangle$ in Fig. 4, when reduced by the vertical viscous diffusion time h^2/ν, tend to be an order of magnitude smaller than qualitative frequency estimates for air based on local temperature measurements.[12] The smallest r at which a singularity in $N(t)$ has been reported for air is 3.3,[12] which is a factor of 1.5 larger than r_T in the present experiment. There seems to be no convincing evidence that the previous observations pertain to the same state of the system as the present measurements. My observations are in rather good agreement with the theoretical results of McLaughlin and Martin.[6] These authors obtain a transition to a turbulent state for $r_T = 1.6$, with an amplitude of about 1.5% of N for $r > r_T$. They find a mean frequency which is within a factor of 2 or 3 of the experimental value, and very little change in the slope of $N(R)$ at r_T.

I am grateful to P. A. Fleury for calling my attention to this problem, to J. E. Graebner for the use of his automatic data acquisition system and for the calculations of the Fourier transforms, and to P. C. Hohenberg for numerous stimulating discussions.

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