Compared to the PRM it is attenuated by a factor

$$R=\frac{\mathcal{J}_0}{\mathcal{J}_0+\mathcal{J}_p}=1-\frac{\langle j_x\rangle}{[I(I+1)-\langle j_z^2\rangle]^{1/2}}. \qquad (13)$$

$\mathcal{J}_p=\langle j_x\rangle/\omega$ is the contribution of the outside particle (see Fig. 2). Taking into account that there was no fit parameter used ($\mathcal{J}_0$ is taken from $K0_c$), we find the agreement of this simple model with the experiment and with the fully self-consistent calculation surprisingly good. The last two columns in Fig. 3 are calculations within the PRM, without attenuation (column 7) and with a fit over four parameters (see Ref. 5).

The attenuation R is caused by the decoupling of the outside particle, which can be described very easily within the cranking model. It is strongly spin dependent (see Table I) and approaches 1 for high spin values. The "favored" states $I=\frac{5}{2}, \frac{9}{2}, \ldots$ are more attenuated than the unfavored ones $I=\frac{7}{2}, \frac{11}{2}, \ldots$. It should be emphasized that the attenuation [see Eq. (13)] is contained within the solution of the cranking model and there is no further parameter needed.

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Thermal Fluctuations and Experiments on the Free Fall of Electrons

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In order to study the gravitational force on an electron Witteborn and Fairbank have measured the acceleration of electrons moving along the axis of a vertical copper tube. I show that in experiments of this type there are large corrections to the motion of the electrons due to thermal fluctuations in the electromagnetic fields in the tube.

In a remarkable series of experiments Witteborn and Fairbank[1-5] (WF) have attempted to measure the gravitational free fall of electrons in vacuum. They studied the motion of electrons along the axis of a vertical "drift tube" of copper. Apart from known or applied electric fields, it was believed that all vertical electric or magnetic potential energy gradients had been reduced to below about 10^{-25} erg cm^{-1}. This was necessary to observe the effects of gravity, since for an electron in the earth's gravitational field the gradient of the gravitational potential energy is only 10^{-24} erg cm^{-1}. It had earlier been predicted by Schiff and Barnhill[6] that within a metal enclosure, itself in the earth's gravitational field, there should be an induced electric field of magnitude mg/e (m is the electron mass) directed downwards. This field arises because the electrons in a metal will be redistributed as a result of gravity. The Schiff-Barnhill field (SB) exerts a force on an electron inside a metal enclosure which exactly cancels the direct effect of gravity, and leads to zero net acceleration for a free electron. In the WF experiment a pulse of $\sim 10^8$ electrons was introduced at the bottom of the tube. These had a continuous distribution of velocities. The experiment consisted of determining the longest possible time τ_{max} for an electron to reach the top of the tube. It was assumed that τ_{max} was the transit time for an electron that started with just sufficient kinetic energy to reach the top of the tube. τ_{max} can then be simply related to the downward acceleration of the electron. Witte and Fairbank found that the acceleration of the electrons was zero to within an uncertainty of $\pm 0.09g$, thus giving agreement with the SB theory. They also made measurements of the change in τ_{max} when an additional force was exerted on the electrons by applying a voltage along the length of the tube. The dependence of τ_{max} on applied voltage was approximately as expected for particles having mass m, and ruled out the possibility that ions were being observed.

It was later pointed out by Dessler, Michel, Rorschach, and Trammell[7] (DMRT) that SB had made an approximation equivalent to assuming that the metal was incompressible. By allowing for compressibility DMRT found an electric field of order of magnitude Mg/e (M is the ion mass in the metal) directed *upwards* along the tube axis. Thus, in the presence of the DMRT field an electron should undergo a downward acceleration of the order of $(M/m)g$, in contrast to the less than $0.09g$ experimentally observed. This discrepancy has still not been resolved. Theoretical work by Herring[8] and others[9-15] has supported the conclusions of Dessler *et al.* Experimental evidence for the existence of the DMRT field has been obtained from measurements of the the contact potential of a metal as a function of applied stress.[16-21] Ways in which the DMRT field might be absent in the WF experiment have been discussed by Peshkin,[10] Craig,[17] Trammel and Rorschach,[12] and Schiff.[13] These proposals assume that the DMRT field is screened by electrons at or near the surface of the metal, and are subject to various assumptions about the details of the surface.

Here I wish to point out that in experiments of the Witteborn-Fairbank type it is necessary to consider carefully the thermal fluctuations in the electric field experienced by the electron. The mean-square value of the z component of the electric field in a cavity containing a thermal-equilibrium distribution of radiation at temperature T is

$$\langle E^2\rangle = 4\pi^3 k_B^4 T^4/45c^3\hbar^3. \qquad (1)$$

This assumes that the linear dimensions of the cavity are much greater than the wavelength λ_{th} of a typical thermal photon. The Witteborn-Fairbank experiment was performed at 4.2°K where $\lambda_{th} \sim 0.1$ cm. At this temperature the rms field is 3×10^{-6} statV cm^{-1}, giving the electron a typical acceleration of $\sim 10^{12}$ cm sec^{-2}. The electric field and the acceleration fluctuate around zero with a characteristic time τ_0 of the order of $\hbar/k_BT \sim 10^{-11}$ sec. To work out the effect of the electric field over macroscopic times I use the Langevin equation[22]

$$m\,dv/dt = -\alpha v + F(t) + F_{app}, \qquad (2)$$

where v is the velocity of the electron, $F(t)$ is the random force equal to $-eE(t)$, and F_{app} is a time-independent applied force such as gravity. The friction constant α is related to the force-force correlation function by

$$\alpha = (1/2k_BT)\int_{-\infty}^{\infty} ds\,\langle F(0)F(s)\rangle$$
$$= (e^2/2k_BT)\int_{-\infty}^{\infty} ds\,\langle E(0)E(s)\rangle. \qquad (3)$$

I consider initially the motion of an electron when $F_{app}=0$.

One may make an estimate of the correlation function for the electric field as follows. Suppose that at $t=0$ the cavity mode i has magnitude E_i and phase φ_i. Then for $t\geq 0$

$$E(t) = \sum_i E_i \cos(\omega_i t + \varphi_i)\exp(-\gamma_i t) + E'(t), \qquad (4)$$

where ω_i and γ_i are the real and imaginary parts of the frequency of mode i. $E'(t)$ represents the new fluctuations that also occur while the fluctuations present at $t=0$ are dying out. I assume that the components in $E'(t)$ have random phase relative to the initial fluctuations. If we now average over the phases φ_i, we obtain

$$\langle E(0)E(t)\rangle = \tfrac{1}{2}\sum_i E_i^2 \cos(\omega_i t)\exp(-\gamma_i t), \qquad (5)$$

$$\int_{-\infty}^{\infty} ds\,\langle E(0)E(s)\rangle = \sum_i E_i^2\gamma_i/(\gamma_i^2+\omega_i^2). \qquad (6)$$

Now introduce the quality factor Q of the modes defined in the usual way as $\omega_i/2\gamma_i$, and assume that this is independent of i. Then

$$\int_{-\infty}^{\infty} ds\,\langle E(0)E(s)\rangle \approx (\hbar/3k_BT)(1/2Q)\sum_i E_i^2$$
$$\approx 4\pi^3 k_B^3 T^3/135c^3\hbar^2 Q. \qquad (7)$$

Consider what happens to an electron placed into this fluctuating field at $t=0$ with zero initial velocity. There are three times of significance in the subsequent motion: τ_0 ($\sim 10^{-11}$ sec), the typical time that the random electric field remains of a given sign; τ_c ($\sim Q\tau_0$), the time over which the oscillation of a mode remains coherent; $\tau_{\rm th}$, the time for the electron to thermalize, i.e., acquire a mean-square velocity of k_BT/m. For times t much less than the thermalization time, the viscosity term in the Langevin equation is small. Then, the mean-square velocity of the electron is

$$\langle v^2(t)\rangle = (e^2/m^2)\int_0^t ds \int_0^t ds' \langle E(s)E(s')\rangle. \qquad (8)$$

If t is macroscopic (i.e., $t \gg \tau_c$) this gives[23]

$$\langle v^2(t)\rangle = (e^2t/m^2)\int_{-\infty}^{\infty} ds \langle E(0)E(s)\rangle = (4\pi^3t/135)e^2k_B{}^3T^3/m^2c^3\hbar^2Q = 1.66\times10^{12}t/Q \text{ cm}^2 \text{ sec}^{-2}. \qquad (9)$$

The thermal velocity of an electron at 4.2°K is $\sim 10^6$ cm sec^{-1}. Hence, the thermalization time $\tau_{\rm th}$ is numerically $\sim Q$ sec. The value for the friction coefficient α at 4.2°K is

$$\alpha = 1.2\times10^{-27}/Q \text{ dyn cm}^{-1} \text{ sec}. \qquad (10)$$

It is difficult to make a serious estimate of the effective Q of the cavity. The tube used in the Witteborn-Fairbank experiment was made by electroforming copper onto an aluminum cylinder that was later dissolved away. The "cavity" thus has open ends and an unknown surface impedance[24] at 300 GHz. In what follows I take $Q = 10^4$ although it is recognized that this may possibly be in error by as much as 2 orders of magnitude in either direction. Note that in the above analysis I have ignored the dependence of the correlation function for the electric field on space coordinates. This is permissible because the spatial range of the correlation function is $\lambda_{\rm th} \sim 10^{-1}$ cm, which is much larger than the distance ξ the electron travels in one coherence time τ_c. Even if $Q = 10^4$ and $v = 10^5$ cm sec^{-1}, ξ is still only 10^{-2} cm.

Consider now the combined effect of fluctuations and an applied force. (1) In the WF experiment electrons were observed after having spent a few tenths of a second in the tube. After 0.2 sec Eq. (9) predicts a rms *random* velocity due to fluctuations of 6×10^3 cm sec^{-1}. This compares with 200 cm sec^{-1} for the systematic part of the velocity if an acceleration of g is assumed, and 20 cm sec^{-1} for an acceleration of $0.1g$. Thus, *if the systematic acceleration of the electron is g* (gravity alone) *or less* (gravity plus Schiff-Barnhill field), *fluctuations completely dominate the motion of the electron.* For fluctuations to be a small effect, one would need a Q of 10^8 or greater. This is unlikely. For a Q of 10^4 and an acceleration of $0.1g$, the temperature must be lowered to 0.1°K before fluctuation effects can be neglected. (2) If the DMRT field is present, the effect of fluctuations is greatly reduced. An electron starting from rest in the DMRT field travels 100 cm in $\sim 10^{-3}$ sec and achieves a velocity of 10^4–10^5 cm sec^{-1}. The random velocity induced by fluctuations in 10^{-3} sec is only $\sim 4\times10^2$ cm sec^{-1}, and is therefore negligible. At a velocity of 10^5 cm sec^{-1} the friction force [see Eq. (10)] is equal to 10^{-26} dyn. This may be neglected since it is much smaller than the force due to the DMRT field ($\sim 10^{-20}$ dyn). Fluctuations could be important in the presence of the DMRT field only if the tube contained some radiation that had leaked in from a much higher temperature (e.g., liquid-nitrogen temperature).[25]

Finally, I try to summarize the implications these results have for the interpretation of the Witteborn-Fairbank experiment. Witteborn and Fairbank have argued that the small value they found for the electron acceleration implies that, for some unknown reason, the DMRT field is not present in the tube that they used. However, difficulties arise with this interpretation when the effects of fluctuations are included. In the absence of the DMRT field the acceleration of the electron is g or less and, as shown in (1) above, thermal fluctuations completely dominate the motion of the electron. The motion is then diffusive and some electrons will take a very long time to pass through the tube. A long transit time is not inconsistent with the results of WF. However, for diffusive motion there exists no definite upper cutoff on the transit time, where WF did observe a cutoff. Another problem concerns the way the cutoff time changes when a voltage is applied along the length of the time. Witteborn and Fairbank found that this dependence was approximately that expected for a *free* particle with mass equal to that of an electron. Even if an approximate cutoff time can be defined when the motion is diffusive, it is very unlikely that this time will have the dependence on applied voltage that was found by WF. At the moment we have no explanation for these differences between theory and experiment.

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[23]The relation between the present calculation and conventional electron-photon scattering theory is interesting. For photons of well-defined energy and momentum (i.e., infinite lifetime), electron-photon scattering can only occur in *second order* (Thomson scattering). For a photon gas at 4.2°K this mechanism gives a very long electron mean free path ($\sim 10^{13}$ cm for an electron of velocity 10^2 cm sec^{-1}). The mechanism considered in this paper corresponds to the normally unallowed *first-order* interaction. In the present case this contributes because of the finite lifetime of the photons, which relaxes the conservation laws. Thus the result obtained for $v^2(t)$ is proportional to e^2 (instead of e^4 for Thompson scattering), but vanishes as the photon lifetime becomes infinite ($Q \to \infty$). It is straightforward to give an alternative derivation of Eq. (9) using the scattering approach. Compared to Thompson scattering one gains a large factor of $m^2c^5\hbar/e^2k^2T^2 \sim 10^{21}$ by considering a first-order process, and loses the relatively insignificant factor of Q through not conserving energy exactly.

[24]The copper surface was believed to be amorphous (Ref. 5).

[25]I should like to thank Dr. Herring for suggesting this possibility.