Attenuation of the Coriolis Interaction within the Cranking Model*

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The description of strongly distorted rotational bands within the cranking model allows an interpretation of the attenuation factors used in the particle-plus-rotor model. It turns out that they are not very much influenced by the residual interaction, but are strongly dependent on the angular momentum. A simple model is proposed to calculate distorted spectra which is in rather good agreement with the experimental data and with the fully self-consistent calculation.

The description of very distorted rotational bands of odd-mass deformed nuclei is possible within the particle-plus-rotor model[1] (PRM) by the coupling of particles to the collective rotation using a Coriolis interaction. Practical calculations, however, allow a reproduction of the experimental data only by reducing the strength of this interaction. The attenuation factors R used for this purpose lie between 0.4 and 0.9.[2] There exist attempts to give an interpretation of these factors by taking into account the coupling of the outside particle to collective vibrations of the core.[3] Recently[4] it has been shown that the application of the cranking model within the framework of the Hartree-Fock-Bogolyubov (HFB) theory, which can be derived from a projection of the angular momentum before the variation,[5] allows a quantitative description of these bands without any fit parameter. In particular, no extra attenuation of the Coriolis term has to be introduced. In the present paper we show to what extent this description can be compared with the PRM and why in the latter model the Coriolis term has to be attenuated. A very simple model is introduced, where the outside particle is coupled to a cranked core. It is justified by the self-consistent calculation and agrees very well with the experiment. As a numerical example the very distorted band with positive parity in ^{159}Dy is investigated.

Within the cranking model the internal wave function φ_α of the odd nucleus is calculated by the variational equation

$$\langle\delta\varphi_\alpha|\hat{H}-\omega\hat{J}_x-E^\alpha|\varphi_\alpha\rangle=0. \tag{1}$$

If one restricts φ_α to the HFB functions, it corresponds to the blocked HFB equations[6] in the rotating frame. One has to look for solutions of this system which have odd particle-number parity[7] and which are eigenfunctions of a rotation about 180° around the x axis:

$$\exp(i\pi J_x)\varphi_\alpha=i(-1)^{I-1/2}\varphi_\alpha. \tag{2}$$

I is the total angular momentum and the cranking frequency ω is determined by the subsidiary condition[8]

$$\langle\varphi_\alpha|J_x|\varphi_\alpha\rangle^2+\langle\varphi_\alpha|J_z^2|\varphi_\alpha\rangle=I(I+1). \tag{3}$$

Equation (1) is solved directly in Ref. 4 for ^{159}Dy. For comparison with the particle-plus-rotor model, however, it is useful to decompose φ_α,

$$\varphi_\alpha=\gamma_\alpha^\dagger|\varphi_0\rangle=\textstyle\sum_K C_K^\alpha\beta_K^\dagger|\varphi_0\rangle, \tag{4}$$

where φ_0 is the underlying HFB wave function of the even core and $\beta_K^\dagger$ are the quasiparticle operators corresponding to this core, which diagonalize the Hamiltonian (H^{11} diagonal). For $\omega=0$, K is a good quantum number (it corresponds to the eigenvalue of J_z) because of the axial symmetry of the core. For higher ω this is not true. In the numerical calculation of ^{159}Dy (see Fig. 1) however, it turns out that for a large region of spin values ($I<\frac{21}{2}$) the core stays nearly axially symmetric and K is a rather good quantum number. The variation (1) is therefore decomposed into a variation of the core function φ_0 and a variation of the mixing coefficients C_K^α,

$$\langle\delta\varphi_0|\gamma_\alpha(H-\omega J_x)\gamma_\alpha^\dagger|\varphi_0\rangle=0, \tag{5}$$

$$\textstyle\sum_{K'}\{[\langle\varphi_0|H-\omega J_x|\varphi_0\rangle+E_K(\omega)]\,\delta_{KK'}-\omega(j_x^{11})_{KK'}\}C_{K'}^\alpha=E^\alpha C_K^\alpha; \tag{6}$$

j_x^{11} is the one-quasiparticle part of J_x corresponding to the operators $\beta_K^\dagger$. Equation (5) corresponds to blocked HFB equations for the determination of the core wave function φ_0 with even

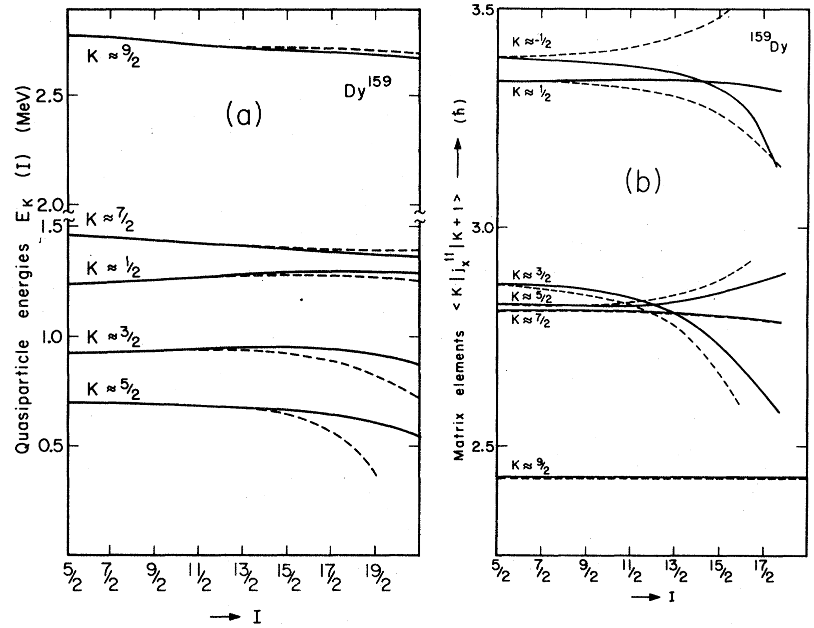

FIG. 1. The dependence of (a) the quasiparticle energies E_K, and (b) the matrix elements $j_{K\,K+1}{}^{11}$ of Eq. (6) on the angular momentum I; $K \approx -\frac{1}{2}$ corresponds to the decoupling parameter. Full lines correspond to the favored solutions, dashed lines to the unfavored solutions (see Ref. 9).

number parity. It is coupled by the blocking of γ_α to Eq. (6) which determines the mixing amplitudes $C_K{}^\alpha$.

The PRM replaces the calculation of φ_0 by assuming a rotor with a fixed moment of inertia. Equation (6) corresponds to the diagonalization of the PRM for the calculation of the mixing amplitudes.

Besides the fact that the cranking model gives energies in the rotating frame, there is a close analogy between Eq. (6) and the PRM concerning the amplitudes $C_K{}^\alpha$. (a) With the neglect of constants, the diagonal elements are in both cases essentially the quasiparticle energies E_K. In the cranking model they depend on ω, but only very weakly, as shown in Fig. 1(a). (b) The nondiagonal elements vanish exactly for $K \neq K' \pm 1$ in the PRM and approximately in the cranking model. In the latter model the frequency is $\omega = \langle\varphi_\alpha | J_x | \varphi_\alpha\rangle / \mathscr{I}_{\mathrm{sc}}$. $\mathscr{I}_{\mathrm{sc}}$ is the self-consistently determined moment of inertia. With regard to Eq. (3) the elements $K' = K+1$ are

$$-\frac{[I(I+1) - \langle J_z{}^2\rangle]^{1/2}}{\mathscr{I}_{\mathrm{sc}}}[j_x{}^{11}(\omega)]_{K\,K+1} \qquad (7a)$$

in the cranking model, and

$$-\frac{[I(I+1) - K(K+1)]^{1/2}}{\mathscr{I}_{\mathrm{rotor}}}[j_x{}^{11}(\omega=0)]_{KK+1} \qquad (7b)$$

in the PRM.[1] Both expressions are very similar. If one neglects the small ω dependence of the matrix elements $j_x{}^{11}$ in the cranking model [see Fig. 1(b)] and the fact that $[I(I+1) - K(K+1)]^{1/2}$ is replaced by $[I(I+1) - \langle J_z{}^2\rangle]^{1/2}$ in the cranking model, there remains only one big difference between both Coriolis interactions, explaining why one needs attenuation factors in the PRM but not in the cranking model: The cranking model uses a self-consistently determined moment of inertia $\mathscr{I}_{\mathrm{sc}}$, which includes the effect of the decoupling particle and which is strongly I dependent (see Fig. 2). For small I values, where the particle is coupled to the core, it is very easy to gain angular momentum in the x direction by decoupling the particle. Therefore, the value of $\mathscr{I}_{\mathrm{sc}}$ is large and the Coriolis interaction is strongly attenuated. This effect can also be seen in the simple Inglis formula for the odd nucleus in the state α:

$$\mathscr{I}_\alpha{}^{\mathrm{Ing}} = \sum_{K,K'\neq\alpha} \frac{|(J_x{}^{20})_{KK'}|^2}{E_K + E_{K'}} + \sum_{K\neq\alpha} \frac{|(j_x{}^{11})_{K\alpha}|^2}{E_K - E_\alpha}. \qquad (8)$$

The first part comes from the core. The second part describes the particle. Because of the small energy denominator it can become much larger than the first part. In the case of ^{159}Dy, we found $\mathscr{I}_{\mathrm{sc}} = 26.82 + 96.53 = 123.35$ MeV^{-1}.

However, for higher spin values a perturbation-theoretic treatment is no longer possible. The

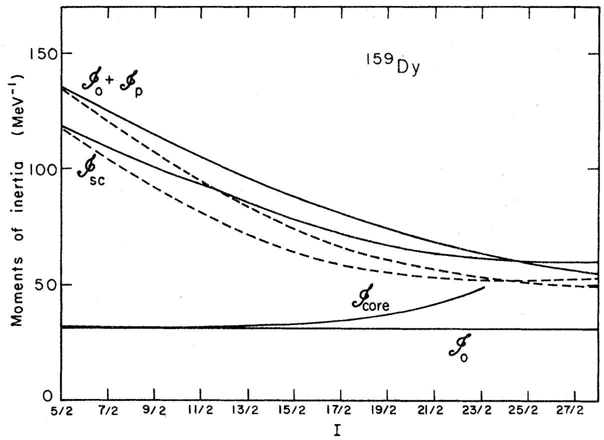

FIG. 2. Moments of inertia dependent on the angular momentum. $\mathscr{I}_{sc}=\langle\varphi_\alpha|J_x|\varphi_\alpha\rangle/\omega$ and $\mathscr{I}_{core}=\langle\varphi_0|J_x|\varphi_0\rangle$ correspond to the many-body wave function ($K0_c$ in Fig. 3). $\mathscr{I}_0$ and $\mathscr{I}_0+\mathscr{I}_p$ correspond to the particle-plus-cranking model [see Eq. (10)].

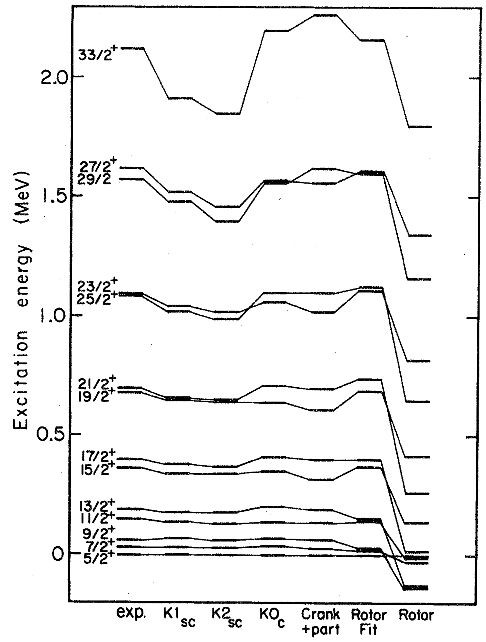

FIG. 3. The positive-parity band in ^{159}Dy: Experiment (see Ref. 10) and different calculations as described in the text.

exact solution (see Fig. 2) shows that the particle is more and more aligned and its contribution to the moment of inertia becomes smaller and smaller. Therefore the self-consistent moment of inertia $\mathscr{I}_{sc}$ diminishes with increasing spin. Only for very high spin values should it increase again because of the antipairing and the stretching effect of the core.

Figure 3 shows the experimental spectrum of the positive-parity band in ^{159}Dy and different calculations. $K1_{sc}$ is the fully self-consistent solution of Eq. (1) as described in Ref. 4. It uses a pairing-plus-quadrupole force including the exchange term of the quadrupole-quadrupole force, its contribution to the pairing potential, and the contributions of the pairing force to the self-consistent field. $K2_{sc}$ uses a similar force, which does not include the latter three terms and which is adjusted to reproduce the same energy gap and the same deformation:

	$Q_p=Q_n$	Q_{pn}	G_p	G_n	ΔE_{n+} (MeV)
K1	-0.034	-0.089	-0.190	-0.139	-0.25
K2	-0.034	-0.089	-0.195	-0.148	-0.30

Units and details are given in Ref. 4.

In the column $K0_c$ the influence of the residual interaction is neglected; i.e., the calculation is done within constant fields Γ and Δ taken from $K2_{sc}$ at $\omega=0$. This procedure changes the behavior of the spectrum at very high spin values. However, the attenuation of the Coriolis interaction is only very little influenced by the residual interaction.

We studied it in the following simple model (column labeled crank + part in Fig. 3) suggested by Eq. (6). One outside particle is coupled to a rotor with moment of inertia $\mathscr{I}_0$:

$$\langle\varphi_0|H-\omega J_x|\varphi_0\rangle=\tfrac{1}{2}\mathscr{I}_0\omega^2. \quad (9)$$

Neglecting the ω dependence of E_K and $j_{KK'}$[11] one has to diagonalize

$$\tfrac{1}{2}\mathscr{I}_0\omega^2+E_K-\omega(j_x^{11})_{KK'}. \quad (10)$$

The subsidiary condition for ω is

$$\mathscr{I}_0\omega+\langle j_x\rangle=[I(I+1)-\langle j_z^2\rangle]^{1/2}. \quad (11)$$

Therefore the Coriolis interaction can be written as

$$H_{\mathrm{Cor}}=-\omega j_x^{11}=-\frac{[I(I+1)-\langle j_z^2\rangle]^{1/2}j_x^{11}R}{\mathscr{I}_0}. \quad (12)$$

TABLE I. Attenuation factors $R(I)$ calculated by Eq. (13).

I	5/2	7/2	9/2	11/2	13/2	15/2	17/2	19/2	21/2	23/2	25/2	27/2	29/2	33/2
$R(I)$	0.23	0.25	0.27	0.32	0.32	0.42	0.39	0.51	0.45	0.58	0.53	0.64	0.58	0.63

Compared to the PRM it is attenuated by a factor

$$R = \frac{\mathcal{g}_0}{\mathcal{g}_0 + \mathcal{g}_p} = 1 - \frac{\langle j_x \rangle}{[I(I+1) - \langle j_z^2 \rangle]^{1/2}}. \qquad (13)$$

$\mathcal{g}_p = \langle j_x \rangle / \omega$ is the contribution of the outside particle (see Fig. 2). Taking into account that there was no fit parameter used ($\mathcal{g}_0$ is taken from $K0_c$), we find the agreement of this simple model with the experiment and with the fully self-consistent calculation surprisingly good. The last two columns in Fig. 3 are calculations within the PRM, without attenuation (column 7) and with a fit over four parameters (see Ref. 5).

The attenuation R is caused by the decoupling of the outside particle, which can be described very easily within the cranking model. It is strongly spin dependent (see Table I) and approaches 1 for high spin values. The "favored" states $I = \frac{5}{2}, \frac{9}{2}, \ldots$ are more attenuated than the unfavored ones $I = \frac{7}{2}, \frac{11}{2}, \ldots$. It should be emphasized that the attenuation [see Eq. (13)] is contained within the solution of the cranking model and there is no further parameter needed.

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Thermal Fluctuations and Experiments on the Free Fall of Electrons

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In order to study the gravitational force on an electron Witteborn and Fairbank have measured the acceleration of electrons moving along the axis of a vertical copper tube. I show that in experiments of this type there are large corrections to the motion of the electrons due to thermal fluctuations in the electromagnetic fields in the tube.

In a remarkable series of experiments Witteborn and Fairbank[1-5] (WF) have attempted to measure the gravitational free fall of electrons in vacuum. They studied the motion of electrons along the axis of a vertical "drift tube" of copper. Apart from known or applied electric fields, it was believed that all vertical electric or magnetic potential energy gradients had been reduced to below about 10^{-25} erg cm^{-1}. This was necessary to observe the effects of gravity, since for an electron in the earth's gravitational field the gradient of the gravitational potential energy is only 10^{-24} erg cm^{-1}. It had earlier been predicted by Schiff and Barnhill[6] that within a metal enclosure, itself in the earth's gravitational field, there should be an induced electric field of magnitude mg/e (m is the electron mass) directed downwards. This field arises because the electrons in a metal will be redistributed as a result of gravity. The Schiff-Barnhill field (SB) exerts a