Thickness of a Moving Helium-II Film*

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The thickness of a superfluid helium film in steady radial horizontal flow to a central sink has been determined at two radii. The thickness difference is found to be consistent with the prediction of Kontorovich, despite conditions which would favor recondensation from the vapor onto the flowing film.

Kontorovich[1] pointed out that a term $\frac{1}{2}(\rho_s/\rho)v_s^2$ should be added to the chemical potential in a moving superfluid film, where ρ_s/ρ is the superfluid density fraction and v_s is the superfluid velocity, so that

$$gh+\tfrac{1}{2}(\rho_s/\rho)v_s^2-\alpha/\delta n=0, \tag{1}$$

where h is the height of the film above bulk liquid and g is the acceleration of gravity. The final term is the potential of the Van der Waals attraction to the substrate, δ being the film thickness and $n \simeq 4$.[2] Equation (1) simply expresses Bernoulli's principle for the superfluid velocity field in a region of no dissipation and predicts a thinning of the moving film. However, an attempt by Keller[3] to observe the decrease in thickness yielded a null result. Several explanations for this have been advanced,[4-6] whose spirit is not to deny the existence of the kinetic energy term, but to propose additional mechanisms which compensate it. Of these, recondensation from the vapor[6] seems to be the most generally accepted. It arose from a suggestion[7] that the vapor over the flowing film would be unstable against condensation into the film until the pressure recovered the static saturation vapor pressure, P_{sat}. In a steady flow this implies a replenishment of the film by a flow in the vapor, driven by a pressure drop $\frac{1}{2}\rho_v v_s^2$, where ρ_v is the vapor density (if $\rho_s/\rho \sim 1$), thus accounting for Keller's result. Very recently, two experiments have been reported[8,9] in which Kontorovich's prediction has been observed, and the positive result is explained as due to an inhibition of the recondensation mechanism, in one case[8] because of large Poiseuille resistance to vapor flow in a capillary, and in the other because of the very low temperatures ($T < 0.9$ K) at which the experiment was done, with correspondingly low vapor density.

The recondensation mechanism is inconsistent with the acceptance of dissipationless superflow as an equilibrium state. Equation (1) follows from Landau's[10] original definition of the chemical potential of the *entire fluid* and not merely the superfluid fraction. Thus the condition for equilibrium at the interface of vapor and flowing film is not $P=P_{sat}$, but rather equality of the chemical potentials. This *requires* a local overpressure $\frac{1}{2}\rho_v v_s^2$ in the saturated vapor which is accomplished by bringing the vapor nearer to the substrate: The film thins as in Eq. (1). The only instability resulting from this overpressure is with respect to the formation of new nuclei: static droplets suspended in the region from which the fluid has withdrawn because of its flow. Even with the most conservative estimates of the parameters involved, this is an overwhelmingly improbable event[11]: $\sim\exp(-10^{11})$.

If the recondensation mechanism is rejected, Keller's result is the most clear-cut exception to the Kontorovich prediction. The importance of this result induced us to refine and repeat the experiment. Our experiment was designed to probe a single flow at two different locations which had clearly different velocities. This led to the use of planar, horizontal, radial flow into a central well, with the film thickness determined by capacitors at two radii; hydrodynamic continuity would then ensure the required velocity difference. Dissipation is expected to occur at the lip of the inner well, where the velocity is greatest. This was well downstream from the probing capacitors. The lack of dissipation between the capacitors was experimentally verified, and is discussed below.

Figure 1 shows a cross section of the cell. A block of Be-Cu (A), 5.08 cm in outer diameter and 2 cm in height, defined the inner wall of the outer, annular reservoir (E), and a central hole in the block (B), one tenth the outer diameter, led to an inner reservoir at the bottom (D), whose cross-sectional area was ~6 times larger than the hole. The top surface of A served as the flow surface between the inner and outer reservoirs. It was polished to a finish of ~1 μm, with a flatness of ~2 μm. The vertical walls were only fine

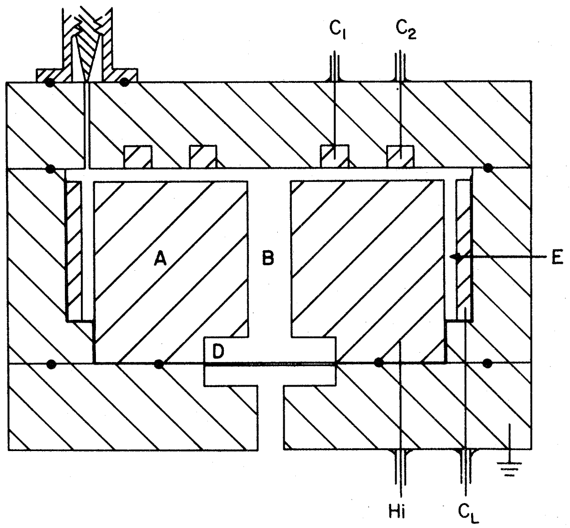

FIG. 1. Schematic view of the cell, roughly to scale except that the gap between the horizontal capacitance surfaces has been greatly exaggerated.

machined. The central block was closely fitted into the bottom of a Be-Cu vessel, being insulated from it by a Mylar film, which formed the floor of the inner well. A mechanical contrivance was fitted to the underside so that the liquid level in the inner well could be displaced. The side walls of the vessel left a radial gap of 0.06 cm for the outer reservoir, E, and the top plate facing the flow surface was separated from it by a gap of ~40 μm. Liquid helium could be admitted into the cell from a condensing chamber above via a needle valve and a 250-μm hole in the top plate. The central block, A, served as the "high" terminal for three capacitors. Two ring-shaped electrodes were fitted into the upper surface facing the flow surface, at approximately 2 and 4 times the radius of the central hole. These formed the thickness capacitors (C_1 and C_2 in Fig. 1). This surface was then polished to the same degree as the flow surface itself. An annular sleeve fitted to the outer wall of the cell formed a coaxial capacitor (C_L) with the central block, which allowed the level of the liquid in the outer well to be measured. Any two of these capacitors could be monitored simultaneously, by using two identical three-terminal capacitance bridges (General Radio type 1615-A). The cell was directly immersed in the liquid-helium bath.

The experiment consisted of the measurement of Δ, the film thickness difference at the two capacitors during a steady flow between the reservoirs. The flow was measured from the rate of change of the level capacitor C_L and the known geometry of the cell. It is expressed as Q, the flow rate per centimeter of perimeter at the outer radius of A. From Eq. (1), Δ is given by

$$\Delta = \frac{\rho_s}{\rho}\frac{Q^2}{n\delta h}\left[\left(\frac{1}{r_1^2}-\frac{1}{r_2^2}\right)\frac{R^2}{2g}\right], \qquad (2)$$

where r_1 and r_2 are the root-mean-inverse-square radii of the capacitor plates and R is the outer radius of A. The quantity in square brackets is a constant of the apparatus (6.97×10^{-3} sec^2 cm^{-1}). δ, the film thickness at $Q=0$, is determined in the first instance from the adsorption isotherm during the initial filling of the cell, with appropriate corrections for the vapor contribution to the capacity and the effect of distortion of the cell by the vapor pressure. This yielded a value of δ of order 450 Å, C_1 and C_2 giving values consistent with each other to ~20 Å. After formation of the saturated film, δ is a weak function of h; for the limited range of heights available (~1 cm), the value of n was usually approximately 3.7 ± 0.5. Measurement of Δ has been made at temperatures from 1.22 K to 2.0 K for both inflow to and outflow from the central well, and with both bare metal and argon substrates. In every case a finite value of Δ has been observed which agrees with Eq. (2) to within experimental error. The error is due mainly to the uncertainty in the absolute values of Q, δ, and n, rather than in the determination of Δ itself. Thus no compensatory mechanism operating on the film thickness need be postulated.

Because of the long flow time involved, the simplest case to present in detail here is that of the inflow to the central well following the initial charges of bulk liquid admitted to the outer well from the condensing chamber. Beginning at the point when hardly any bulk liquid had yet condensed in the cell, the needle valve was opened to introduce sufficient liquid to raise the level in the outer well from zero to ~1 mm in 2 to 3 min. The supply was then shut off and, because the outer well lies at a higher level than the inner well, the outer well emptied to the center at a rate which was constant for several minutes. The apparent value of Q for these initial fills was 1.03×10^{-5} cm^2/sec and did not vary by as much as 1% over several weeks of experiments. This value of Q is approximately half the true rate of emptying, because of the edge-effect correction of the level capacitor when the liquid level is low.

At the beginning of this filling, either thickness capacitor indicated a rapid rise in film thickness

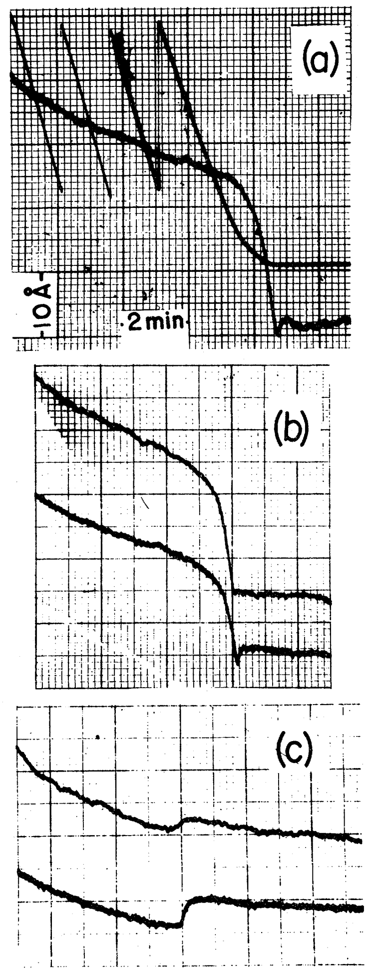

FIG. 2. Responses of the capacitors while the outer well is emptying at 1.3 K. The horizontal scale is 2 min/large division. Scale on C_1 and C_2 is 10 Å/large division in terms of a *single* film thickness. (a) The level capacitor, with 0.02-pF steps inserted (equivalent to 0.25 mm height changes), and the inner capacitor C_1. When the outer well becomes empty, the film readjusts itself to the lower inner level. (b) The outer (upper trace) and inner capacitors C_2 and C_1. Both readjust themselves as in (a); but the thickness is not the same for the flow and static situations. (c) As in (b) with liquid remaining in the outer well when flow stops. The film recovers thickness with a step.

of ~35 Å, and then with the supply shut off and as the level fell in the outer well the thickness would also slowly fall. When the well became empty and flow ceased there was a rapid fall in thickness to a final constant value, little more than the initial one. This is shown in Fig. 2(a). The final change in thickness is interpreted as being due to the change in the reference point of the chemical potential of the film from the level in the outer well during flow to that in the inner well when flow ceased. The change, ~30 Å average for the outer capacitor, is in good agreement with the change of *two* film thicknesses (top and bottom plates) for the difference in heights of the bottoms of the inner and outer wells (2.5 mm). Similarly, the fall of the capacity during the flow is accounted for by the change in δ with h of the *outer* well as it empties, according to $\dot{\delta} = 2\delta\dot{h}/nh$. Since both thickness capacitors fall at this same rate, as shown in Fig. 2(b), the chemical potential at both radii is being referred to the outer well. If the inner capacitor were referring to the height in the inner well, its rate of change would be an order of magnitude less, and this has sometimes been observed for outflow from B. Thus we conclude that for cases such as those shown in Fig. 2 the flow is pure potential flow to beyond the inner capacitor.

The existence of a time-independent profile is apparent in Fig. 2(b), where $\Delta \sim 7$ Å. If a finite level remains in the outer well when flow ceased, the profile simply recovers with a stepwise response as shown in Fig. 2(c). A similar step was always observed from C_1 in cases when E was emptied as well, as in Figs. 2(a) and 2(b), and never for C_2. The mechanism of the recovery of uniform thickness depends on the kinetic energy distribution in the film and is most apparent at the inner radius.

In summary this experiment contradicts Keller's result and agrees with the Kontorovich prediction. Thus the rationale supporting the recondensation mechanism is removed. In this context, it may be noted that the shortest time involved in this experiment, the flow time between the two thickness capacitors, is always at least an order of magnitude greater than the vapor replenishment time calculated by William and Packard.[9] In this experiment, as in Keller's, the stationary film on the capacitor plate facing the flow would be a local source of vapor which could replenish the material lost via recondensation while itself flowing at a negligible rate of order Δ/δ.

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Relaxation of Toroidal Plasma and Generation of Reverse Magnetic Fields

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The spontaneous generation of reversed fields in toroidal plasmas is shown to be a consequence of relaxation under constraints. With perfect conductivity a topological constraint exists for each field line and the final state is not unique. With small departures from perfect conductivity, topological constraints are relaxed and the final state becomes unique. The onset of the reversed field and other features of this model agree well with observations on ZETA.

One of the best known methods for magnetic confinement of plasma is the toroidal pinch as in ZETA.[1,2] In such experiments a toroidal field B_0 is created by external coils and a toroidal current I is induced in the plasma. The pinch effect associated with this current produces the plasma compression whose magnitude depends on the ratio $2I/aB_0 \equiv \theta$ (where a is the minor radius of the torus).

A remarkable feature of these experiments is that after an initial, violently unstable, phase the plasma frequently relaxes into a "quiescent" state in which it appears to be largely stable. Furthermore, when the pinch ratio θ exceeds some critical value this relaxation is accompanied by the generation of a reversed toroidal field in the outer regions of the plasma.

This Letter outlines a theory of the relaxation of toroidal plasma which appears to account well for this remarkable behavior and which predicts the critical value of θ for the generation of the reversed field. It can also account for other phenomena observed in toroidal pinches.

In this theory the plasma is regarded as a conducting but viscous fluid enclosed in a rigid, perfectly conducting, toroidal vessel. The initial state is arbitrary except that both the magnetic field and current are tangential to the conducting wall. The system is *not* in stable equilibrium and when released will therefore move (usually violently) and dissipate energy before coming to rest. Only when its energy is a minimum is it incapable of further rapid movement. Hence the final state must be one which makes the energy a minimum subject to any constraints which are imposed on the allowed motion.[3] The major problem, of course, lies in determining and applying these constraints.

For simplicity it is assumed here, as is indeed the case in most experiments, that the plasma internal energy is negligible compared to the magnetic energy $W_m = \int (B^2/2) d\tau$ which is therefore to be minimized. The inclusion of plasma energy will be discussed elsewhere

The constraints which must be applied to the variations in $\vec{B}$ (without which the minimum would be $\boldsymbol{B}=0$) arise from the fact that in a perfectly conducting fluid variations in the magnetic field must satisfy

$$\partial \vec{B}/\partial t - \nabla \times (\vec{v} \times \vec{B}) = 0, \quad (1)$$

where $\vec{v}$ is the fluid velocity.

As is well known, Eq. (1) means that lines of force may be labeled by the fluid elements on them and so be regarded as moving with the fluid velocity. Since this velocity is continuous, field lines cannot break or coalesce (except where $B = 0$ which we exclude). All topological properties of the field lines are therefore invariant; e.g., if two closed field lines are initially linked n times, then they must remain so linked at all times.

These topological constraints can be expressed