If the function f is such that arbitrarily large $|y|$ are admissible in our coordinate system, then we fix the multiplicative factor in f (and, therefore, in Δ, ξ, and $\exp\chi$) by specifying that the asymptotic values be $|\Delta y|^{3/4}$ or $|\Delta y|^{1/4}$. Then, if the asymptotic form is $|\Delta y|^{3/4}$, the limiting case $\Delta = 0$ corresponds to a Minkowski space with Cartesian coordiantes x_α which are related to ours by $2u = x_3 - x_4$, $u\rho = x_1$, $u\sigma = x_2$, and $2v = 2(x_3 + x_4) + u(\rho^2 + \sigma^2)$, where $v = 4\xi^{7/4}/7$. Similar relations hold when the asymptotic form is $|\Delta y|^{1/4}$.

I thank Dr. Fred Ernst for verifying the solution, for the numerical integration of Eq. (15), and for the relations of the coordinates to Cartesian coordinates when $\Delta = 0$.

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Longitudinal, Massive Photon in an External Magnetic Field

R. A. Cover
KMS Fusion, Inc., Ann Arbor, Michigan 48106

and

G. Kalman
Boston College, Chestnut Hill, Massachusetts 02167
(Received 21 February 1974)

The study of the vacuum polarization for strong magnetic fields reveals the existence of a massive, longitudinal photonlike resonance for magnetic fields exceeding $B \simeq m^2c^4/e^3$.

The existence of strong magnetic fields in the vicinity of collapsed stars has brought into focus the question of the dispersive properties of the vacuum in the presence of strong magnetic fields.[1-7] Such dispersive features become significant when the magnetic field reaches and exceeds the critical field value $B_* = m^2c^3/\hbar e = 4.41 \times 10^{13}$ G. In this Letter a novel feature of the electromagnetic vacuum is pointed out: When the magnetic field strength exceeds a second critical value B_0 of the order of $(\hbar c/e^2)B_*$, a massive, longitudinal photonlike resonance appears whose mass value roughly coincides with the energy of an electron-positron pair occupying the lowest Landau level. For magnetic field values in the vicinity of the critical field the photon is heavily damped, but for increasing field strengths the lifetime rapidly increases: Typically for $B \simeq 10B_0$, $\tau \simeq 2\pi \times 15 \times \omega^{-1} \simeq 7 \times 10^{-18}$ sec.

The existence of the massive photon is inferred from the study of the longitudinal dispersion relation

$$\epsilon(\vec{k}, \omega) = 0, \tag{1}$$

with $\epsilon(\vec{k}, \omega)$, the longitudinal "dielectric function," and $\alpha(\vec{k}, \omega)$, the longitudinal polarizability of the vacuum, being related to the regularized longitudinal vacuum polarization $\overline{\Pi}_{33}(\vec{k}, \omega)$ by

$$\epsilon(\vec{k}, \omega) \equiv 1 + \alpha(\vec{k}, \omega) = 1 - \omega^{-2}\overline{\Pi}_{33}(\vec{k}, \omega). \tag{2}$$

We calculated the vacuum polarization in the presence of an arbitrarily strong, uniform magnetic field, to lowest order in e^2. The photon momentum is restricted to be parallel to $\vec{B}$. For small momenta the k dependence of Π_{33} is not significant and in this Letter we consider the $k \to 0$ limit only. Π_{33} can be obtained by standard methods, using the appropriate Green's functions for electrons in a magnetic field.[8] The unrenormalized value of Π_{33} is given by

$$\Pi_{33}(0, \omega) = 8\frac{e^3B}{2\pi}\sum_{n=0}^{\infty} a_n \int_{-\infty}^{\infty} \frac{m^2 + 2neB}{\epsilon_{pn}(\omega^2 - 4\epsilon_{pn}^2)}\, dp, \tag{3}$$

where

$$\epsilon_{pn}{}^2 = m^2 + p^2 + 2neB,$$
$$a_n = \begin{cases} 1 \text{ for } n = 0, \\ 2 \text{ for } n > 0, \end{cases} \qquad (4)$$

and $\hbar = c = 1$, and B and e stand for the absolute values of the magnetic field and the electron charge.

This expression is regularized by subtracting out $\Pi_{33}(0,0;B=0)$ and $\omega^2(\partial\Pi_{33}/\partial\omega^2)(0,0;B=0)$ to yield the regularized Π_{33}.[6] This can be accomplished in two steps, by first regularizing with respect to the $\boldsymbol{B}$-dependent vacuum, and then correcting for the difference between vacuum with and without magnetic field:

$$\overline{\Pi}_{33}(0,\omega;B) = \Pi_{33}(0,\omega;\boldsymbol{B}) - \Pi_{33}(0,0;B) - \omega^2(\partial\Pi_{33}/\partial\omega^2)(0,0;\boldsymbol{B}) - \omega^2\alpha(0,0;\boldsymbol{B}), \qquad (5)$$

where $\alpha(0,0;\boldsymbol{B})$ is the (regularized) static, uniform polarizability of the vacuum,

$$\alpha(0,0;\boldsymbol{B}) = (\partial\Pi_{33}/\partial\omega^2)(0,0;\boldsymbol{B}=0) - (\partial\Pi_{33}/\partial\omega^2)(0,0;B). \qquad (6)$$

This latter is obtained by evaluating the difference

$$\alpha(0,0;B) = \frac{e^3\boldsymbol{B}}{3\pi}\sum_{n=0}^{\infty} a_n \frac{1}{m^2+2neB} - \frac{e^2}{4\pi^2}\int d^3p \frac{m^2+\frac{2}{3}p^2}{(p^2+m^2)^{5/2}}. \qquad (7)$$

The result is

$$\alpha(0,0;B) = -(e^2/3\pi)[\tfrac{1}{2}b = \ln b + \psi(1/b)], \qquad (8)$$

where $\psi(x)$ is Euler's ψ function, and $b = 2e\boldsymbol{B}/m^2$. Equation (8) is a generalization for arbitrarily strong magnetic fields of known results valid for weak fields only.[6,10,11] Thus

$$\mathrm{Re}\overline{\Pi}_{33}(0,\omega) = -2\frac{e^3\boldsymbol{B}}{2\pi}\Bigg(\sum_{n=0}^{\infty} a_n \frac{-\frac{2}{3}[3(m^2+2neB)+2\omega^2/4]}{m^2+2ne\boldsymbol{B}} + \sum_{n=n_\omega+1}^{\infty} a_n \frac{2(m^2+2neB)}{(\omega/2)(m^2+2neB-\omega^2/4)^{1/2}}\tan^{-1}\frac{\omega/2}{(m^2+2neB-\omega^2/4)^{1/2}}$$
$$+\sum_{n=0}^{n_\omega} a_n \frac{m^2+2neB}{(\omega/2)[\omega^2/4-(m^2+2ne\boldsymbol{B})]^{1/2}}\ln\left|\frac{\omega/2-[\omega^2/4-(m^2+2ne\boldsymbol{B})]^{1/2}}{\omega/2+[\omega^2/4-(m^2+2ne\boldsymbol{B})]^{1/2}}\right|\Bigg) - \omega^2\alpha(0,0;B), \qquad (9)$$

$$n_\omega = \begin{cases} -1 \text{ for } \omega < 2m, \\ (\frac{1}{4}\omega^2 - m^2)/2eB \text{ for } \omega > 2m, \end{cases}$$

while the imaginary part is

$$\mathrm{Im}\overline{\Pi}_{33}(0,\omega) = -2\frac{e^3B}{\omega}\sum_{n=0}^{n_\omega} a_n \frac{m^2+2ne\boldsymbol{B}}{[\omega^2/4-(m^2+2neB)]^{1/2}}. \qquad (10)$$

The structure of $\overline{\Pi}_{33}$ is determined largely by the singularities at the pair energies

$$\omega = \omega_n \equiv 2(m^2+2ne\boldsymbol{B})^{1/2}, \quad n = 0, 1, 2, \ldots. \qquad (11)$$

On the low-frequency side of each singularity, $\overline{\Pi}_{33} \to -\infty$; on the high-frequency side of each singularity, $\overline{\Pi}_{33}$ is finite. The value of α at and above a given singularity (say, the sth) can be estimated by separately evaluating the contributions coming from the $n=0$, $0<n<s$, $n=s$, and $n>s$ terms in Eq. (5). With increasing value of B, α decreases, becoming negative in the vicinity of $s=0$ for $B>0.24B_*$, and for $B>B_*$ it equals roughly $(-4/3\pi)e^3B/m^2$. This behavior is plotted in Fig. 1.

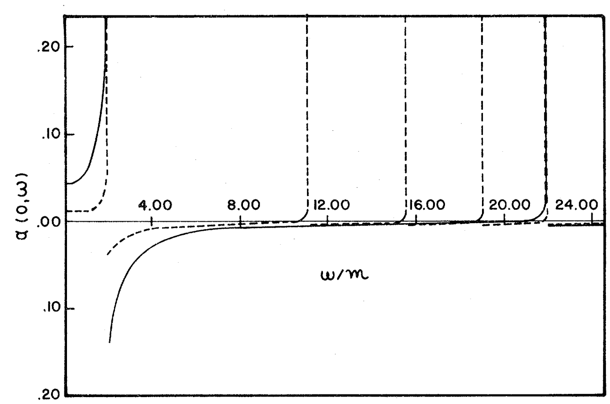

FIG. 1. The structure of the real part of the longitudinal polarizability of the vacuum $\alpha(0,\omega) \equiv \omega^{-2}\overline{\Pi}_{33}(0,\omega)$ for two subcritical magnetic fields values, $B = 15B_*$ (dashed line) and $B = 60B_*$ (solid line).

When $\boldsymbol{B}$ is sufficiently large α can decrease below -1. If this happens, then there exists a solu-

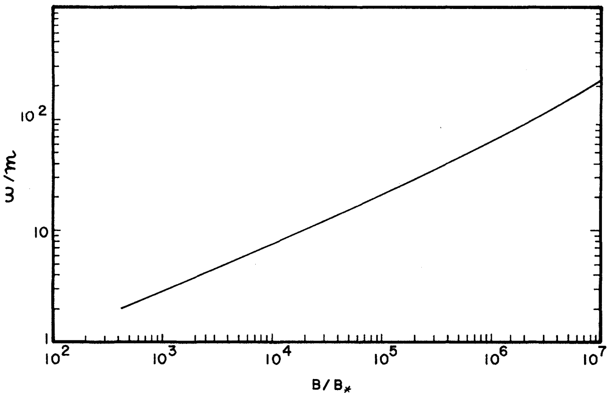

FIG. 2. Mass values of the longitudinal photon versus the magnetic field $B > B_0$.

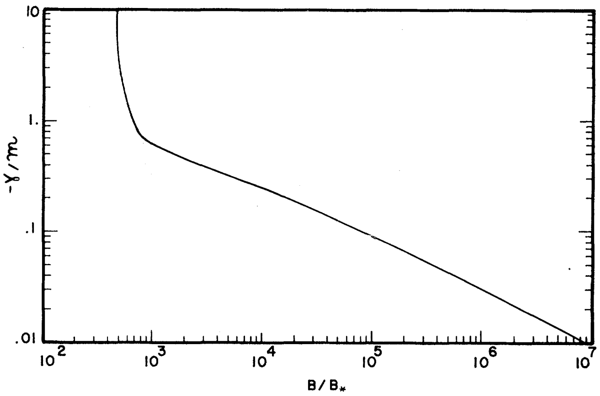

FIG. 3. The imaginary part of the photon energy.

tion ω to Eq. (1) between the first and the second singularity, which starts out at $\omega = 2m$ and for increasing $\boldsymbol{B}$ values rapidly moves away toward the $s = 1$ singularity.

The critical value $\boldsymbol{B}$ has been determined by computer evaluation of the series (9):

$$\boldsymbol{B}_0 = 429\boldsymbol{B}_*.$$

A graph of the photon mass versus $\boldsymbol{B}$ is given in Fig. 2. The photonlike resonance has a finite lifetime, caused by its decay into electron-positron pairs. The inverse lifetime γ is evaluated by

$$\gamma_s = -\frac{\mathrm{Im}\overline{\Pi}_{33}(0,\omega)/\omega^2}{(\partial/\partial\omega)[\omega^{-2}\,\mathrm{Re}\overline{\Pi}_{33}(0,\omega)]_{\omega=\omega_s}}. \tag{12}$$

Since $\mathrm{Im}\overline{\Pi}_{33}$ diverges as $[\omega - 2(m^2 + 2seB)^{1/2}]^{-1/2}$ in the vicinity of the sth singularity, for magnetic field values near the threshold γ is large: $\gamma \gtrsim \omega$. However, magnetic fields higher than the threshold field lead to rapidly diminishing values of γ, typically resulting in $\gamma \simeq 0.07\omega_0$ for $\boldsymbol{B} = 10B_0$. This result is illustrated in Fig. 3.

Whether the ultrahigh magnetic fields required for the generation of the massive photon exist in the universe cannot be ascertained at the present time. The usually assumed pulsar magnetic fields are weaker by several orders of magnitude. Ferromagnetic or differentially rotating[12] neutron stars could, however, generate high enough fields. Also, asymmetric field configurations in the neighborhood of black holes can result in very high field values. Should the threshold magnetic field strength be exceeded, the Cherenkov-synchrotron radiation of high-energy particles would be substantially affected and so would be, by the decay of the massive photon, the rate of spontaneous generation of electron-positron pairs.[1,8,12]

A more detailed description of the properties of the massive photon would include the full energy-momentum relation, including anisotropy effects. It is also not clear to what extent higher-order radiative corrections would affect the pole structure of $\overline{\Pi}_{33}$: Since the field B_0 is of the order $\alpha^{-1}B_*$, radiative corrections of the order α may well have important effects. Self-energy effects are probably negligible, since for $\boldsymbol{B} \gtrsim \boldsymbol{B}_*$ it has been shown recently by different authors[12,13] that the self-energy correction is $\sim \ln(B\delta\mu/m)$, where $\delta\mu$ is the anomalous magnetic moment. In contrast it is only for low fields that the self-energy correction is linear in $\boldsymbol{B}$. It is, however, more difficult to estimate the effect of vertex corrections. Finally, if the massive resonance is regarded as due to positron-electron binding, the existence of *transverse* massive photons is also suspected. These questions are under study and will be reported on at a later time.

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Conversion of Electromagnetic to Gravitational Radiation by Scattering from a Charged Black Hole

D. W. Olson* and W. G. Unruh†‡
Department of Physics, University of California, Berkeley, California 94720
(Received 16 September 1974)

When a purely electromagnetic incoming wave is scattered by a charged black hole, the electromagnetic-gravitational coupling can lead to conversion to gravitational radiation in the outgoing wave. Analytic estimates and numerical results are given for this effect, which proves to be significant only for maximally or near-maximally charged black holes.

Gerlach[1] has pointed out that in the presence of a background electromagnetic field an electromagnetic wave will gradually convert into a gravitational wave and vice versa. In particular he notes that an electromagnetic wave scattering from a charged black hole could convert entirely into gravitational radiation. Zerilli[2] and Moncrief[3,4] have found the coupled and decoupled wave equations obeyed by electromagnetic and gravitational perturbations on a Reissner-Nordstrøm charged-black-hole-background space-time for both odd- and even-parity waves.

A purely electromagnetic incoming wave is a linear combination of two normal modes. Because the normal modes obey different wave equations, their relative phase will be shifted by scattering from the black hole. The outgoing wave will be a combination of both electromagnetic and gravitational radiation. In this note we give the results of computer calculations of this effect for the odd-parity perturbations.

After separation of harmonic time and angular dependence, the gauge-invariant Moncrief radial equations for odd-parity normal modes are

$$(d^2/dr^{*2}+\omega^2-V_\pm)R_\pm=0, \qquad (1)$$

where

$$V_\pm=\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\left(\frac{L(L+1)}{r^2}-\frac{3M}{r^3}+\frac{4Q^2}{r^4}\pm\frac{\sigma}{r^3}\right),$$

and where $\sigma=[9M^2+4Q^2(L-1)(L+2)]^{1/2}$, M and Q are the mass and charge of the black hole,[5] ω is the angular frequency of the wave, and L is the total angular momentum index in the Regge-Wheeler tensor harmonic expansion. The r^* coordinate is defined by $dr/dr^*=1-2M/r+Q^2/r^2$. V_+ and V_- are the effective potentials, which vanish at infinity and on the horizon at $r=M+(M^2-Q^2)^{1/2}$ (or $r^*=-\infty$). The barrier maximum for each effective potential occurs near $r=[3M+(9M^2-8Q^2)^{1/2}]/2$.

The normal modes R_+ and R_- are defined by

$$R_+=F\cos\Psi+G\sin\Psi,$$
$$R_-=-F\sin\Psi+G\cos\Psi,$$
$$\sin 2\Psi=2Q[(L-1)(L+2)]^{1/2}/\sigma,$$

in terms of Moncrief's gauge-invariant amplitudes. F represents a purely electromagnetic perturbation, and G represents a combination which near infinity is purely gravitational.[6] With