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Proton-Proton Total Cross Section above 10^4 GeV: Can Cosmic Rays Give the Answer?

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From Glauber calculations of the absorption cross section of nucleons on air (σ_{abs}) we find that present extensive-air-shower data cannot resolve the question of the asymptotic energy behavior of the nucleon-nucleon total cross section [$\sigma_t(NN)$]. Relative differences between various asymptotic extrapolations for $\sigma_t(NN)$ are reduced by about a factor of 3 in σ_{abs} through the Glauber conversion. Novel techniques or conventional experiments with a better determination of electron and muon numbers in extensive air showers will be required in order to obtain useful information on proton-proton cross sections in the asymptotic region.

The discovery[1] of a rising proton-proton total cross section at the CERN intersecting storage rings generated a great deal of interest in the eventual asymptotic behavior. Although the CERN data may provide clues as to the asymptotic regime, measurements at higher energies are necessary to determine the ultimate trend. A wide range of theoretical speculations accounting for the CERN data give very different asymptotic extrapolations. The proposals for the rise of $\sigma_t(NN)$ include (i) a $\ln^2 s$ growth inferred from studies of massive quantum electrodynamics,[2] (ii) an empirical $\ln s$ growth from geometrical scaling with a logarithmically growing radius,[3] (iii) an asymptotic constant behavior from Regge-cut models,[4] (iv) a threshold behavior from particle production,[5] (v) an oscillating behavior from complex Regge poles associated with dynamical thresholds,[6] and (vi) asymptotically growing cross sections from s-[7] and t-channel[8] unitarity arguments. The wide divergence of high-energy expectations of $\sigma_t(NN)$ is illustrated in Fig. 1.

Until a new generation of accelerators is constructed the only hope of resolving the question of asymptotic growth lies with cosmic-ray experiments. Whereas new generations of accelerators and cosmic-ray techniques might eventually compete between 10^4 and 10^6 GeV, the 10^6–10^9 GeV energy range is exclusively accessible to extensive-air-shower (EAS) techniques. We address the

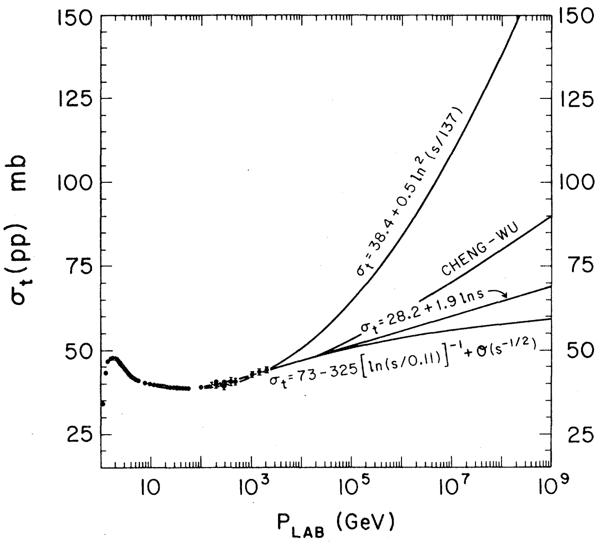

FIG. 1. Typical high-energy model extrapolations of the proton-proton total cross section to the energy range accessible to extensive-air-shower experiments.

question: Can cosmic-ray experiments reveal the truly asymptotic behavior of $\sigma_t(NN)$?[9] At first glance, the enormous difference in the various extrapolations of $\sigma_t(NN)$ in Fig. 1 suggests that a rough measurement of the absorption cross section of protons on air around 10^9 GeV might be fruitful in distinguishing different approaches.

From measurements of the absorption cross section (σ_{abs}) of protons on air nuclei, the proton-proton total cross section is deduced from a Glauber calculation.[10] In the framework of the Glauber model we find that σ_{abs} data are unlikely to be useful until present EAS techniques are considerably improved. The basic reasons are simple: (i) For a large proton-proton total cross section the air nucleus is a black disc and the absorption cross section is independent of $\sigma_t(NN)$. (ii) The Glauber conversion from σ_{abs} to $\sigma_t(NN)$ depends on specific assumptions on the impact-parameter profile of the proton-proton elastic amplitude and is therefore model dependent.[11-13] A qualitative argument for point (i) can be illustrated by using the following simplified Glauber relation:

$$\sigma_{abs} = \int d^2b\{1 - \exp[-\sigma_t T(b)]\}, \qquad (1)$$

where $T(b)$ represents the impact profile of the air nucleus. For small $\sigma_t(NN)$

$$\sigma_A \simeq \sigma_t(NN)\int d^2b\, T(b) \simeq A\sigma_T(NN), \qquad (2)$$

and a value of σ_A implies a value of $\sigma_t(NN)$. For large σ_t, however,

$$\sigma_{abs} \simeq \int_0^{R_A} d^2b \simeq \pi R_A{}^2 \simeq CA^{2/3}, \qquad (3)$$

where R_A is the nuclear radius and A is the atomic number. This result is independent of $\sigma_t(NN)$! Our detailed calculations using the full Glauber formalism show that σ_{abs} is relatively insensitive to $\sigma_t(NN)$ above 10^4 GeV; therefore, the limit in Eq. (3) is more relevant to the actual experimental situation. Indeed, the empirical A dependence of nuclear absorption cross sections between 20 and 400 GeV,[14]

$$\sigma_{abs} = C(s)A^{0.691}, \qquad (4)$$

is approximately in accord with Eq. (3), suggesting that one is close to the black-disc limit even at accelerator energies.

In addition to the insensitivity of σ_{abs} to increasing values of $\sigma_t(NN)$ our ignorance of the impact profile of the nucleon introduces model-dependent uncertainties.[11-13] To illustrate this aspect we need the full Glauber formalism[10] which takes into account the impact profile of the proton as well as that of the nucleus:

$$\sigma_{abs} = \int d^2b[1 - |1 - \Gamma_A(\vec{b})|^2]; \qquad (5)$$

Γ_A is the profile of the nucleus,

$$\Gamma_A(\vec{b}) = 1 - [1 - \int \Gamma_N(\vec{b} - \vec{b}')\rho(z, \vec{b}')\, dz\, d^2b']^A. \qquad (6)$$

Here $\rho(z, \vec{b}')$ is taken to be a Gaussian distribution of nucleons in the nucleus,

$$\rho(r) = (\sqrt{\pi}R)^{-3}\exp(-r^2/R^2), \qquad (7)$$

with

$$R \simeq 2 \text{ fm}. \qquad (8)$$

The profile of the nucleons, Γ_N, is model dependent. In many cases Γ_N can be adequately represented by the Gaussian form

$$\Gamma_N(b) = \sigma_t(NN)\frac{\exp(-b^2/2B)}{4\pi B}. \qquad (9)$$

Here B is the forward slope of the differential cross section, defined as $(d/dt)(\ln d\sigma/dt)$ at $t = 0$. We consider the following representative models (see Fig. 1): (a) an energy-independent profile with

$$\sigma_t(NN) = 42 \text{ mb}, \quad B = 12 \text{ GeV}^{-2}; \qquad (10)$$

(b) the Cheng-Walker-Wu impact picture[2] which has the asymptotic behavior

$$\sigma_t \to \ln^2 s, \quad B \to \ln^2 s; \qquad (11)$$

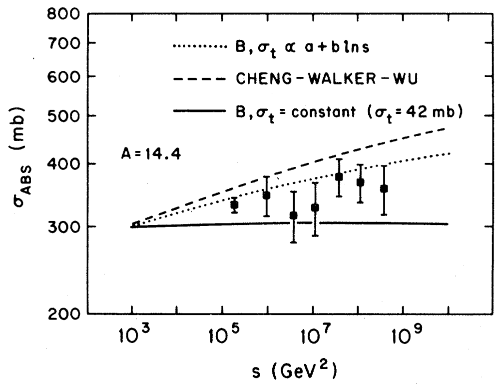

FIG. 2. Glauber calculation of the absorption cross section of protons on air nuclei for various asymptotic trends of $\sigma_t(NN)$. Also shown are EAS data from Ref. 13 with statistical errors only. The parameters for B and $\sigma_t(NN)$ for the various models shown in the figure have been adjusted to describe the accelerator data [see Eqs. (10)–(12)].

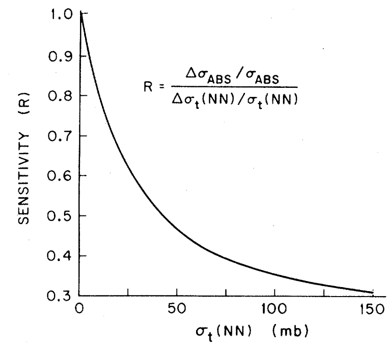

FIG. 3. Sensitivity R of determination of $\sigma_t(NN)$ from measurement of σ_{abs} [c.f. Eq. (13)].

(c) a geometrical scaling model[3] consistent with accelerator data,

$$\sigma_t = 28.2(1 + 0.068 \ln s),$$
$$B = 8.32(1 + 0.068 \ln s). \quad (12)$$

The corresponding results for σ_{abs} are shown in Fig. 2. The relative difference between the various asymptotic extrapolations for $\sigma_t(NN)$ are about a factor of 3 smaller in σ_{abs} than in $\sigma_t(NN)$ around 10^8 GeV primary energy. The sensitivity of σ_{abs} to the value of $\sigma_t(NN)$ can be quantitatively represented by the ratio

$$R = \frac{\Delta\sigma_{abs}/\sigma_{abs}}{\Delta\sigma_t(NN)/\sigma_t(NN)} = \frac{\sigma_t(NN)}{\sigma_{abs}} \frac{\partial\sigma_{abs}}{\partial\sigma_t(NN)} \quad (13)$$

for a given slope parameter B. Results for R with $B = 12$ are shown in Fig. 3. At large values of $\sigma_t(NN)$ the sensitivity of a determination of $\sigma_t(NN)$ from σ_{abs} is a factor of 3 down from a direct measurement. Hence relatively precise measurements of σ_{abs} would be required even to separate a constant 42 mb cross section from one that grows like $\ln^2 s$. Furthermore $\ln s$ and $\ln^2 s$ behaviors are practically indistinguishable. In the energy range under consideration the Cheng-Walker-Wu impact picture has not reached its asymptotic regime given by Eq. (11). B is relatively constant, reducing the effect of the $\ln^2 s$ increase on σ_{abs}. This is the reason why models (a) and (b) yield relatively similar results on σ_{abs} despite different growth for $\sigma_t(NN)$.

The existing EAS data[15] are shown in Fig. 2 with statistical errors only. These cross sections for protons on air have been extracted from the measurements of Fukui *et al.*[16] of N_μ-N_e frequency distributions for showers at various zenith angles. In addition to the statistical errors quoted in Ref. 15 there are systematic uncertainties, which tend to require a revision of σ_{abs} upward by some 15–30%.[13] These are due to zenith-angle dependence of fluctuations in the number of muons actually detected, and they make the data of Ref. 15 largely irrelevant to the resolution of the asymptotic behavior of $\sigma_t(NN)$.

It is clear from inspection of Fig. 2 that in order to distinguish between an asymptotically constant cross section and, for example, the Cheng-Walker-Wu model one has to determine σ_{abs} with an error substantially better than 10%. Therefore conventional experiments must measure muons in the shower front with sufficient statistical accuracy to overcome systematic effects such as the one mentioned above. A novel approach that may be able to give the required statistics up to 10^{10} GeV is the "fly's eye" experiment proposed by the Utah group.

Although existing EAS data cannot distinguish among the models shown in Fig. 2, they can provide some information on the growth of a fringe component of the NN amplitude. This is so because the fringes lead to an expanding nuclear disc, causing σ_{abs} to increase.[12] A possible dynamical origin of a growing fringe is the $\pi\pi$ exchange contribution to the NN amplitude.[17,18] Present estimates of the $\pi\pi$ fringe[17,18] yield absorption cross sections in air close to the model calculations with growing $\sigma_t(NN)$ already shown in Fig. 2. Significant sensitivity of σ_{abs} to the fringe of the proton only occurs if the fringe represents an extremely small fraction of $\sigma_t(NN)$

that is responsible for the bulk of its growth with energy. As a specific case of this extreme we considered the parametrization of Leader and Maor[19] in which the NN amplitude is a sum of two Gaussians with

$$\sigma_1 = 38.4 \text{ mb}, \quad \sigma_2 = 0.49[\ln(s/122)]^2 \text{ mb},$$

$$B_1 = 10.8 \text{ GeV}^{-2}, \quad B_2 = 5[\ln(s/122)]^2 \text{ GeV}^{-2}.$$

This model yields σ_{abs} values of the order of 10^3 mb around 10^9 GeV which is a factor of 2 above existing data, even when systematic uncertainties are included.

Further sources of uncertainties in a cosmic-ray determination of $\sigma_t(NN)$ include corrections for quasielastic effects, model dependence of other NN parameters, and uncertainties in nuclear densities. We will discuss them in a forthcoming paper.[13]

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