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Longitudinal and Transverse Resonance in the *B* Phase of Superfluid He^3

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I report measurements of the longitudinal resonance in the *B* phase of superfluid He^3 at melting pressures over a broad temperature interval. I also present the results of a transverse NMR study in the *B* phase which is designed to test the theory of textures in the *B* phase and to provide accurate measurements of the *B*-phase susceptibility. Finally, I report the first measurements of a field-dependent frequency shift in the transverse *B*-phase NMR associated with the anisotropy energy of the rotation axis of the Balian-Werthamer state.

In the context of the spin-fluctuation theory of Brinkman, Serene, and Anderson[1,2] the *B* phase of superfluid He^3 must be identified as the Balian-Werthamer (BW) or "isotropic" state[3] described by Leggett[4] in which the spin and orbit coordinates are rotated by $\cos^{-1}(-\frac{1}{4})$ about an arbitrary axis $\hat{n}$. Yet to date most experimental results have not been interpreted as supporting this identification.[5,6] Only the work of Osheroff and Brinkman[7] (OB) seems to clearly support the Brinkman-Serene-Anderson result. As a consequence of the OB interpretation of their data as evidence for domain structure or a "texture"[8] in the *B* phase, Brinkman, Smith, Osheroff, and Blount[9] (BSOB) have developed a Ginzburg-Landau theory to determine the effects of surfaces and magnetic fields on the spatial orientation of the axis $\hat{n}$ and the effects of this spatial orientation upon the transverse NMR spectrum.

In this work I present measurements of longitudinal resonance in the *B* phase at melting pressures. I also present results of a careful transverse-resonance study in the *B* phase designed to test the BSOB theory. I finally use the BSOB theory to derive the reduced *B*-phase susceptibility $\chi(B)/\chi(\mathrm{F.L.})$ from present NMR absorption measurements and use these values to compare the longitudinal resonant frequencies in the *B* phase with those measured by OB in the *A* phase.

The compressional apparatus used in this work has been described by Osheroff and Anderson[10] (OA) and by OB as have the NMR and thermometry techniques. This work utilized the NMR tail piece used by OA, which included a longitudinal resonance coil not mentioned by OA. This coil had a smaller diameter than the equivalent coil of OB which improved the He^3 filling factor for the new coil by about a factor of 4.

Longitudinal resonance in the *B* phase was observed in a manner similar to the measurements made in the *A* phase by sweeping the cell pressure (temperature) and plotting the rf level across the tank circuit as a function of both time and pressure. The temperature of the resonance at a given frequency was obtained by averaging the apparent resonant temperature on equal numbers

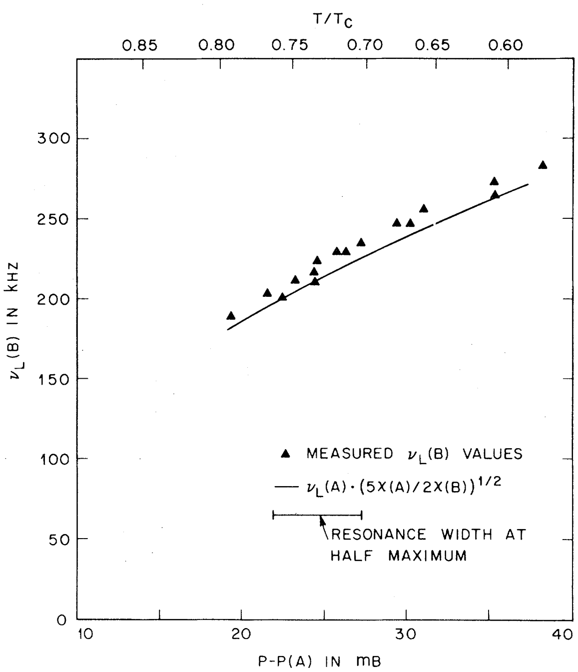

FIG. 1. Longitudinal resonant frequencies in the superfluid He^3 B phase. The solid curve is a calculation based on other A- and B-phase measurements using Leggett's theory.

of increasing and decreasing temperature sweeps. The rate of pressurization in these sweeps was typically 1.5 mbar/min.

The results are shown in Fig. 1. From the pressure dependence of ν_L I estimate that the resonance linewidths of about 4 mbar are equivalent to frequency widths of 20 kHz. This is in good agreement with recent theoretical estimates by Combescot and Ebisawa.[11] The broad linewidth and poor quality of the present signals make quantitative analysis of the line shape impossible.

The observed dependence of the amplitude of the longitudinal resonance on magnetic field strength is consistent with the BSOB theory. I failed to observe a resonance at zero field on some occasions which might be considered evidence supporting the low-field texture of BSOB, although a convinging probe of this texture must await more carefully designed experiments.

Leggett has shown[4] that the longitudinal resonant frequencies for the Anderson-Brinkman-Morel and BW states are related by the following:

$$\nu_L^2(B)=\tfrac{5}{2}\nu_L^2(A)\frac{\chi(A)}{\chi(B)}\left(\frac{\Delta(B)}{\Delta(A)}\right)^2, \tag{1}$$

where $\chi(B)$ is the susceptibility of the BW state, $\chi(A)$ is equal to the Fermi-liquid susceptibility $\chi(\mathrm{F.L.})$, and the ratio of the energy gaps is nearly unity. Using the values of $\nu_L(A)$ from OB and values of $\chi(B)/\chi(\mathrm{F.L.})$ obtained in the transverse-resonance study to be discussed, Eq. (1) is used to predict $\nu_L(B)$ assuming $\Delta^2(A)=\Delta^2(B)$. This last assumption should be valid to within a few percent.[12] The result so derived is shown as the solid curve in Fig. 1. The agreement of this curve with the data (to within ~2%) is excellent considering the broad resonance linewidths, and might be improved upon if $\Delta^2(B)>\Delta^2(A)$. This agreement is much less satisfactory when one uses values of $\chi(B)/\chi(\mathrm{F.L.})$ obtained by extrapolating the magnetization studies of Paulson and co-workers[5] to $T/T_c\sim 0.78$, a procedure which yields an estimate of ν_L at $T/T_c=0.78$ of about 250 kHz, which is nearly 35% above the observed frequency. While I cannot explain the results of Paulson and co-workers, I feel they do not chararcterize the spin system of the B phase. Their data and my own can be made to agree if one assumes that their magnetometer was about 1.5 times as sensitive as their calibration showed, and suggest that it would be desirable for them to be able to calibrate at low temperatures and working pressures.

OB have pointed out that misorientation of the rotation axis $\vec{n}$ by an angle θ away from $\vec{H}_0$ in the BW state gives rise to a transverse-frequency shift:

$$\nu_0^2-(\gamma H_0)^2=\nu_L^2\sin^2\theta. \tag{2}$$

BSOB, by considering the anisotropy energies which orient $\vec{n}$ along $\vec{H}_0$ in the bulk and away from $\vec{H}_0$ near surfaces, and the bending energy required to vary $\vec{n}$ spatially, have obtained expressions for the $\vec{n}$ texture in cylindrical geometries such as those used by OB and OA. At high fields a texture is expected in which $\vec{n}$ points along $\vec{H}_0$ on the cylinder axis and flares outward at nonzero radii to be nearly orthogonal to $\vec{H}_0$ at the cylinder walls. BSOB then solve for the transverse NMR line shape using their $\vec{n}$ texture and Eq. (2). To make contact with experiment, BSOB integrate their line-shape expression from γH_0 to $\gamma H_0+\Delta\omega$ to determine the fraction of spins resonating in $\Delta\omega$. Their result is very accurately approximated by

$$N(\Delta\omega)=\left[1-\frac{H_BR_c}{2H_0R}\ln\left(\frac{(2\pi\nu_LA_0)^2}{2\Delta\omega\gamma H_0}\right)\right]^2. \tag{3}$$

Here H_B is a characteristic magnetic field and

R_c is a characteristic cylinder radius. R, the cylinder radius, is 0.3 cm for this work, and the longitudinal resonant frequencies ν_L can be obtained from Fig. 1. A_0, the angle between $\vec{n}$ and $\vec{H}_0$ at the cylinder walls, is assumed to be $\pi/2$. By dropping the term in Eq. (3) quadratic in the logarithm BSOB obtain an approximate result which shows that if the variation in the log term is ignored, $1-N(\Delta\omega)$ is linear in H_0^{-1}.

To test Eq. (3) I have measured the integrated B-phase absorption signal over a frequency interval 700 Hz wide at a series of magnetic fields. The field gradients across the sample were adjusted by using external coils so that the Fermi-liquid absorption signal had the same shape and a constant width of ~400 Hz at each field. These gradients were maintained during the B-phase measurements. $\chi''(\omega)$ was recorded at a series of temperatures in fields corresponding to resonant frequencies of 1.667, 2.500, and 5.000 MHz and integrated afterward. I insured that no solid signal was included in the resulting integrals. χ''(F.L.) measured in this way both before and after obtaining B-phase data reproduced itself to $\pm 0.5\%$. By keeping the normal-phase linewidth fixed, but nonzero, I had to assume an average $\Delta\omega$ for Eq. (3) which was estimated to be 350 Hz, but insured that the average $\Delta\omega$ would be field independent. Since $\Delta\omega$ enters into Eq. (3) only in the logarithm, the exact value need not be known accurately, although the estimate is probably good to $\pm 20\%$.

The B-phase absorption integrals, normalized by dividing by the Fermi-liquid absorption integrals, are shown plotted as a function of ν_0^{-1} in Fig. 2. The data were first plotted as a function of T/T_c at constant field and smoothed slightly (± 0.005). The field dependence of the data strongly supports the BSOB result. The straight lines in Fig. 2 are used to extrapolate the data to $\nu_0^{-1} = 0$, where $N(\Delta\omega) \simeq 1$, to approximately obtain $\chi(B)/\chi$(F.L.). By considering the full expression in Eq. (3), one expects these intercepts to overestimate $\chi(B)/\chi$(F.L.) by about 0.02. It is remarkable that the intercept values obtained in this work agree with those obtained by reanalyzing the OB results to within ± 0.01. This agreement suggests that variations in the log term are indeed unimportant.

To estimate $\chi(B)/\chi$(F.L.) better I have used Eq. (3) to fit the data. It is these values that I have used to generate the solid curve in Fig. 1. I feel that this analysis allows one to estimate $\chi(B)/\chi$(F.L.) with an overall uncertainty of less than 1%.

Figure 3 shows the values of $\chi(B)/\chi$(F.L.) obtained both from the analysis of Eq. (3) and the extrapolation plotted as a function of T/T_c. The solid curves are estimates of χ(BW)/χ(F.L.) for a BW state using the analysis of Leggett[13] for $Z_0 = -3.0$ and -3.2. The effects of spin fluctuations on the energy gap should cause the BW susceptibility to drop more rapidly near T_c and to level off at a higher temperature than is calculated by

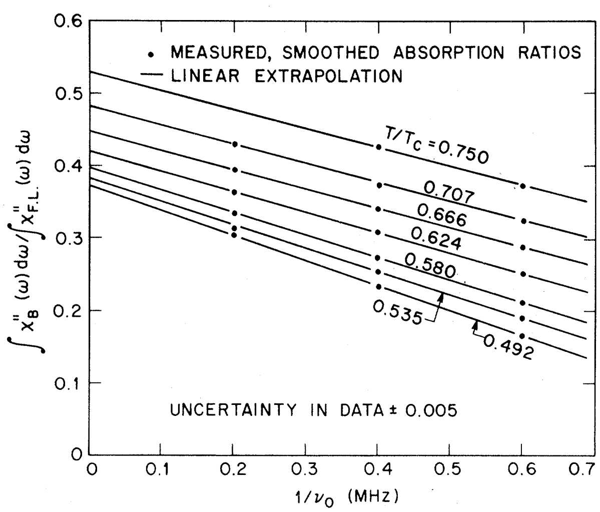

FIG. 2. Normalized B-phase absorption integrals at fixed temperatures versus $1/\nu_0$ in $(\mathrm{MHz})^{-1}$. The solid lines are used to extrapolate to $\nu_0^{-1}=0$.

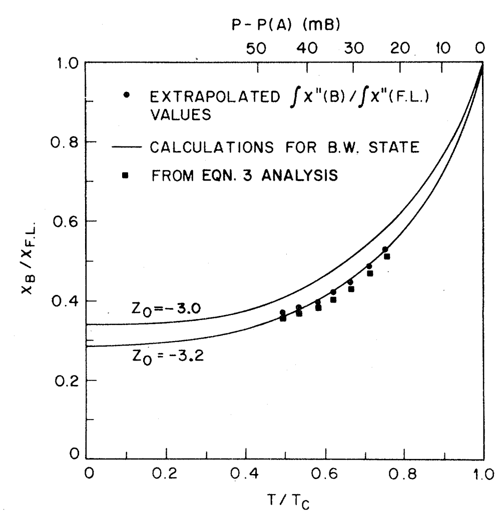

FIG. 3. A comparison of normalized B-phase susceptibilities obtained from extrapolation and from analysis using Eq. (3) of the text with estimates for a BW state assuming $Z_0 = -3.0$ and -3.2.

the Leggett analysis. The present data are probably consistent with a $Z_0 = -3.0$ BW curve suitably corrected for spin-fluctuation effects.

From the susceptibility-data analysis using Eq. (3) I also obtain values of R_cH_B. The three values of R_cH_B obtained at each temperature from fitting the data with the equation agree with each other to $\pm 1\%$. (Errors in choosing $\Delta\omega$ may cause the absolute values of R_cH_B to all be off by as much as 20% but not the ratios of the various values.) Starting at the highest temperature in Fig. 2, the values of R_cH_B for all temperatures shown are 10.1, 10.9, 12.4, 13.7, 15.4, 16.8, and 18.7 Oe cm. These values are nearly proportional to τ, and to values of $\rho_s^{1/2}$ obtained from the present susceptibility estimates using the accepted Fermi-liquid parameters.

By maximizing the magnetic field homogeneity, I was able to verify semiquantitatively the line-shape predictions of BSOB (taking into account a frequency shift not considered by BSOB), provided $\Delta\omega$ was greater than a few hertz. On the low-frequency edge of the observed line shape a peculiar peaking was apparent which might be evidence of collective-mode effects. This peaking became more pronounced as the temperature was lowered.

In performing transverse resonance in the B phase one effectively rotates $\vec{n}$ away from $\vec{H}_0$. This causes an added torque on $\vec{M}$, the magnetization. The energy responsible for this torque is proportional to $(\vec{n}\cdot\vec{H})^2$ and is estimated by Engelsberg, Brinkman, and Anderson[14] to be about 10^{-12} °K/atom at 1 kOe. Curiously, this energy is nearly temperature independent. Leggett[4a] first recognized that this would shift the transverse resonant frequency so that $\nu_0^2 - (\gamma H_0)^2$ would be of the order $2(\mu H_0/\Delta)^2(\nu_L)^2$, but dismissed this tiny shift as being "very much too small to observe in practice." I have measured this phenomenon at 1.1, 2.8, and 5.0 MHz, where I observe temperature-independent shifts of roughly 13.5, 35, and 55 Hz, respectively. These values are equivalent to a g-value shift in the superfluid of 1.2×10^{-5}. Leggett[15] obtains a value of 0.85×10^{-5} for the effective g-value shift using the results of Engelsberg, Brinkman, and Anderson, in reasonable agreement with the present experimental observations. Collective-mode effects might explain the discrepancy between the two values.

I feel that these results strongly support the identification of the B phase as the BW state. The wealth of characteristic behavior observed here in the B phase may in fact make that identification more certain than the Anderson-Brinkman-Morel A-phase identification. At the same time these results support many aspects of the BSOB theory and the assumptions upon which it is based.

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