Saturation of the Ion-Acoustic Instability in a Weakly Ionized Plasma*

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The spatial saturation and the frequency dependence of the energy spectrum of ion-acoustic waves in a low-pressure discharge in helium in the pressure range 0.03–0.2 Torr has been investigated. In the high-pressure region the spectrum is peaked around the most unstable frequency predicted by linear theory [i.e., $\omega_{\text{peak}} \sim (1/\sqrt{3})\omega_i$], and the wave energy saturates at the modest value of $W/nT_e \simeq 10^{-5}$. At low pressures the wave energy increases and varies with frequency according to $W(\omega) \propto \omega^{-\alpha}$, with α assuming values between 2.2 and 2.8.

In this Letter we present measurements of the nonlinear saturation level and the turbulent frequency spectrum of ion-acoustic waves that are excited spontaneously in a low-pressure discharge. The growing interest in the nonlinear development of unstable ion-acoustic waves and the resulting turbulent state has resulted in the formulation of a number of theoretical models that invoke different nonlinear interactions and arrive at vastly different values for the saturation level as well as the wave-number or frequency dependence of the turbulent energy spectrum.[1-9] Experiments in the collisionless regime with drift velocities comparable to the electron thermal speed have confirmed the ω^{-1} dependence of the energy spectrum first proposed by Kadomtsev.[2,3] The present communication addresses itself to the regime where the excitation of the instability is collision dominated and where the electron drift velocity is typically much smaller than the electron thermal speed ($V_{de}/V_e \sim 0.1$). Under these conditions we find a substantially lower saturation value and an energy spectrum that decreases more rapidly with frequency than the Kadomtsev spectrum. The results are in agreement with the idea that amplitude saturation is produced by a competition between electron trapping by the waves and collisional untrapping.[4]

The experiments were carried out on a low-pressure hot-cathode discharge in helium in the pressure range 0.03–0.2 Torr. The discharge tube was 1 m long and 6.4 cm in diameter; $T_e/T_i \simeq 10^2$, $B=0$. The ion-acoustic waves were detected as current fluctuations to plane molybdenum probes, 1 mm in diameter, biased at space potential and movable in the radial and axial directions. In the pressure regime of the present experiment, the excitation of ion-acoustic waves is determined by the competing effects of inverse Landau damping of electrons and ion-neutral collisions. The ion-acoustic instability sets in at a critical current for which the condition $(\omega_i/\nu_i)\times(v_d/v_e)=(27/2\pi)^{1/2}$ is satisfied.[10] Here ω_i is the ion plasma frequency, ν_i the ion-neutral collision frequency, v_d and $v_e=(T_e/m)^{1/2}$ are the drift velocity and the thermal speed of the electrons. The details of the experiment and measurements of the linear phase of the instability were reported elsewhere.[10]

At onset, the observed frequency of the most unstable mode is given by $\omega_{\max}=\omega_i/3^{1/2}$ in agreement with theory. This frequency shifts slightly toward higher values as one moves into the unstable region.[10] The unstable waves grow exponentially in space and saturate at distances of the order 5–20 cm. The amplitude then stays relatively constant over the length of the discharge. Measurements of the spatial growth rate will be discussed in a subsequent paper; in this Letter we restrict ourselves to the investigation of the saturated region.

The spectrum of the unstable waves in this region exhibits a different character depending on the pressure and the discharge current. For pressures larger than about 0.06 Torr the spectrum is peaked around the most unstable frequency predicted by linear theory [i.e., $\omega_{\text{peak}} \sim (1/\sqrt{3})\omega_i$] in the current regime that has been explored. For lower pressures a sudden transition occurs above a certain discharge current, in which the spectrum shifts towards lower frequencies. We shall refer to these two regimes as the high- and low-pressure regimes.

In the high-pressure regime the unstable wave spectrum extends over a well-defined frequency range. This allowed us to measure the total wave energy at saturation. With the probe biased at space potential, the fluctuating part of the electron probe current and the wave potential are related to a good approximation by $i(\omega)/i_0=e\varphi(\omega)/$

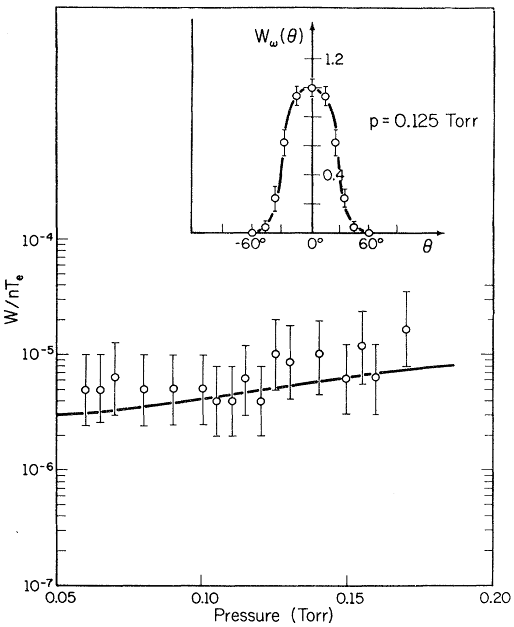

FIG. 1. Total wave energy at saturation in the high-pressure regime, and angular distribution of the wave energy. The peak frequency of the spectrum was 17 MHz; $T_e = 6$ eV, and $n_e = 6\times10^{10}/\text{cm}^3$. The electron-neutral collision frequency was taken as $\nu_{en} = 1.6\times10^9 p$ sec^{-1}, where p is the pressure in Torr. The solid curve represents the condition $\omega_B = (\nu_c)_{\text{eff}} + \nu_{en}$ [Eq. (2)].

T_e, where i_0 is the dc probe current. The total wave energy is then given by

$$W/nT_e = (e^2/T_e^2)\int |\varphi(\omega)|^2\, d\omega. \quad (1)$$

In Fig. 1 measured values of the wave energy are plotted for various pressures. The absolute values are probably not more accurate than a factor of 2 because of uncertainties in the determination of the plasma potential and the effects of probe geometry. The angular distribtuion of the unstable waves was determined in an approximate fashion by changing the orientation of the plane probe with respect to the tube axis. In this way it was determined that in the high-pressure regime the wave propagation is typically restricted to a narrow cone of angle 20–30° around the tube axis. An example for the angular distribution is also shown in Fig. 1.

On the basis of ideas put forth by Nishikawa and Wu,[4] the magnitude of the saturation level can be understood as being determined by a competition between trapping of charge particles in the potential troughs of the waves and collisional processes, which tend to scatter them out of the trapped region of velocity space. Since the wave energy is rather low and $v_d \ll v_e$, only electron trapping merits consideration. Saturation would set in when the bounce frequency ω_B is of the order of the effective collision frequency for untrapping, which in the present case is determined by electron-neutral and Coulomb collisions. The condition $\omega_B = (\nu_c)_{\text{eff}} + \nu_{en}$ then leads to the relation

$$\omega(M/m)^{1/2}(e\varphi/T_e)^{1/2} = \nu_c(e\varphi/T_e)^{-1} + \nu_{\text{en}}. \quad (2)$$

Here we have used[4,11,12] $(\nu_c)_{\text{eff}} = \nu_c(v_e^2/v_t^2)$, with v_t the trapping velocity $= (e\varphi/m)^{1/2}$, ν_{en} the electron-neutral collision frequency, and ν_c the Coulomb collision frequency for 90° deflection of the thermal electron, given by[13]

$$\nu_c = 3.26\times 4\pi n e^4(\ln\Lambda)m^{-1/2}(2T_e)^{-3/2}.$$

The saturation level as determined from Eq. (2) is plotted in Fig. 1 for comparison. The agreement with the experimental data is very satisfactory, considering that Eq. (2) is only an order-of-magnitude estimate. In this context it should be noted that collisionless trapping predicts saturation values that are much too low.[9]

As mentioned before, the spectrum of the unstable waves shifts dramatically towards lower frequencies when the current is increased in the low-pressure region. It was determined experimentally that the transition from the high-pressure to the low-pressure behavior occurred when the effective collision frequency for Coulomb collisions became of the order of the electron-neutral collision frequency. For the region in which the electron-neutral collision frequency can be neglected, Nishikawa and Wu calculated the frequency dependence of the turbulent wave spectrum as $W(\omega) \propto \omega^{-7/3}$.

An example of the measured frequency dependence of the wave energy in the low-pressure regime is shown in Fig. 2. In the low-frequency region, $\omega \lesssim 0.6\omega_i$, the wave energy decreases with the frequency according to $W(\omega) \propto \omega^{-\alpha}$ indicated by the dashed line. α was found to be independent of discharge current but varied somewhat with pressure. Figure 3 shows measured values of α for various discharge currents and two different pressures. The results are in good agreement with the theoretical value $\alpha = 7/3$. Similar results have been reported previously for ion-acoustic turbulence excited by an electron beam.[14] The region around the most unstable mode in Fig. 2 still shows the effect of the

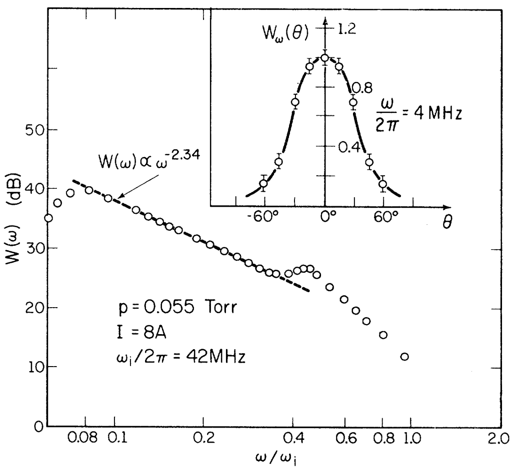

FIG. 2. Example of the frequency dependence and angular distribution of the wave energy in the low-pressure regime. $n_e = 1.6\times 10^{11}/\text{cm}^3$, $T_e = 7$ eV, $v_d = 1.2\times 10^7$ cm/sec.

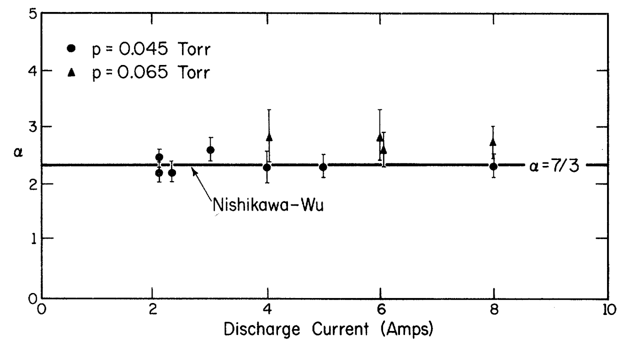

FIG. 3. The coefficient α, defined by $W(\omega)\propto \omega^{-\alpha}$, as a function of discharge current for two different pressures.

source term of the instability that feeds energy into the spectrum at this frequency and gives rise to the slight peak. Beyond this peak the energy falls off rapidly with frequency and no general power law seems to exist. It is clear that the region around $\omega \sim \omega_i$ will show a complicated behavior since the spectrum has a cutoff at this frequency. Also shown in Fig. 2 is an example of the angular distribution of the frequency spectrum. Compared with the high-pressure region, the unstable waves extend over a wider angular range. Associated with the shift towards lower frequencies is an increase of W/nT_e to values of the order of 10^{-3}–10^{-2}. The absolute magnitude of the wave energy in the low-pressure regime could only be determined approximately because the low-frequency part of the spectrum, which contributes most to the wave energy, showed a complicated structure and could not be measured with sufficient accuracy. This observed increase in the wave energy can be understood qualitatively from Eq. (2), which predicts a higher saturation value for lower frequencies.

In summary, we conclude that the saturation of the ion-acoustic instability in a weakly ionized plasma, at both high and low pressures, is adequately described by a competition between electron trapping and collisional untrapping. In the high-pressure regime, electron–neutral-atom collisions are responsible for the untrapping mechanism, and in the low-pressure regime the "effective" Coulomb collision, proposed by Nishikawa and Wu, is responsible for it. The transition between high- and low-pressure behavior occurs when the electron-neutral collision frequency is comparable with the effective Coulomb collision frequency. Finally, the turbulent spectrum predicted by Nishikawa and Wu, $W(\omega)\propto\omega^{-2.3}$, was obtained in the low-pressure regime.

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Sum-Rule Analysis of Collective Modes in a Turbulent Plasma

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An explicit relationship between the dispersion relations of collective modes and the energy spectrum of turbulent fluctuations is derived from a sum-rule analysis of the dielectric response function. It is pointed out that turbulence produces significant modification in the dispersion relation so that an experimental investigation of such an effect offers a promising method of probing a turbulent plasma.

In the study of a complex many-particle system, an exact theoretical formula, derived from first principles, has been instrumental in clarifying the relationship between experimentally observable quantities and microscopic properties of the system. Thus, the dynamic form factors of a many-particle system can be revealed by scattering experiments with the aid of the Born-approximation formula for the cross section.[1,2] In this Letter, we wish to point out an explicit relationship between the dispersion relations of collective modes, measurable through interferometric technique for example, and the energy spectrum of density fluctuations in a plasma. The relationship is derived from a sum-rule analysis of the dielectric response function; the result is valid as long as the plasma is in a stationary state.

The effects of enhanced fluctuations on the decay rate of a collective mode have been considered in the literature[3] with various degrees of approximation; to the best of our knowledge, this work is the first to establish an exact dependence of the real frequency on the spectral function of fluctuations in a turbulent plasma. For a plasma in a turbulent stationary state, an independent assessment of the turbulence spectrum, either theoretical or experimental, can be correlated through such a formula with the dispersion relation of the collective mode measured in the plasma. Such an investigation should thus provide a useful clue to clarification of fundamental properties of plasma turbulence.

We consider a charged-particle system with average density n, whose Hamiltonian (per unit volume) is expressed as

$$H=\sum_{j=1}^{n} p_j^2/2m+\tfrac{1}{2}\sum_{\vec{q}}{}'\Phi(q)(\rho_{\vec{q}}\rho_{\vec{q}}^{\dagger}-n). \quad (1)$$

Here, $\Phi(q)$ and

$$\rho_{\vec{q}}=\sum_{j=1}^{n}\exp(-i\vec{q}\cdot\vec{r}_j)$$

are the Fourier components of the interaction potential and of density fluctuations, respectively; $\vec{p}_j$ and $\vec{r}_j$ are the momentum and the position of the jth particle. According to the standard linear-response formalism,[2] the frequency- and wave-vector-dependent dielectric response function $\epsilon(\vec{q},\omega)$ is calculated in terms of a statistical average of the retarded commutator of density fluctuations. In the spectral representation, the susceptibility $\chi(\vec{q},\omega)$, defined through $[\epsilon(\vec{q},\omega)]^{-1}-1=\Phi(q)\chi(\vec{q},\omega)$, takes the form

$$\chi(\vec{q},\omega)=\int_{-\infty}^{\infty}(d\omega'/\pi)\,\mathrm{Im}\chi(\vec{q},\omega')/(\omega'-\omega-i\eta), \quad (2)$$

where η is a positive infinitesimal, and

$$\mathrm{Im}\chi(\vec{q},\omega)=-(1/2\hbar)\int_{-\infty}^{\infty}dt\,\langle[\rho_{\vec{q}}(t),\rho_{\vec{q}}^{\dagger}(0)]\rangle\exp(i\omega t) \quad (3)$$