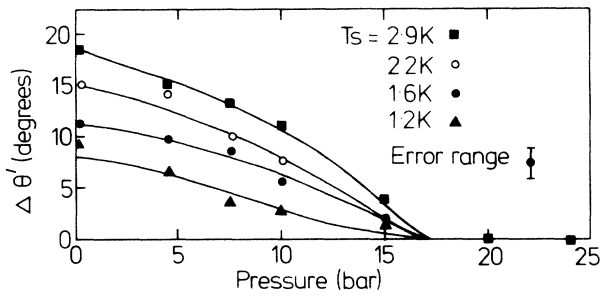

FIG. 3. Variation of the angular spreading of the slit source beam half-width at 1% peak height as a function of pressure for various source temperatures.

observed. However, the increase in $\Delta\theta'$ as the mean phonon energy increases and as the pressure decreases is in qualitative agreement with the expected behavior for a three-phonon decay process when γ is negative at $p \lesssim 17$ bars and $|\gamma|$ increases with decreasing pressure. The broadening is certainly not due to interactions with the background thermal phonons since even at the lowest pressure the width of the distribution is independent of the background helium temperature below ~0.25 K, and the fact that the distribution goes to the geometric limit at $p > 17$ bars rules out the possibility of broadening due to four-phonon decay processes or impurity scattering. We therefore conclude that the broadening we observe at $p \lesssim 17$ bars is consistent with three-phonon decay processes and that this is evidence for γ being negative in this pressure region.

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Experimental Determination of the Viscosity and Density of the Normal Component of Superfluid ^{3}He at the Melting Curve

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The viscosity and the density of the normal component of liquid ^{3}He in the A and B phases have been determined at the melting curve with a vibrating-wire viscometer. The results show that resistive flow of the liquid is accompanied by a flow of zero viscosity. The data thus prove superfluidity both in the A and in the B phase.

The first clear indication of superfluidity in liquid ^{3}He was the drastic change[1] in the damping of a vibrating-wire viscometer at the A and B transitions. Using the same technique with improved resolution, we have now been able to establish quantitative values for η_n and ρ_n, the viscosity and the density of the normal component of liquid ^{3}He, respectively, as a function of temperature. Our results show that viscous flow in the A and B phases is accompanied by frictionless flow, i.e., the two phases of ^{3}He behave as superfluids.

Several vibrating-wire experiments[2-4] in ^{4}He and in normal ^{3}He have successfully been interpreted in terms of Stokes's[5] theory of an infinite wire of circular cross section oscillating in an

unbounded fluid. He obtained

$$\vec{F} = -\pi a^2 \rho [k(m)\, d\vec{v}/dt + \omega k'(m)\vec{v}] \quad (1)$$

for the force per unit length of a wire vibrating with a small amplitude at an angular frequency ω ($2\pi \times 1900$ sec^{-1} in our case). Here a is the radius of the wire and ρ is the density of the fluid; $\vec{v} = \mathrm{Re}(\vec{v}_0 e^{i\omega t})$ is the velocity of the wire. The parameter m is given by $m = a/2\delta$, where $\delta = (\eta/\rho\omega)^{1/2}$ is the characteristic length of decay for the velocity field around the wire, i.e., the viscous penetration depth. Functions $k(m)$ and $k'(m)$, as introduced and tabulated by Stokes, determine the hydrodynamic mass and the damping of the wire.

Stokes's model can be modified to describe a two-fluid system with an isotropic superfluid component.[3] In this case we obtain from Eq. (1)

$$\vec{F} = -\pi a^2 \{[\rho_n k(m) + \rho_s] d\vec{v}/dt + \omega \rho_n k'(m)\vec{v}\}, \quad (2)$$

where ρ_s is the density of the superfluid component. The total density of the fluid ρ is the sum of ρ_n and ρ_s; now we have $\delta = (\eta_n/\rho_n\omega)^{1/2}$. Equation (2) shows that the nonviscous component of the fluid influences the hydrodynamic mass of the wire only, but cannot cause damping of the oscillations. We assume that this description is also valid in the case of an anisotropic superfluid, provided that η_n, ρ_n, and ρ_s are replaced by the suitably averaged quantities $\bar{\eta}_n$, $\bar{\rho}_n$, and $\bar{\rho}_s$.

An approximate solution for the shift of the resonance line of a vibrating-wire viscometer is then given by

$$\Delta\omega_R = \tfrac{1}{2}\omega_0 [\bar{\rho}_n k(m) + \bar{\rho}_s]/\rho_V, \quad (3)$$

where ω_0 is the resonant frequency in vacuum and ρ_V is the density of the viscometer wire (NbZr, $\rho_V = 7.79$ g/cm^3). In the same approximation the width of the resonance at half-maximum becomes

$$\Delta\omega_W = \omega_0 \bar{\rho}_n k'(m)/\rho_V. \quad (4)$$

After combining Eqs. (3) and (4) we find

$$\frac{k-1}{k'} = \frac{2\Delta\omega_R - (\rho/\rho_V)\omega_0}{\Delta\omega_W}. \quad (5)$$

If both $\Delta\omega_R$ and $\Delta\omega_W$ are measured, Eq. (5) can be used to obtain m and, hence, $\bar{\eta}_n/\bar{\rho}_n$; Eq. (4) may then be employed to solve for $\bar{\eta}_n$ and $\bar{\rho}_n$ separately. We emphasize that Eqs. (3)–(5) are not exact and were not used in our final calculations; iterative processes were actually employed.

If the oscillation amplitude r_0 of the wire is of the same order of magnitude as the radius a of the wire, Stokes's solution[5] may fail through the omission of quadratic terms in the Navier-Stokes equation. In our experiment, however, r_0 is always several orders of magnitude smaller than a. In this case Fisk[6] has shown, by using the relevant Reynolds and Strouhal numbers, that Stokes's solution remains valid as long as the inequality $r_0/L \ll 1$ is fulfilled. Here L is a characteristic length, either the radius a (0.15 mm in our case) of the wire or the viscous penetration depth δ, whichever is smaller. At our lowest experimental temperatures, below 1.3 mK, $r_0/a \simeq 10^{-4}$ and $r_0/\delta \simeq 10^{-5}$; Fisk's condition is thus amply fulfilled. Further, the correction to Stokes's solution due to the finite size of the ^{3}He sample[7] is estimated to be small in the experiment; the radial distance of the wire to the cell wall is 2 mm in our case.

We refrigerated the ^{3}He specimen by means of Pomeranchuk's method.[8] After ^{3}He had been brought to a temperature well below the B point, pressurization was stopped and the sample was allowed to warm through the B' and A transitions under the influence of the stray heat leak. Warming-up rates of about 10 μK sec^{-1} were typical. Sometimes the pressure was slowly released during warming. The cycle was then repeated. Resonance curves were measured while traversing the A and B phases and the normal Fermi-liquid region in both directions; $\Delta\omega_R$ and $\Delta\omega_W$ were then determined from the data. The measuring technique required that the sample be exposed to a magnetic field of 0.149 T during the experiment; other experimentalists[9] as well as ourselves[10] have shown that a field of this magnitude does not alter the conclusions derived from these results.

The values of $\Delta\omega_R$ and $\Delta\omega_W$ from our measurements in the normal Fermi-liquid region were first analyzed to find the viscosity of ^{3}He from about 15 mK to the A transition. The data are consistent with the T^{-2} law expected for a normal Fermi liquid.[3,4,11] We obtain for the viscosity at the A point $\eta_A = 0.127$ P.[10]

Next, the measured resonance shifts, both in the normal ^{3}He and in the A and B phases, were plotted against the corresponding resonance widths. The difference in $\Delta\omega_R$ between the normal Fermi-liquid phase and the superfluid phases at equal damping, $\delta\Delta\omega_R$, was then used to determine $\bar{\eta}_n$ and $\bar{\rho}_n$ as described above; the maximum difference, observed at ~1.2 mK, is about

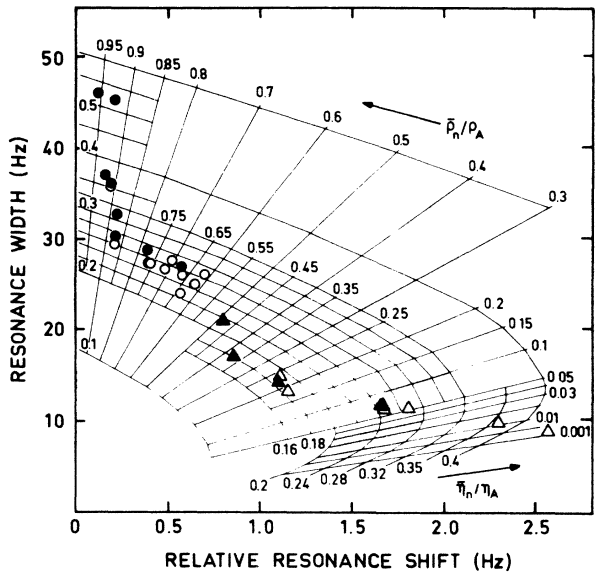

FIG. 1. Width of the resonance curve at half-maximum as a function of the relative resonance shift between the resonance curves of equal damping in the normal Fermi-liquid region and the superfluid phases of ^{3}He. Several lines of constant $\bar{\eta}_n$ and $\bar{\rho}_n$ have also been plotted in the figure. Open symbols represent points taken upon cooling and filled ones during warming; circles are for in the A phase and triangles for in the B phase.

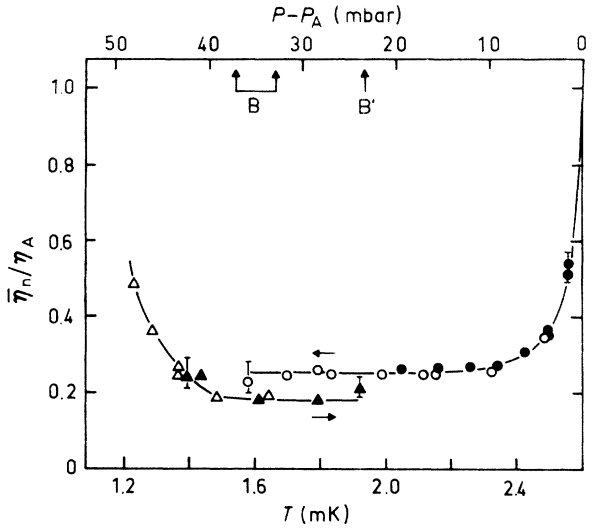

FIG. 2. Reduced average viscosity $\bar{\eta}_n/\eta_A$ of the normal component of superfluid ^{3}He at the melting curve as a function of temperature; η_A is the viscosity of normal ^{3}He at the A point. The tentative temperature scale was plotted by assuming $T_A = 2.6$ mK (Refs. 10 and 12) and $dP/dT = -35$ bar/K on the melting curve at $T < T_A$. The pressure difference relative to the A transition is also indicated. For an explanation of the data symbols we refer to Fig. 1. The curves and arrows have been drawn as visual aids only. The error bars reflect the estimated uncertainty of ± 0.15 Hz in the determination of the shifts of the resonance curves.

2.5 Hz. The data were also found to be self-consistent, within the framework of Stokes's model, through an examination of the relation connecting the amplitude and the width of the resonance curves.

In Fig. 1 we have plotted $\Delta\omega_W$ as a function of $\delta\Delta\omega_R$. The data were measured during a run in which the A and B phases were traversed three times. Lines of constant $\bar{\eta}_n$ and $\bar{\rho}_n$ are also shown in the figure. Figure 2 illustrates our results on $\bar{\eta}_n/\eta_A$ and Fig. 3 on $\bar{\rho}_n/\rho$.

We observe from Fig. 2 that, as the sample cools through the A transition, $\bar{\eta}_n$ decreases rapidly, within 0.3 mK, to approximately 25% of η_A and then becomes essentially constant. The decrease is much faster than $(T/T_A)^8$, estimated by Greytak *et al.*[13] for the B phase from heat-flow measurements at lower pressures. At the B transition, below 1.7 mK, $\bar{\eta}_n$ drops discontinuously to about 20% of η_A but starts to increase again as the temperature is reduced. Upon warming back to the A phase $\bar{\eta}_n$ returns to 25% of η_A via the lower set of points which thus seem to characterize the B phase. We note in passing that the viscosity of both ^{3}He and ^{4}He behave in the su-

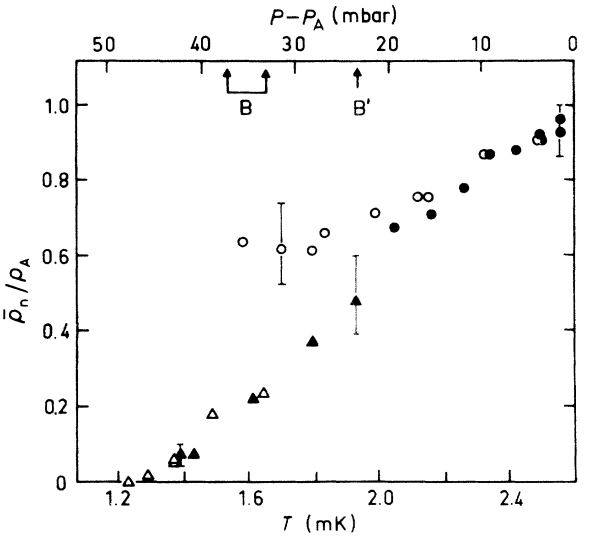

FIG. 3. Reduced average density $\bar{\rho}_n/\rho_A$ of the normal component of superfluid ^{3}He at the melting curve as a function of temperature; ρ_A is the total density of ^{3}He at the A point. The slight variation of ρ, as a function of pressure, was taken into account in the normal Fermi-liquid region by extrapolating the data of Grilly (Ref. 12). For an explanation of the data symbols we refer to Fig. 1.

perfluid region in a rather similar way. A calculation of viscosity for an isotropic Fermi liquid by Shumeiko[14] is in fair agreement with our results. His theory gives $\eta = \text{const}$ ($< \eta_c$) for $T \ll T_c$ and $\eta = \eta_c(1 - \alpha\Delta/T)$ in the vicinity of T_c. Here α is a constant and Δ is the energy gap; in our comparison we assumed $T_c = T_A$ and $\eta_c = \eta_A$. Seiden's calculation[15] for $T \ll T_c$ predicts η to be a weak function of temperature. Quantitative comparisons between these theories and our measurements are, however, difficult to make.

Figure 3 shows that in the A phase $\bar{\rho}_n$ decreases relatively slowly with temperature, approximately as T/T_A. This is in fair agreement with the fourth-sound measurements of Kojima, Paulson, and Wheatley.[16] However, at the B transition $\bar{\rho}_n$ drops abruptly from about 60 to 25% of ρ and then decreases rapidly upon further cooling; at our lowest experimental temperatures, below 1.3 mK, $\bar{\rho}_n$ is $<1\%$ of ρ. The jump in $\bar{\rho}_n$, if any, in warming through the B' transition is small.

Weak-coupling theories predict that ρ_n decreases most rapidly for the isotropic states. In this case ρ_n/ρ can be described by the Yoshida function[17] which gives the same value for $\bar{\rho}_n$ at $T/T_c \simeq 0.3$ as we find experimentally at 1.3 mK. However, our experimental temperature corresponds to $T/T_c \geq 0.5$, assuming for T_c a maximum temperature of the A transition, 2.6 mK. The agreement between theory and experiment becomes even poorer if the Fermi corrections, through Leggett's formula,[18] are introduced.

If it is possible that the wire has an orienting effect on the liquid, which tends to rotate the orbital angular momentum perpendicular to the wire's surface, a proper angular average could conceivably emphasize the directions where the gap is largest. However, it is not probable that this effect could be so large as to explain the discrepancy, because, e.g., in the Anderson-Morel state ρ_n/ρ, even in the most favorable direction,[19] does not decrease more rapidly than in the isotropic states.

A more likely explanation follows from the experimental observation[20] that the specific heat below the A transition is larger than weak-coupling theory would predict. If this is true also in the B phase, the gap decreases actually faster than predicted by conventional BCS-type calculations and consequently also ρ_n vanishes more rapidly.

The rapid variation of $\bar{\rho}_n$ as a function of temperature below the B transition seems to be in disagreement with the data of Kojima, Paulson, and Wheatley[16]; they observed no appreciable change in the temperature dependence of $\bar{\rho}_n$ near B'. The experimental discrepancies may indicate that the bulk properties of the ^{3}He superfluids, particularly those of the B phase, are changed when the liquid is contained inside a porous medium necessary for the observation of fourth sound; this could easily lead to spurious conclusions. Major changes in the bulk properties of the superfluid phases have been observed in NMR measurements[21] when ^{3}He was intermixed with fine platinum powder.

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Excitation Spectrum and Thermodynamic Properties of Liquid Films in Cylindrical Pores*

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We examine the dynamic and thermodynamic properties of adsorbed liquid films in cylindrical cavities. We find a dynamical stability limit for the thickness of films adsorbed uniformly on the walls. This limit is shown to coincide with a thermodynamic stability limit which depends on both substrate and capillary forces. The Kelvin equation describing capillary condensation is generalized to include the substrate interaction. The low-temperature properties of the films are calculated.

Considerable attention has been directed recently toward understanding the excitation spectrum and thermal properties of multilayer He^4 films.[1-6] The film differs from the bulk liquid for several reasons, including explicit size effects associated with the boundary conditions for the velocity and superfluid order parameter, and exchange of particles, energy, and momentum with both vapor and substrate. As one or more dimensions of the film becomes comparable to or smaller than the thermal wavelength of the excitations in bulk He^4, the excitation spectrum begins to take on discrete character, leading to pronounced modification of the thermal behavior.

In this paper we report studies of volume and surface excitations of liquid films in partially and completely filled cylindrical cavities, an idealization of the geometry in a porous material. Anomalous behavior of the calculated spectrum leads us to consider the thermodynamics of adsorption. We then generalize the Kelvin equation to include the substrate potential, obtaining results applicable to the adsorption of He^4 and those normal liquids which wet the substrate walls.

We take the liquid to be compressible, inviscid, and irrotational with velocity $\vec{v}=\nabla\varphi$. We consider motion of a single component (superfluid) alone, neglecting effects of the normal fluid in the boundary healing region. This is plausible for a relatively thick film of He^4 at low temperature. By linearizing the continuity and Euler equations, we find for motion at frequency ω

$$\nabla^2\varphi+(\omega/s)^2\varphi=0, \tag{1}$$

where we take the speed of sound s to be constant. The modes of the system are determined by solving (1) subject to appropriate boundary conditions.[7] The solutions are of the form

$$\varphi=f_{km}(r)\exp[i(kz+m\theta)]$$

in terms of variables r, θ, and z appropriate to the cylindrical symmetry. In all cases f_{km} is expressed in terms of Bessel functions.

The most interesting case is that of partially filled pores [illustrated in Fig. 1(a)]. The eigenfrequencies, when $l^2\equiv k^2-\omega^2/s^2>0$, are given by

$$\omega_{km}{}^2=[-g(a)+(\sigma/\rho_0a^2)(1-m^2-k^2a^2)]\times l(d\ln f_{km}/dr)_{r=a}. \tag{2}$$

Here $Mg(a)=-(dU/dr)_{r=a}$, where M is the atomic mass and $U(r)$ is the Van der Waals potential due to the substrate; σ is the surface tension.[8] We identify these modes as ripplons in the sense that such surface oscillatory modes occur in the limit of an incompressible fluid, in which case l is replaced by k.

The case $m=0$, $lR\ll1$ is of particular interest. Using the small-argument limit of the Bessel