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Evidence for Upward Phonon Dispersion in Liquid ^{4}He from the Angular Spreading of Phonon Beams*

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Measurements have been made of the angular spreading of initially well-defined beams of phonons of energies $\hbar\omega/k_B \lesssim 12$ K in liquid ^{4}He at $T \sim 0.1$ K and at pressures between saturated vapor pressure and 24 bars. The spreading is consistent with three-phonon decay processes and the results strongly suggest that in the dispersion relation $\omega = cq(1-\gamma\hbar^2q^2)$ the constant γ is negative at $p \lesssim 17$ bars and positive at higher pressures.

There is considerable current interest in the exact shape of the dispersion curve for elementary excitations in liquid ^{4}He at low temperatures ($T < 1$ K) and at different hydrostatic pressures p. In particular the question of whether at low pressures the low-q ($q \lesssim 0.2$ Å^{-1}) phonon region curves upwards above the asymptotic linear form $\omega = cq$ before bending over towards the roton minimum is still unresolved. The present situation is well summarized in the recent papers of Svenson, Woods, and Martel[1] and Narayanamurti, Andres, and Dynes,[2] and in the comprehensive review by Woods and Cowley.[3] The first attempt to obtain detailed information about the shape of the dispersion curve using heat-pulse techniques was made by Guernsey and Luszczynski.[4] Unfortunately this work was before the advent of the dilution refrigerator and so the repetitive experiments necessary for signal averaging to measure small signals with good accuracy were not possible. This work did, however, indicate that at the saturated vapor pressure (svp) any increase in phonon group velocities due to upward dispersion was small ($\lesssim 1\%$), and the authors concluded that there was no evidence for such an effect to within the accuracy of their experiment.

The simplest representation of the dispersion curve in this low-q region is the expansion

$$\omega(q) = cq(1 - \gamma\hbar^2 q^2), \tag{1}$$

where $c = 238$ m sec^{-1} is the low-frequency first-sound velocity, the possibility of a significant linear deviation from linearity having been ruled out by the elegant experiment of Roach *et al.*[5] At the svp, neutron-scattering measurements give $\gamma = (0 \pm 2)\times 10^{36}$ g^{-2} cm^{-2} sec^2 but other experiments imply that γ may be negative (i.e., upwards deviation from linearity) with values of $|\gamma|$ up to $\sim 10^{38}$ g^{-2} cm^{-2} sec^2.[3] The situation is considerably clearer at $p = 24$ bars (just below the solidification pressure) with all experiments implying that γ is positive, although with little agreement over its numerical value. (Recent direct measurements[6] of the group velocity of 90-GHz phonons at 24 bars do, however, strongly support the neutron-scattering[1] value of $\gamma = 6.2 \times 10^{37}$ g^{-2} cm^{-2} sec^2.) At intermediate pressures there is some evidence from both ultrasonic attenuation and specific-heat measurements that γ changes sign at $p \simeq 16$ bars.[3]

The interest in the exact shape of the dispersion curve arises because of the very important consequences it has for the lifetime of the excitations (phonons) in this region at low temperatures. Since the liquid ^{4}He is isotropic and there is only one branch of the dispersion curve, phonon decay by a three-phonon process $(\omega_1, \vec{q}_1) \to (\omega_2, \vec{q}_2) + (\omega_3, \vec{q}_3)$ is prohibited if γ is positive. In this case ω and $\vec{q}$ cannot be simultaneously conserved and the dominant decay is by the much weaker four-phonon process. If γ is zero or negative, however, the three-phonon decay process can take place, the decay phonon wave vectors $\vec{q}_2$, $\vec{q}_3$ making increasingly large angles θ to the incident $\vec{q}_1$ as $|\gamma|$ and $\vec{q}_1$ increase. It is readily shown that with the dispersion relation of Eq. (1)

the maximum value of θ is $\theta_m = \hbar q_1(6|\gamma|)^{1/2}$ in the limit of small θ. Thus an experiment which directly measures the occurrence or nonoccurrence of three-phonon decay processes enables the question of the sign of γ at any pressure to be answered. In this Letter we describe such experiments where we find convincing evidence that γ is negative at the svp with γ decreasing monotonically with increasing pressure up to $p \simeq 17$ bars. At $p \gtrsim 17$ bars the results are consistent with the positive γ implied by other experiments.[3,6]

The basic idea of the experiments to be described is to measure the angular spreading of a beam of phonons radiated from a thin-film heater and collimated to produce a geometrically defined beam whose angular distribution would, in the absence of phonon scattering or decay processes, be conserved as the beam propagated through the helium. Any departures from the geometric beam width could thus be interpreted in terms of angular spreading due to such scattering or decay processes.

From a variety of experiments performed in this and other laboratories,[2,4,6-9] it is known that at least some phonons radiated from a pulse-heated metallic film into liquid ^{4}He at $T \lesssim 0.2$ K propagate ballistically with velocity $\simeq c$, and that the mean free paths of these phonons increase with p. The first experiment was therefore designed to place as small a limit as possible on any beam spreading due to phonon scattering or decay at high pressures. A resistive gold film was evaporated uniformly over a 3.7-mm length of a 0.75-mm-diam glass rod which was then positively located with its axis 0.60 mm behind an aluminum alloy bar 1.57 mm wide. (See Fig. 1 inset for sketch plan.) The bar therefore cast a shadow in the distribution of phonons radiated from the source, the width of this shadow being calculated from the measured geometry and also measured directly by watching the occlusion of the heater behind the bar as it was rotated on a spectrometer table. (These two determinations were in agreement within the 1° error range.) This source assembly was mounted on needle bearings so that it could be rotated about an axis parallel to that of the cylindrical heater while in a pressure cell of liquid ^{4}He maintained at $T \lesssim 0.1$ K by a dilution refrigerator. The rotation was produced by the alignment of a permanent magnet attached to the source assembly in an external field of ~1 mT, the source angle thus being the same as that of the applied external field. The source could be rotated continuously at angular speeds up to 0.1 rad min^{-1} with no detectable heating of the cell and only ~2° lag between the source angle and the applied field angle. For static positioning any error due to rotational hysteresis was $\lesssim 0.5°$.

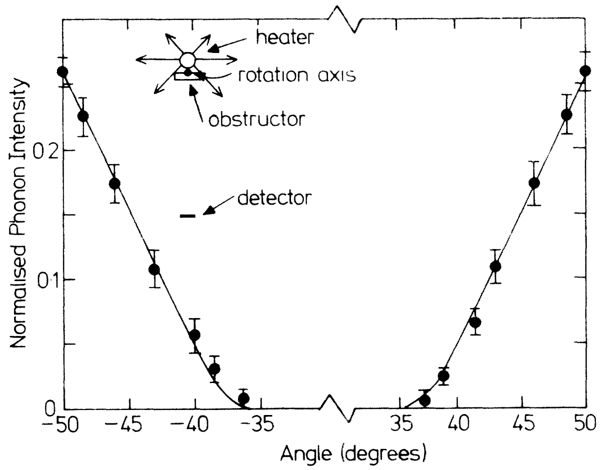

FIG. 1. Theoretical angular distribution and measured phonon intensities for shaded cylinder source (inset) at $p = 24$ bars. Each experimental point and error bar embraces the results for six different source temperatures in the range 1.0 to 3.3 K.

The source (resistance 23 Ω) was pulse heated to give energy fluxes w in the range 0.04–4 W cm^{-2}. The pulse length was 5 μsec with an overall repetition period in the range 10 msec to 1 sec. To estimate the source temperature T_s we assume that phonons are radiated into the helium and that the relevant mean phonon transmission coefficient $\bar{\alpha}$ is the same as that obtained from steady-state Kapitza conductance measurements.[10] This gives $T_s = (4w/h_K)^{1/4}$, where h_K is the coefficient of T^3 in the conventionally defined Kapitza conductance $\dot{q}/A = h_K T^3 \Delta T$.[10] Using $h_K = 0.1$ W cm^{-2} K^{-4} (a typical value for most solids at temperatures approximately a few Kelvin[10]) gives $1.0 \lesssim T_s \lesssim 3.5$ K for the energy fluxes used. We note that although these assumptions have not been verified for heat-pulse transmission into liquid He they are valid for heat pulses in solids[11] and that the fourth-root dependence of T_s on h_K makes it very insensitive to the value of h_K assumed. Thus we consider it unlikely that the error in these values of T_s exceeds 50%; and in any case it will most probably be largely systematic, so some real significance can be attached to relative values.

The phonon pulse was detected by a narrow (0.9 mm wide) graphite film bolometer ~10 mm away from the source. The bolometer has a (thermal) time constant ~50 μsec and so the received signal rises linearly during the first 5 μsec after the propagation time (30–40 μsec depending on p) and then decays. (A typical signal shape for one of these detectors is shown in Ref. 7.) The slope S of the initial rise is thus a measure of the incident energy flux and for $w \lesssim 0.5$ W cm^{-2}, S increases linearly with w. (At higher values of w the rate of increase of S with w is less than linear as observed by Guernsey and Luszczynski.[4]) By restricting attention to the first 5 μsec of the received signal any phonons arriving by paths involving reflections in the cell (4 cm diam×8 cm long) were eliminated while still allowing a sufficient time window to include those which have undergone decays of angle up to ~30°. At the lower heater powers detection was achieved by accurately setting and maintaining a 5-μsec boxcar gate over the first 5 μsec of the received signal and averaging in a single-point mode. This technique is very convenient as continuous traces of S versus source angle can be made. At high powers where the repetition period is long the boxcar technique is unsatisfactory because of baseline drifts, so the received signal as a function of time was averaged digitally (using a transient recorder and signal averager) at a number of discrete source angles and S obtained by measuring the slope of the initial rise. In the overlap region both methods of measurement were in excellent agreement and in all cases the (averaged) signal rms noise levels were ≲ 2% of maximum (much less for the higher power signals).

In Fig. 1 we show the theoretical angular distribution of this source (allowing for broadening due to the finite width of the bolometer) and normalized signals at different source temperatures for a number of angles in the regions of interest. (Both theoretical and experimental signals have been normalized to unity at large angles where there is no shadowing so that a direct comparison between signals of widely different sizes can be made.) It is seen that for the full range of input energy fluxes w (i.e., the full range of source temperatures) the experimental points are in excellent agreement with the theoretical curve down to the noise level of 1% peak height. Thus we conclude that, down to this level at least, there is negligible angular spreading over the propagation distance of 1 cm, outside a maximum experimental error of 2°, due to *any* phonon scattering or decay processes for phonons of energy $\hbar\omega/k_B \lesssim 12$ K.

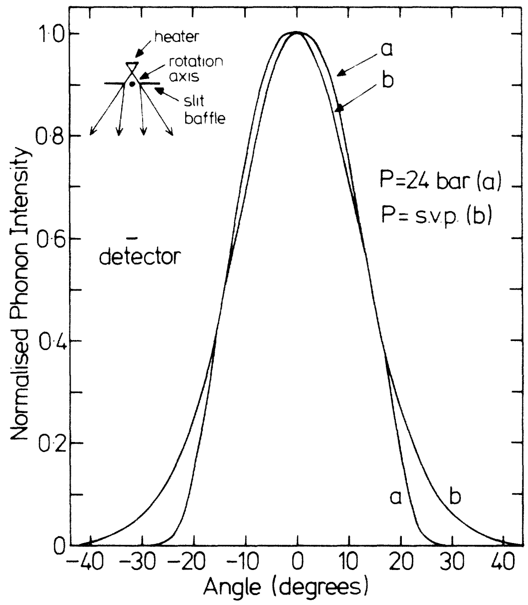

FIG. 2. Measured angular distributions from a slit source (inset) at $p = 24$ bars and p = svp for a calculated source temperature of 2.2 K. The curves are normalized tracings of the boxcar output as a function of source angle (see text) and have an rms noise level < 0.5% of peak height.

At pressures below 16 bars definite spreading was observed and a second series of experiments was devised to look at this in more detail. In these a simple slit collimator was used (Fig. 2, inset) where the heater and slit widths were ~0.5 mm and the separation ~1 mm. It was now no longer necessary to be able to measure the geometry with any great precision, the angular widths being defined by the measured phonon angular distribution at $p = 24$ bars. Typical measured angular distributions at $p = 24$ bars and at the svp for a calculated heater temperature $T = 2.2$ K are shown in Fig. 2. These curves, which have been normalized to a peak height of 1 for convenience of display, were obtained directly from the boxcar output in its single-point mode (see above) versus field angle. In Fig. 3 we illustrate the effects of T_s and p on the beam spreading. (Here we define $\Delta\theta'$ as half of the increase of the total beam width measured at 1% of peak height over the geometrical beam width at the same height.) The detailed interpretation of the curves in Figs. 2 and 3 does of course require a very detailed model of the system since the mean free paths of the decaying phonons and the possibility of multiple decays both affect the amount of broadening

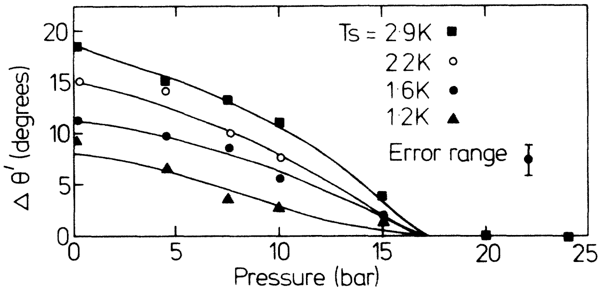

FIG. 3. Variation of the angular spreading of the slit source beam half-width at 1% peak height as a function of pressure for various source temperatures.

observed. However, the increase in $\Delta\theta'$ as the mean phonon energy increases and as the pressure decreases is in qualitative agreement with the expected behavior for a three-phonon decay process when γ is negative at $p \lesssim 17$ bars and $|\gamma|$ increases with decreasing pressure. The broadening is certainly not due to interactions with the background thermal phonons since even at the lowest pressure the width of the distribution is independent of the background helium temperature below ~0.25 K, and the fact that the distribution goes to the geometric limit at $p > 17$ bars rules out the possibility of broadening due to four-phonon decay processes or impurity scattering. We therefore conclude that the broadening we observe at $p \leq 17$ bars is consistent with three-phonon decay processes and that this is evidence for γ being negative in this pressure region.

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Experimental Determination of the Viscosity and Density of the Normal Component of Superfluid ^{3}He at the Melting Curve

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The viscosity and the density of the normal component of liquid ^{3}He in the A and B phases have been determined at the melting curve with a vibrating-wire viscometer. The results show that resistive flow of the liquid is accompanied by a flow of zero viscosity. The data thus prove superfluidity both in the A and in the B phase.

The first clear indication of superfluidity in liquid ^{3}He was the drastic change[1] in the damping of a vibrating-wire viscometer at the A and B transitions. Using the same technique with improved resolution, we have now been able to establish quantitative values for η_n and ρ_n, the viscosity and the density of the normal component of liquid ^{3}He, respectively, as a function of temperature. Our results show that viscous flow in the A and B phases is accompanied by frictionless flow, i.e., the two phases of ^{3}He behave as superfluids.

Several vibrating-wire experiments[2-4] in ^{4}He and in normal ^{3}He have successfully been interpreted in terms of Stokes's[5] theory of an infinite wire of circular cross section oscillating in an