with $y \lesssim y_3$.

[20]Equation (7) gives a reliable value at all large E for the ratio of secondaries with $y \gtrsim y_3$. The evolution of the cascade for $y \lesssim y_3$ is somewhat ambiguous when K is not an integer; fortunately this is a very small portion of phase space.

Improved Limit on Photon Rest Mass

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Recent observations of Alfvén waves in the interplanetary medium provide an improved upper limit on the photon rest mass. We find a reliable upper limit $\mu \leq 3.6\times10^{-11}$ cm^{-1}, $m_{ph} \leq 1.3\times10^{-48}$ g, and a stronger, but less certain upper limit $\mu < 3.1\times10^{-12}$ cm^{-1}, $m_{ph} < 1.1\times10^{-49}$ g. These represent improvements on the heretofore best reliable estimate by 0.5 and 1.5 orders of magnitude, respectively.

One of the various methods for experimentally determining if the photon has a nonzero rest mass involves the propagation of low-frequency waves in an electrically conducting medium. It is well known[1] that a highly conducting medium can support a variety of propagating waves at frequencies below the proton gyrofrequency. The mode of particular interest here is the intermediate, or Alfvén, wave, with a dispersion relation given by

$$k^2\cos^2\theta = \omega^2/v_A^2, \tag{1}$$

where k is the wave number, ω the (circular) frequency, θ the angle between the direction of propagation and the unperturbed magnetic field, and v_A the Alfvén speed in the medium which supports the wave. It is easy to show[2,3] that if the photon rest mass m_{ph} is different from zero, then Eq. (1) is modified to read

$$k^2\cos^2\theta = \omega^2/v_A^2 - \mu^2, \tag{2}$$

where $\mu = m_{ph}c/\hbar$ is the inverse reduced Compton wavelength associated with m_{ph}. Thus if $|\omega| < \mu v_A$, the intermediate mode does not propagate, but becomes evanescent. Conversely, if propagating waves of frequency ω are observed, than one must have

$$\mu < \omega/v_A. \tag{3}$$

This fact has already been used[3,4] in connection with early spacecraft data to establish that $\mu \leq 10^{-9}$ cm^{-1} (i.e., $m_{ph} \leq 4\times10^{-47}$ g).

Our purpose here is to use recent observations of propagating intermediate waves in the interplanetary medium to set an improved upper limit on μ. It is now well established[5] that propagating intermediate waves represent a major contribution to the fluctuations of the interplanetary plasma detected by spacecraft at 1 astronomical unit (A.U.). This identification is based on observations of the correlation between fluctuations in the magnetic field and plasma velocity, which are observed[5,6] to have a phase shift of 0 or 180°. Such a phase shift is consistent with propagating Alfvén waves, but not consistent with evanescent waves, for which the phase shift would be ±90°. These waves are of extremely low frequency,[6] with period $T' \leq 1$ day $= 8.6\times10^4$ sec in the spacecraft frame. To get the frequency ω in the local rest frame of the plasma, we must use the Doppler formula

$$2\pi/T' = \omega + \vec{k}\cdot\vec{v}, \tag{4}$$

where $\vec{v}$ is the solar-wind velocity. There is evidence[7,8] that the waves with periods of several hours or less are propagating nearly parallel to the average interplanetary magnetic field, so that for these waves $\theta \cong 0°$. There is no evidence concerning the direction of propagation of waves with period near 1 day, however. But geometrical "optics" provides an explanation[9] for the observation that $\theta \cong 0°$ which is independent of the wave period, and so we can assert that $\theta \cong 0°$ also for the longer period waves which have $T' = 1$ day. Thus, since the average magnetic field at 1 A.U. is nearly at 45° to $\vec{v}$, we have $\vec{k}\cdot\vec{v} = 0.707kv$. To obtain k we can make either of two extreme assumptions. First we can assume that it is possible to neglect any effects of μ at even the lowest frequency observed [this assumption is consistent with all of the data, if we assume that the absence of waves with periods longer than 1 day is not due to the presence of μ in Eq. (2)]. Then $k = \omega/v_A$ and $2\pi/T' = \omega(1+0.707v/v_A)$. With representative values at 1 A.U., i.e., $v = 300$ km sec^{-1} and $v_A = 20$ km sec^{-1}, we obtain $\omega = 6.3\times10^{-6}$ sec^{-1}, and thus

$$\mu < \omega/v_A = 3.1\times10^{-12} \text{ cm}^{-1}, \tag{5}$$

$$m_{ph} < 1.1\times10^{-49} \text{ g}. \tag{6}$$

Alternatively, we can assume that the absence of the intermediate mode at periods longer than 1 day is in fact due to the presence of μ in Eq. (2). Then at $T' = 1$ day we would have $\mu = \omega/v_A$, $k=0$, and $\omega = 2\pi/T' = 7.2\times10^{-5}$ sec^{-1}. With $v_A = 20$ km sec^{-1} we then have

$$\mu = \omega/v_A = 3.6\times10^{-11} \text{ cm}^{-1}, \tag{7}$$

$$m_{ph} = 1.3\times10^{-48} \text{ g}. \tag{8}$$

There is at present no certain way in which we can decide between inequalities (5) and (6), or Eqs. (7) and (8), or an intermediate situation. However, we must emphasize that there is a simpler explanation for the absence of waves with periods longer than 1 day, which does not invoke the presence of μ in Eq. (2). This explanation notes that the waves may be generated by the photospheric supergranulation,[10] which has a time scale of about 1 day, so that longer periods would not be expected to be present. We thus feel that the evidence is consistent with (5) and (6) as new upper limits on μ and m_{ph}, but only (7) and (8) (with $\leq$ replacing =) can be considered well established.

How accurate are the limits (5) and (6) or (7) and (8)? Observed values of v_A and v in the solar wind are only rarely less than the values we have used, so that our estimates of μ based on these quantities are probably conservative. Specifically, v is less than 300 km sec^{-1} only 5% of the time,[11] while v_A is less than 20 km sec^{-1} only 6% of the time,[12] so that our use of these values is probably well justified, although examination of a specific data set would be preferable. We believe that our estimate of the angle between $\vec{k}$ and $\vec{v}$ is also in the conservative sense, since $\vec{k}$ and $\vec{v}$ would become parallel in the absence of solar-wind stream structure, and thus the angle between k and v is possibly somewhat less than the value used above.[8] The least reliable quantity is T'. The power spectra in Ref. 6 do not show only a pure intermediate mode, but are confused by other types of fluctuations. However, $T' = 1$ day is probably reliable to within a factor of 2, so that (5)–(8) are also reliable to within a factor of 2.

How does our new limit compare with previous work? The best reliable upper limit reported in the recent review by Goldhaber and Nieto[3] is $\mu < 10^{-10}$ cm^{-1}, $m_{ph} < 4\times10^{-48}$ g. Our (7) and (8) (with $\leq$ replacing =) represents an improvement by 0.5 order of magnitude, while our less reliable limits (5) and (6) represent an improvement by 1.5 orders of magnitude. But this is several orders of magnitude worse than a limit reported by Williams and Park,[13] based on a discussion of the galactic magnetic field. However, as mentioned by Goldhaber and Nieto, the galactic limit can not be regarded as well established, since it may be possible to construct configurations for the galactic magnetic field which were not considered by Williams and Park, and which lead to less stringent upper limits on μ. In addition, Williams and Park based their discussion on the classical collisional electrical conductivity of the galactic plasma, but it should be pointed out that wave-particle interactions could drastically alter the conductivity, and thus Williams and Park's limit on μ; but we doubt that this effect alone could make the Williams-Park limit worse than ours since their limit depends only on the square root of the electrical conductivity. We thus feel that the limit on μ reported in this paper is the lowest available limit which may be considered well established by experiment, but the Williams-Park limit, although more speculative, is probably lower.

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