the resulting fit parameters was negligible. The averages of the values shown in Table II are presented in Table I. They constitute the final results of our measurements of these resonance parameters.

Thus, we have fitted the data shown in Fig. 1 using several different approaches and have tabulated the results in Table II. The goodness of these fits depended crucially on the existence of a negative-parity resonance of mass ~2200 MeV/c^2. We have determined the mass, width, and the product of the elasticity and $(J+\frac{1}{2})$ for this resonance as well as the $\Delta(2420)$, $\Delta(2850)$, and $\Delta(3230)$. These values are summarized in Table I.

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Nuclear Collisions and Factorization*

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Total cross sections for nucleon-α and $\alpha\alpha$ collisions are calculated for $86 < s < 2776$ GeV^2 in the Glauber approximation. The results are compared with those obtained by assuming factorization for the Pomeron. Calculations for $s > 2776$ GeV^2, with simple assumptions regarding nucleon-nucleon elastic collisions, are also given. For $100 < s < 2776$ GeV^2, where NN data exist, the Glauber-approximation results differ from the factorization predictions by ~4–10%. At higher energies the factorization predictions are approached slowly.

One of the important ideas in Regge-pole theory is the factorizability of the residues of the contributing Regge poles.[1] The classic test of factorization for the Pomeron residue is the asymptotic total-cross-section relation[1]

$$\sigma_{\pi\pi}\sigma_{NN}/\sigma_{\pi N}^2=1 \tag{1}$$

for $s\to\infty$. However, since the pion is unstable, $\sigma_{\pi\pi}$ is difficult to measure and no experimental tests of Eq. (1) have been made.

If factorization can be applied to nuclear scattering, asymptotic cross-section relations such as

$$\sigma_{NN}\sigma_{AA}/\sigma_{NA}^2=1 \tag{2}$$

would apply, where σ_{AA} represents the total cross section for collisions between nuclei with baryon number A, and σ_{NA} represents the total cross section for collisions between nucleons and nuclei with baryon number A. For stable nuclei these cross sections are measurable. At Berkeley, high-energy nucleus-nucleus cross sections will soon be measured, and relations such as Eq. (2) can be tested.

One of the most successful approaches for calculating total cross sections for high-energy nuclear collisions has been via the Glauber approximation.[2-4] How do the Glauber-approximation results differ from Eq. (2)? Using NN total-cross-section and elastic-scattering data, we have calculated the left-hand side of Eq. (2) for the ^{4}He nucleus ($A=4$). The cross sections were calculated at fixed values of s. The ratio $\sigma_{NN}(s)\sigma_{\alpha\alpha}(s)/\sigma_{N\alpha}^2(s)$, for $86<s<2776$ GeV^2 where NN data exist, was formed. In the calculations I assumed that pp, pn, and np elastic-scattering amplitudes are equal and have the form

$$f_{NN}(q)=k\sigma_{NN}(i+\rho)\exp(-bq^2/2)/4\pi, \tag{3}$$

where $\hbar^2q^2=-t$ and $\hbar k$ is the incident momentum. The parameters σ_{NN}, ρ, and b were obtained from NN data. In cases where both pp and np measurements exist, the average values of the parameters were used. Interpolations of the data between energies were made in order to obtain the values of the parameters at the particular energies needed. The results we obtained were rather insensitive to small variations in these parameters.

For the ^{4}He nucleus we took the wave function to be a product of single-particle harmonic-oscillator wave functions. The ^{4}He form factor is therefore

$$S(q)=\exp(-R^2q^2/4). \tag{4}$$

The parameter R^2 was taken to be 1.95 fm^2 to fit the ^{4}He rms radius[5] of 1.71 fm (when c.m. and finite-nucleon-size corrections are made).

The nucleon-α total cross section $\sigma_{N\alpha}$ is given simply by[3]

$$\sigma_{N\alpha}=2\pi(R^2+2b)\,\mathrm{Re}\sum_{j=1}^{4}(-1)^{j+1}\binom{A}{j}\frac{1}{j}\left(\frac{(1-i\rho)\sigma_{NN}}{2\pi(R^2+2b)}\right)^j. \tag{5}$$

This result contains four terms, each corresponding to a particular order of scattering, from first through fourth.

The total cross section $\sigma_{\alpha\alpha}$ is rather more complicated. It is calculated from[6]

$$\begin{aligned}\sigma_{\alpha\alpha}=2\,\mathrm{Re}\int d^2b\,\Big(1-\int&\rho(s_1)\cdots\rho(s_4)\rho(s_1{}')\cdots\rho(s_4{}')\,d^2s_1\cdots d^2s_4\,d^2s_1{}'\cdots d^2s_4{}'\\ &\times\prod_{i=1}^{4}\prod_{j=1}^{4}\{1-(2\pi ik)^{-1}\textstyle\int\exp[-i\vec{q}\cdot(\vec{b}-\vec{s}_i+\vec{s}_j{}')]f_{NN}(q)\,d^2q\}\Big),\end{aligned} \tag{6}$$

where $\rho(s)=(\pi R^2)^{-1}\exp(-s^2/R^2)$. This integral has an analytic solution which may be expressed as a finite sixteenfold sum.[7]

We should point out that since the left-hand side of Eq. (2) is being calculated at fixed s, the momenta involved in the determination of σ_{NN}, $\sigma_{\alpha\alpha}$, and $\sigma_{N\alpha}$ are all different. Furthermore, since the optical theorem introduces a factor k^{-1} in the total-cross-section relation, at finite energies a kinematic factor should multiply the left-hand side of Eq. (2). This factor consists of ratios of the appropriate k's and rapidly approaches unity as s increases, being less than a 1% correction for $s=275$ GeV^2. The actual quantity calculated is

$$\frac{s[(s-4m_N{}^2)(s-4m_\alpha{}^2)]^{1/2}}{[s-(m_N+m_\alpha)^2][s-(m_N-m_\alpha)^2]}\;\frac{\sigma_{\alpha\alpha}(s)\sigma_{NN}(s)}{\sigma_{N\alpha}{}^2(s)}, \tag{7}$$

where m_N and m_α are the masses of the nucleon and α particle, respectively. This quantity is shown in Fig. 1 as a function of s for values of s in the range $86<s<2776$ GeV^2 where the appropriate NN data exist. Generally, the results fall below the factorization prediction by ~4–10%. The particularly low values of the ratio at $s=86$ and 100 GeV^2 are due in part to the kinematic factors which equal 0.86 and 0.91, respectively.

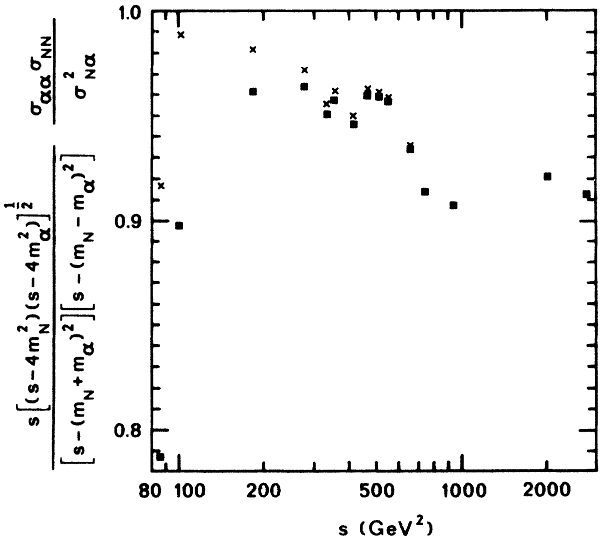

FIG. 1. Cross-section ratio as a function of s. Factorization predicts unity for $s\to\infty$. The crosses give the cross-section ratio without the kinematic factor. The squares include the kinematic factor. The predictions are obtained from Eqs. (5) and (6).

If the slope parameter b is an increasing function of s, such as for example $b_0+2\alpha'\ln s$ with s expressed in GeV^2, then as $s\to\infty$ we find that $\sigma_{N\alpha}\to4\sigma_{NN}$ and $\sigma_{\alpha\alpha}\to16\sigma_{NN}$, and Eq. (2) is satisfied. However, this asymptotic agreement is attained very slowly. To illustrate this we have calculated $\sigma_{NN}\sigma_{\alpha\alpha}/\sigma_{N\alpha}{}^2$ as a function of s for $10^2<s<10^{160}$ GeV^2 assuming $\sigma_{NN}=40$ mb (constant),

$\rho = 0$, and with $\hbar^{-2}b = 8.23 + 0.556 \ln s$ $(\mathrm{GeV}/c)^{-2}$ as found recently by Bartenev *et al.*[8] for $100 \lesssim s \lesssim 750\ \mathrm{GeV}^2$. We find that the ratio is equal to 0.829, 0.834, 0.853, and 0.984 at $s = 10^2$, 10^8, 10^{24}, and $10^{160}\ \mathrm{GeV}^2$, respectively. The approach to unity is quite slow.

After this paper had been submitted for publication, a paper by P. M. Fishbane and J. S. Trefil on the same subject was published.[9]

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The lower limit on the integral of Eq. (6) should be $\tilde{\mu}$, where $\tilde{\mu} = (\mu^2 + k_{\mathrm{F}}^2)^{1/2}$ is defined following Eq. (6).