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$^{12}C(e,e'p)$ Results as a Critical Test of an Energy Sum Rule

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The reaction $^{12}C(e,e'p)$ at 497 MeV in conjunction with a distorted-wave impulse-approximation analysis was used to determine kinetic and separation energies of bound protons. The spectral function for separation energies less than 74 MeV provides only half of the total binding energy; i.e., the data do not satisfy Koltun's sum rule. The momentum distributions are compatible with elastic electron scattering.

In a Letter by Koltun[1] a sum rule relating kinetic and separation energies of bound protons to the total binding energy has been established and successfully applied to $(p,2p)$ data.[2] We present results from the reaction $^{12}C(e,e'p)$ which by far do not satisfy this sum rule whose only model assumption is that of two-body forces. Our data also show that the often discussed incompatibility[3] of (e,e) and $(e,e'p)$ data does not exist.

The experiment was performed with electrons of $T_e=497$ MeV from the Saclay linac. In the focal plane of a first spectrometer the positions and directions of outgoing protons with energies $78<T_p<94$ MeV were measured; those of the coincident electrons, with a second spectrometer at a fixed angle $\theta_{e'}=-52.9°$. For each event, the recoil momentum $\vec{k}=\vec{k}_e-\vec{k}_{e'}-\vec{k}_p$ of the $A-1$ nucleons and the missing energy $E_m=T_e-T_{e'}-T_p-k^2/2M_{A-1}$ were determined with a resolution of $\Delta k=6$ MeV/c and $\Delta E_m=1.2$ MeV, respectively. By varying $T_{e'}$ and the proton scattering angle, a range $0<E_m<74$ MeV and $0<k<300$ MeV/c was covered. A more detailed description including data on ^{28}Si, ^{40}Ca, and ^{58}Ni will be published elsewhere.

The estimated absolute uncertainty of the differential cross sections is 20%. The magnitude of the radiative corrections[4] is seen in Fig. 1 which contains the cross section averaged over the recoil momenta $0<k<60$ MeV/c versus the missing energy. For $E_m>25$ MeV, one notices

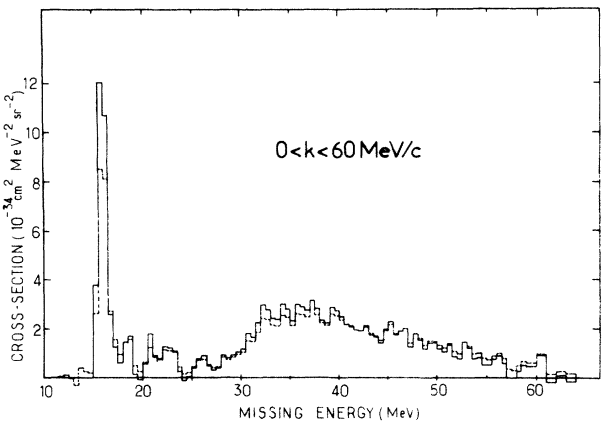

FIG. 1. A "spectrum" (see text) of the reaction $^{12}C(e,e'p)$ with (solid line) and without (dashed line) radiative corrections. The cross section is zero within the error bars for $64 < E_m < 74$ MeV.

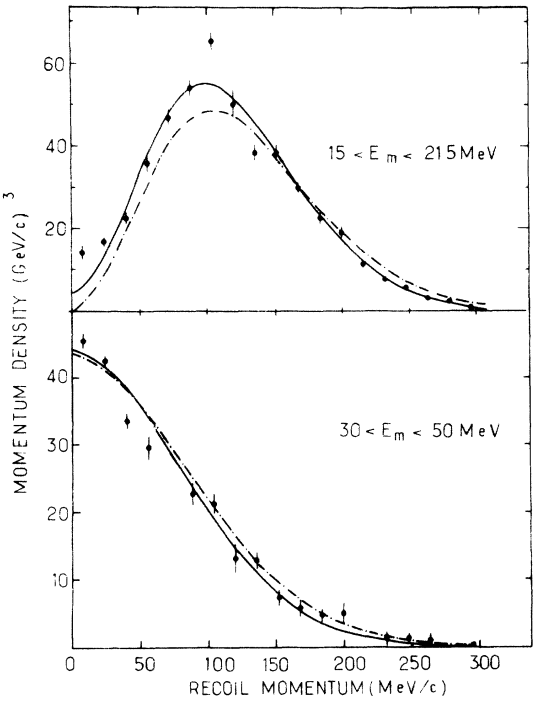

FIG. 2. Recoil momentum distributions in different regions of separation energies. The DWIA (solid line) and PWIA results (dashed line) were normalized by least-squares fits to the data.

a broad peak of slightly asymmetric shape characterized by an $l = 0$ momentum distribution. As in the reaction[5] $^{12}C(d,{}^3He)^{11}B$, the low-energy region is dominated by a $l = 1$ transition to the clearly resolved ground state of ^{11}B.

Dividing the cross section by the electron-proton cross section and kinematical factors yields the "distorted spectral function" $G(E_m, k)$ which differs from the true spectral function[6] by the effects of final-state interactions of the proton. In Fig. 2 we show "distorted momentum distributions" obtained by projecting certain energy bands of $G(E_m, k)$ onto the momentum axis. The pronounced $l = 1$ and $l = 0$ distributions for the regions below and above $E_m = 25$ MeV confirm the existence of proton shells in ^{12}C. The $l = 0$ behavior persists up to the maximum energies observed; a "background" above 50 MeV with a different momentum distribution, as seen in $(p, 2p)$ experiments,[2,7] is not observed. This suggests that this background is not a property of the spectral function as assumed in Ref. 1, but is due to multiple scattering contributions to the $(p, 2p)$ cross sections.

The first columns of Table I contain the results for the occupation numbers n, the mean kinetic energies $\langle T \rangle = \langle k^2/2m \rangle$, and the mean separation energies $\langle E_m \rangle$ obtained with Eq. (13) of Ref. 1 by taking $G(E_m, k)$ as the spectral function [partial-wave impulse approximation (PWIA)]. Also given are corrections for the distortions of the outgoing proton waves and the corrected "distorted-wave impulse-approximation (DWIA)" results. The dis-

TABLE I. Occupation numbers n, average kinetic energies $\langle T \rangle$, and separation energies $\langle E_m \rangle$. All energies are in MeV.

Energy region (MeV)	Experimental results[a]			DWIA corrections			Corrected values		
	n	$\langle T \rangle$	$\langle E_m \rangle$	nl_j	η	ΔT	n	$\langle T \rangle$	$\langle E_m \rangle$
15–25	1.7	16.3	16.9	$1p_{3/2}$	0.66	2.14	2.6	18.4	16.9
25–74	0.56	11.5	38.7	$1s_{1/2}$	0.52	1.85	1.1	13.4	38.7
15–74							3.7	16.9	23.4

[a] The estimated relative error is 20% for n; the statistical ones are 3% for $\langle T \rangle$ and 1.5% for $\langle E_m \rangle$.

tortion effects were calculated with a DWIA program[8] using the partial-wave methods of Epp and Griffy.[9] Electron distortions may be neglected.[10] Good agreement with the data (Fig. 2) was obtained with bound-state wave functions[11] and optical potentials[12] compatible with elastic electron and proton scattering, respectively. The calculations show that the final-state interactions lead to (i) a reduction of the integrated cross section by a factor η, and (ii) a shift of the momentum distributions towards smaller momenta, thus lowering the kinetic energies by ΔT. The values of η and ΔT are insensitive to variations of the bound-state potentials. Even changing the energy eigenvalues in the full range of E_m produces variations of at most 5%. Therefore we simply applied the corrections η and ΔT to the experimental occupation numbers and kinetic energies, thus saving a maximum of model independence. The values of $\langle E_m \rangle$ need not be corrected.

Even with distortion corrections the total occupation number is 40% lower[13] than the limit $Z = 6$. This deficiency is larger than expected from an extension of the strength into the unobserved region. But in contrast to $\langle E_m \rangle$ and $\langle T \rangle$, the values of n are strongly affected by uncertainties of the absolute cross sections and of the optical potential for the highly excited states of ^{11}B.

Koltun's sum rule,[1] appropriately corrected for recoil energies, is

$$W_Z/Z = \tfrac{1}{2}[\langle T \rangle - \langle E_m \rangle - m_p \langle T \rangle / M_{A-1}], \quad (1)$$

where the binding energy per proton $W_Z/Z = -6.93$ MeV is obtained[1] from nuclear masses and appropriate Coulomb corrections. Instead, with the values of $\langle T \rangle$ and $\langle E_m \rangle$ from the last line of Table I, we find only -4.0 ± 0.5 MeV. The seeming success of Koltun's analysis of $(p, 2p)$ data[2] results partly from neglecting the distortions and partly from including the multiple-scattering background at large E_m. The discrepancy between our data and the sum rule value may have two causes: (i) The sum rule is exact for two-body interactions in the nuclear Hamiltonian only. The contribution of three-body forces to the binding energy per proton would have to be $\langle H_3 \rangle / Z = 5.9$ MeV to eliminate the discrepancy, which is unlikely. (ii) More probably, the experimental cutoff in separation energy simulates a too small value of $\langle E_m \rangle$. The discrepancy would be explained, for example, if 5% of the total events were associated with average separation energies of 150 MeV, everything else unchanged.

Since $G(E_m, k)$ contains no events with an apparent origin other than the $1s$ and $1p$ shells, and since the ratio of the $1p$ to $1s$ occupation numbers is close to 2, the "orbital sum rule"[1] presents the same difficulties. If we attribute the results in lines 1 and 2 of Table I to the $1p$ and $1s$ shells, respectively, we find average separation energies in good agreement with $(p, 2p)$ experiments.[2,7] The (lower limits of) kinetic energies in the two orbits which we have determined are close to those in a Woods-Saxon well.[11] Recent relativistic calculations[14] have yielded kinetic energies which are several MeV lower. Rather the data confirm the model estimates of kinetic energies by Köhler[15] and Elton[16] who hence concluded the impossibility of a first-order Hartree-Fock description[17] of ^{12}C.

The DWIA analysis of the momentum distributions has shown a perfect compatibility with wave functions deduced from elastic electron scattering. The statistical accuracy and energy resolution, greatly improved over those of previous $(e, e'p)$ experiments,[13,18] and the large momentum range investigated have enabled us to determine kinetic energies of bound protons for the first time. The clean reaction mechanism, characterized by weak distortion effects and the absence of a multiple-scattering background, has rendered the data suitable for an application of Koltun's sum rule. The $1p$ and $1s$ shells of ^{12}C appear well within the experimental range of separation energies, but the corresponding part of the spectral function provides only 60% of the total binding energy. This implies necessarily a failure of first-order Hartree-Fock calculations with two-body forces. To the extent that three-body contributions to the binding energy may be neglected, the data do not satisfy the sum rule.

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Continuum–Shell-Model Description of Compound Resonances in the System $^{16}O + n$†

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Bound and continuum properties of the system $^{16}O+n$ are obtained from a microscopic model by means of an extended R-matrix formalism. The model includes configurations arising from excitation of up to four particles from the closed ^{16}O core and provides a useful dynamical description of the observed low-energy spectrum.

We wish to report some early encouraging results of a continuum–shell-model calculation of elastic neutron scattering from ^{16}O in which excitations of up to four-particle, four-hole (4p-4h) states in the target nucleus are included. The need for such a calculation arises because the existing descriptions of the system $^{16}O+n$ are incomplete. R-matrix fits to neutron cross-section data and their reduction into the various partial waves such as recently reported by Johnson,[1] though extremely useful in providing a schematization of experimental data, shed no light on the dynamics of the compound system. In view of the sensitivity of the cross section to the internal structure of the total system (target + projectile), as suggested by the numerous peaks in the resonance curve, a microscopic description would be most desirable. Unfortunately, the microscopic models which have been employed so far are incapable of accounting for the compound resonances in the spectrum.

There have been several attempts to calculate the neutron phase shifts in terms of forces that were successful in predicting bound-state properties of nuclei. Dover and Van Giai[2] use the Hartree-Fock field derived from Skyrme's effective interaction in the no-polarization approximation. MacKellar, Reading, and Kerman[3] use forces due to Davies, Krieger, and Baranger and Tabakin with perturbative corrections to the matrix elements. The results of there calculations are very similar to the results obtained by one of us,[4] who assumed the ^{16}O nucleus to be a closed core, and similar to the result represented by the dashed line in Fig. 1(b). The nonresonant background phase shifts and the resonance in the $d_{3/2}$ partial wave near 1 MeV are well reproduced. In view of the rather simple configurations employed by the above authors, their failure to predict the numerous peaks in the experimental resonance curve is not surprising.

Following Brown's suggestion in 1964 of the possible interpretation of the excited states in ^{16}O in terms of rotational bands, a number of model calculations[5,6] have appeared. The results unanimously point to the importance of configurations consisting of four holes in the ^{16}O core and four particles in the upper orbits. For the first excited $J^\pi = 0^+$ state at 6.05 MeV, the 4h-4p configurations carry a strength of about 85%. This finding is in excellent agreement with an earlier calculation by Unna and Talmi[7] who used simple shell-model arguments and the known energy-level separations in neighboring nuclei to deduce that excitation of two nucleons in ^{16}O from the $p_{1/2}$ to $s_{1/2}$ orbit requires an energy of at least 13.8 MeV while excitation of two neutrons and two protons requires only 6.9 MeV.