2^- Threshold State in ^{8}Be

L. G. Arnold and R. G. Seyler
Department of Physics, The Ohio State University, Columbus, Ohio 43210

and

Louis Brown and T. I. Bonner*
Department of Terrestrial Magnetism, Carnegie Institution of Washington, Washington, D. C. 20015

and

Erich Steiner†
Department of Physics, University of Basel, Basel, Switzerland
(Received 26 February 1973)

The 2^- level in ^{8}Be at the threshold for the reaction ^{7}Li(p,n)^{7}Be is shown to be a virtual state relative to threshold. A description of the state as an S-matrix pole in the scattering-length approximation gives a good account of its properties as observed in different reaction channels, and, in particular, removes an apparent contradiction about its width as observed through particle and γ-ray channels. The width of the 2^- state is found to be about 50 keV.

Effects due to the presence of an excited state near a threshold for particle emission have received considerable study[1] in scattering theory. The most striking effect, known as the Wigner cusp,[2] is expected when there is an excited state of appropriate spin and parity near a neutral-particle threshold, and a number of states in light nuclei have been correlated with such thresholds.[3,4] For the most part, however, the connection between theory and the observed features of these states is tenuous. Application of the Breit-Wigner formalism is potentially misleading. Indeed, for the case considered here, where there is a wealth of experimental data, it is not obvious that the features observed through different reaction channels are due to a single isolated energy level.

Without question the 2^- level at the neutron threshold in ^{8}Be is the classic example of a state near threshold. The cross section for the reaction ^{7}Li(p,n)^{7}Be rises rapidly to about the 2^- partial-wave unitary limit as the proton energy crosses the threshold at $E_p \approx 1881$ keV, and remains large for at least 400 keV above threshold; the cross section for the inverse reaction at thermal neutron energies is a remarkable 50 000 b. Hanna[5] concluded that these features are due to a level in ^{8}Be with a width not greater than 30 keV. The ^{7}Li(p,p')^{7}Li* cross section[6-8] shows a pronounced drop just above the (p,n) threshold that is obscured somewhat away from threshold by a rapidly varying d-wave penetration factor for inelastic protons in the 2^- partial wave and by the background of other partial waves. All of the above features are accounted for by the one-level Breit-Wigner analysis of Newson *et al.*,[6] and their assignment of a 2^- state at $E_p \approx 1900$ keV with $\Gamma > 500$ keV is presently accepted. However, it is difficult to resolve the large ratio of the reduced widths ($\gamma_n^2/\gamma_p^2 \approx 5$, $\gamma_n^2 > \gamma_W^2/3$) extracted from their analysis with the absence of capture γ rays from the 2^- state to the 2^+ first excited state of ^{8}Be. One would expect to see this electric dipole transition unless the 2^- state is isospin zero, in which case $\gamma_n^2 = \gamma_p^2$. Recently, Sweeney and Marion[9] studied the reaction ^{7}Li(p,γ)^{8}Be*(16.63 and 16.90 MeV)$\rightarrow 2\alpha$ and observed a state at the (p,n) threshold with a much narrower width ($\Gamma = 150 \pm 50$ keV). The results of Sweeney and Marion are consistent with a 2^-, $T=0$ state at threshold, but appear difficult to reconcile with the interpretation of earlier experiments.

The excitation curves for ^{7}Li(p,p)^{7}Li elastic scattering[10,11] show a distinct anomaly at the (p,n) threshold. Unfortunately, these data exhibit strong interference effects from a coupled pair of 3^+ states located above threshold, so a direct analysis for information on the 2^- state is not feasible. The key to the matter is a reduction of the elastic scattering data to phase shifts, for which we refer to an analysis in the energy region of interest[12] and an interpretation[13] of the anomaly observed in the $^5S_2$ phase at the (p,n) threshold. The behavior of this phase provides new information about the 2^- state that is important to both the interpretation of the above-mentioned reactions and the nuclear structure of neg-

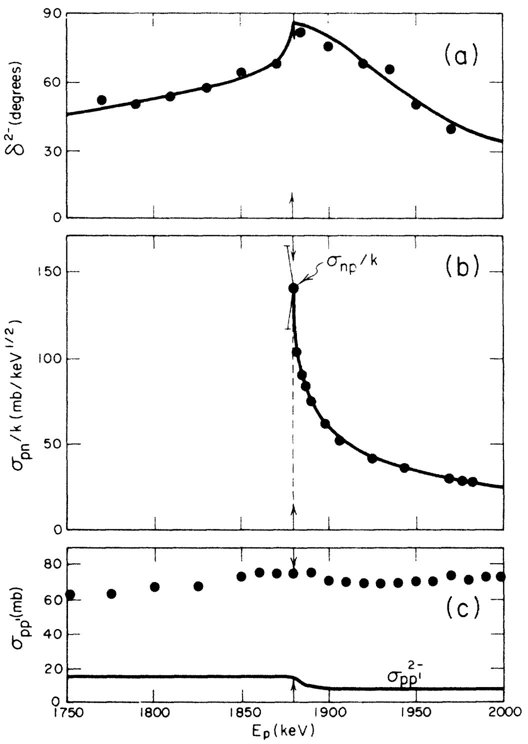

FIG. 1. Scattering-length–approximation fit to the data. (a) ^{7}Li(p,p)^{7}Li 2^- phase shift from Ref. 12. (b) ^{7}Li(p,n)^{7}Be reduced cross section (σ_{pn}/k) from Ref. 14. (c) ^{7}Li(p,p')^{7}Li cross section from Ref. 8. The (p,p') data from Refs. 6 and 7 show a slightly more pronounced drop which starts at the (p,n) threshold. It is the relative absence of structure in the (p,p') channel compared to the (p,n) channel that provides a constraint on the scattering-length–approximation parameters.

ative-parity states in ^{8}Be.

The 2^- state of ^{8}Be is called a threshold state because the data (see Fig. 1) show no features above or below the ^{7}Li(p,n)^{7}Be threshold which can be used to mark the energy of the state; thus, it is not a resonance in the usual sense of the term.

A multichannel scattering-length approximation is used to analyze the 2^- partial wave near the (p,n) threshold. The approximation is exact at threshold, and its use is clearly preferable to the use of the Breit-Wigner resonance form for the intended application.[15,16]

Neutrons emitted from ^{8}Be must be s waves for energies sufficiently close to the ^{7}Be$+n$ threshold, and therefore restricted to the 1^- and 2^- partial waves. Earlier work indicates that emission in the 1^- partial wave is negligible. The 2^- partial-wave S matrix has six channels if d waves are included for the proton channels; of these, we neglect the two d-wave elastic proton channels and treat the two d-wave inelastic proton channels as one. The resulting three-channel S matrix is expanded in rational form about the threshold, with terms linear in the neutron wave number k retained and subjected to the constraints of open-channel unitarity and symmetry above and below threshold. The diagonal elements are

$$S_{pp} = S_{pp}{}^t(1 - ib^*k)/(1 - iak), \tag{1}$$

$$S_{p'p'} = S_{p'p'}{}^t(1 - ibk)/(1 - iak), \tag{2}$$

$$S_{nn} = (1 + iak)/(1 - iak), \tag{3}$$

where $a = a_r + ia_i$ is the scattering length, $b = b_r + ib_i$ is a subsidiary scattering length, and $S_{pp}{}^t = \eta \exp(2i\delta_{pp}{}^t)$ and $S_{p'p'}{}^t = \eta \exp(2i\delta_{p'p'}{}^t)$ are elements evaluated at threshold. The moduli of the off-diagonal elements are given by the permutations of

$$|S_{ij}|^2 = \tfrac{1}{2}(|S_{kk}|^2 - |S_{ii}|^2 - |S_{jj}|^2 + 1). \tag{4}$$

The elements of the open-channel matrix below threshold are obtained from the above expressions by setting $k = i\alpha$. The S matrix has a simple pole at $k = k_r + ik_i = -i/a$, and the state represented by this pole is bound relative to threshold if $a_r < 0$ and unbound (or virtual) if $a_r > 0$.[17] The only virtual state that has been established with certainty in nuclear physics is the $^1S_0$ state of the deuteron.

The scattering-length–approximation S matrix depends on seven parameters, of which only $\delta_{p'p'}{}^t$ cannot be determined from existing data. The other six parameters can be obtained as follows: The ^{7}Be destruction cross section for thermal neutrons, $\sigma_{np} + \sigma_{np'} = (50 \pm 8) \times 10^3$ b,[5] yields $a_i = 0.12 \pm 0.02$ keV$^{-1/2}$; the ratio, $\sigma_{np'}/\sigma_{np} = 0.02 \pm 0.01$,[18] yields $b_i\eta^2/a_i = 0.96 \pm 0.02$; the magnitude and shape of σ_{pn}[14] require $a_r{}^2/a_i{}^2 \ll 1$[19]; the shape of $\sigma_{pp'}$ below threshold[8] requires $b_r/a_r = 1.00 \pm 0.04$; the magnitude of the drop in $\sigma_{pp'}$ above threshold, 8 ± 3 mb,[8] yields $\eta^2 = 0.95 \pm 0.03$, with the result that $b_i/a_i = 1.01 \pm 0.05$. It is worth noting at this point that the reaction data fix only one of the two parameters needed to specify the location of the pole. The other is obtained from the $^5S_2$ phase shift. In particular, the decrease of the $^5S_2$ phase shift above threshold [see Fig. 1(a)] requires that $a_r > 0$, or, equivalently, that the 2^- state is virtual relative to threshold. The behavior of the $^5S_2$ phase below threshold yields

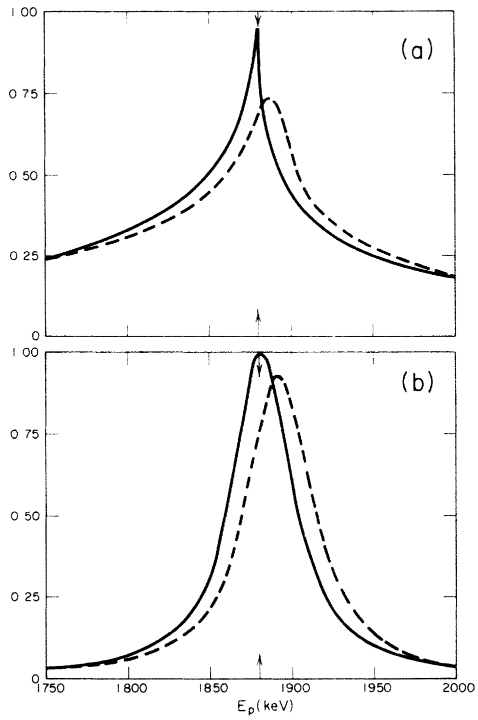

FIG. 2. Comparison of (a) scattering-length–approximation S-matrix denominator with (b) a Breit-Wigner denominator of the same full width Γ centered at threshold (solid lines). The dashed lines show the degradation for a 20-keV-thick target.

$\delta_{pp}{}^t = 85° \pm 5°$ and $a_r = 0.04 \pm 0.02$ keV$^{-1/2}$. A comparison with the data for $b = a = 0.05 + 0.12i$ keV$^{-1/2}$, $\eta^2 = 0.95$, and $\delta_{pp}{}^t = 87°$ is shown in Fig. 1. This choice for the scattering length corresponds to a pole of the S matrix at $k = -7 - 3i$ keV$^{1/2}$. The parameters obtained from this analysis show that the 2^- S matrix is a slightly perturbed ($\eta^2 = 0.95$) two-channel S matrix, and that our truncation of the six-channel S matrix to three channels is a reasonable approximation.

Now that the pole parameters of the S matrix have been determined, it is possible to consider the characteristics of the 2^- state. The spectral line shape $|1 - iak|^{-2}$ is shown in Fig. 2(a), and, for comparison, the line shape of a Lorentzian with similar width is shown in Fig. 2(b). The dashed curves in each case show the degradation due to a 20-keV-thick target.[20] Two differences between the threshold and Lorentzian line shapes are important for the purpose of determining the width of a threshold state from experimental data. First, much of the strength of the threshold state is retained in the wings where it cannot be judiciously subtracted from a real background. Second, the peak height is substantially reduced (25% versus 5% for the Lorentzian) by a target of comparable thickness. These two features of this threshold state suggest that an experimentally relevant definition of a full width at half-maximum, valid for all threshold states, is somewhat impractical; nevertheless, it is clear that the width of the 2^- state is about 50 keV, and much less than 500 keV in any event. The uncertainty in the 50-keV width is ± 20 keV, and is due primarily to the uncertainty in σ_{np} for thermal neutrons. It should be noted that the pole of the S matrix is located $2k_r k_i = 42$ keV off the real axis in the complex energy plane, and this distance from the axis corresponds to a Breit-Wigner width[21] of 84 keV. The analysis by Newson *et al.*,[6] which is equivalent to the present work with $a_r = 0$, would result in the pole being on the real axis and a negligible Breit-Wigner width. The 150 ± 50 keV width observed by Sweeney and Marion[9] is larger than the 50-keV width obtained here, but this difference can be attributed to the uncertainty in extracting a width from the ^{7}Li$(p, \gamma)^8$Be$^* \to 2\alpha$ data.[22]

The scattering-length analysis does not provide new information on the isospin of the 2^- state. However, Bassi *et al.*[18] have measured the ^{7}Be$(n, \gamma)^8$Be$^*(16.63 + 16.90$ MeV$) \to 2\alpha$ cross section[23] for thermal neutrons. Their result, $\sigma_{n\gamma} = 155$ mb, when compared with the measurement of $\sigma_{p\gamma} = 1.43 + 0.85 = 2.28$ μb by Sweeney and Marion,[9] provides a rough estimate of the $T = 1$ isospin impurity of the 2^- state. Following Barker and Mann,[24] we have

$$\frac{\sigma_{p\gamma}}{\sigma_{n\gamma}} = \frac{k_n}{k_p} C_0{}^2(\eta_p)\left(\frac{\alpha_1 + \alpha_0}{\alpha_1 - \alpha_0}\right)^2 = 1.5 \times 10^{-5},$$

where the kinematic factor is 3.5×10^{-5} for thermal neutrons. We find a $T = 1$ isospin impurity of less than 4% in intensity, and note that it may be much smaller because the uncertainty of the background subtraction and degradation for $\sigma_{p\gamma}$ mentioned above can only increase the experimental ratio and, therefore, reduce the impurity content.

All experimental data on the 2^- level at 18.9 MeV in ^{8}Be are consistent with a state that is narrow and isospin zero.

*Fellow of the Carnegie Institution of Washington.
†Fellow of the Swiss National Foundation and the Carnegie Institution of Washington. Present address:

Schweizerisches Institut für Nuklearforschung, Gloriastrasse 35, Zürich, Switzerland.

[1]Rather complete bibliographies of theoretical work on threshold states are contained T. I. Bonner, Phys. Rev. C 1, 1710 (1970), and in H. A. Weidenmüller, Nucl. Phys. 69, 113 (1965).

[2]E. P. Wigner, Phys. Rev. 73, 1002 (1948).

[3]T. I. Bonner, Ph. D. thesis, University of Wisconsin, 1968 (unpublished).

[4]Bonner, Ref. 1.

[5]R. C. Hanna, Phil. Mag. 46, 381 (1955).

[6]H. W. Newson, R. M. Williamson, K. W. Jones, J. H. Gibbons, and H. Marshak, Phys. Rev. 108, 1294 (1957).

[7]G. Bardolle, J. Câbe, J. F. Chrétien, and M. Laurat, J. Phys. (Paris), Colloq. 1, 46 (1966).

[8]G. Presser and R. Bass, Nucl. Phys. A182, 321 (1972).

[9]W. E. Sweeney, Jr., and J. B. Marion, Phys. Rev. 182, 1007 (1969).

[10]S. Bashkin and H. T. Richards, Phys. Rev. 84, 1124 (1951).

[11]P. R. Malmberg, Phys. Rev. 101, 114 (1956).

[12]L. Brown, E. Steiner, L. G. Arnold, and R. G. Seyler, Nucl. Phys. A206, 353 (1973).

[13]L. G. Arnold and R. G. Seyler, to be published.

[14]R. L. Macklin and J. H. Gibbons, Phys. Rev. 109, 105 (1958).

[15]Weidenmüller, Ref. 1.

[16]C. Mahaux and H. A. Weidenmüller, *Shell-Model Approach to Nuclear Reactions* (North-Holland, Amsterdam, 1969), pp. 129–137.

[17]C. J. Goebel and K. W. McVoy, Nucl. Phys. A115, 504 (1968).

[18]P. Bassi, B. Ferretti, G. Venturini, G. C. Bertolini, F. Cappellani, V. Mandl, G. B. Restelli, and A. Rota, Nuovo Cimento 28, 1049 (1963).

[19]The expression for σ_{pn} obtained from Eqs. (1)–(4) reduces to the expression given in Ref. 6 when $a_r = 0$ and $b_i\eta^2/a_i = 1$.

[20]A 20-keV-thick target was used in Ref. 9.

[21]The width of an isolated resonance far from threshold is twice the distance from the S-matrix pole to the real axis.

[22]J. B. Marion, private communication.

[23]It was assumed in Ref. 17 that the measured cross section is for decay only to the 16.63-MeV state in ^{8}Be. Both the 16.63- and 16.90-MeV states should be populated in order to be consistent with the results of Ref. 9. The two states are not resolved in Ref. 17, and we have taken the quoted cross section to be the sum of the cross sections for both states.

[24]F. C. Barker and A. K. Mann, Phil. Mag. 2, 5 (1957).

^{12}C$(e,e'p)$ Results as a Critical Test of an Energy Sum Rule

M. Bernheim, A. Bussière, A. Gillebert, J. Mougey, Phan Xuan Ho, M. Priou, D. Royer, I. Sick,* and G. J. Wagner†

Centre d'Etudes Nucléaires de Saclay, 91190 Gif-sur-Yvette, France

(Received 17 January 1974)

The reaction ^{12}C$(e,e'p)$ at 497 MeV in conjunction with a distorted-wave impulse-approximation analysis was used to determine kinetic and separation energies of bound protons. The spectral function for separation energies less than 74 MeV provides only half of the total binding energy; i.e., the data do not satisfy Koltun's sum rule. The momentum distributions are compatible with elastic electron scattering.

In a Letter by Koltun[1] a sum rule relating kinetic and separation energies of bound protons to the total binding energy has been established and successfully applied to $(p,2p)$ data.[2] We present results from the reaction ^{12}C$(e,e'p)$ which by far do not satisfy this sum rule whose only model assumption is that of two-body forces. Our data also show that the often discussed incompatibility[3] of (e,e) and $(e,e'p)$ data does not exist.

The experiment was performed with electrons of $T_e = 497$ MeV from the Saclay linac. In the focal plane of a first spectrometer the positions and directions of outgoing protons with energies $78 < T_p < 94$ MeV were measured; those of the coincident electrons, with a second spectrometer at a fixed angle $\theta_{e'} = -52.9°$. For each event, the recoil momentum $\vec{k} = \vec{k}_e - \vec{k}_{e'} - \vec{k}_p$ of the $A-1$ nucleons and the missing energy $E_m = T_e - T_{e'} - T_p - k^2/2M_{A-1}$ were determined with a resolution of $\Delta k = 6$ MeV/c and $\Delta E_m = 1.2$ MeV, respectively. By varying $T_{e'}$ and the proton scattering angle, a range $0 < E_m < 74$ MeV and $0 < k < 300$ MeV/c was covered. A more detailed description including data on ^{28}Si, ^{40}Ca, and ^{58}Ni will be published elsewhere.

The estimated absolute uncertainty of the differential cross sections is 20%. The magnitude of the radiative corrections[4] is seen in Fig. 1 which contains the cross section averaged over the recoil momenta $0 < k < 60$ MeV/c versus the missing energy. For $E_m > 25$ MeV, one notices