The main question regarding this work is whether the frequency of their pump light swept *through* a transition of the system—a necessary requirement for adiabatic inversion. In NH_3, where the vibrational-rotational spectrum is known [see F. Shimizu, J. Chem. Phys. 52, 3572 (1970)], we found that this condition could not be satisfied under their experimental conditions. In the 10.3-μm region where their experiment was performed, there are numerous CO_2 lines ($R2$ through $R12$) and NH_3 transitions [$sQ(J,K)$ where $JK=1,2,\ldots,12$]. Neither the laser line nor the NH_3 transition was specified by the authors. However, among these lines, the minimum frequency separation between *any* CO_2 line and *any* NH_3 transition is that between the $R4$ CO_2 line and the $sQ(7,5)$ NH_3 transition, and is 690 MHz, which is more than ten times larger than the 60-MHz frequency sweep of their laser light.

[3]R. G. Brewer and R. L. Shoemaker, Phys. Rev. Lett. 27, 631 (1971), and 28, 1430 (1972), and Phys. Rev. A 6, 2001 (1972).

[4]D. C. McKean and P. N. Schatz, J. Chem. Phys. 24, 316 (1959); T. Shimizu, F. O. Shimizu, R. Turner, and T. Oka, J. Chem. Phys. 55, 2822 (1971). See also, R. G. Brewer and J. D. Swalen, J. Chem. Phys. 52, 2775 (1970) which gives a larger value of μ_{ν_2}.

[5]Shimizu, Ref. 2.

[6]T_1 of $C^{13}H_3F$ has recently been measured by the two-pulse nutation method using the Stark switching technique. J. Schmidt, P. R. Berman, and R. G. Brewer, Phys. Rev. Lett. 31, 1103 (1973). The values of the pressure-independent $\tilde{T}_1$ in their and my experiments, which should scale according to the pumping beam diameters and molecular weights, agree to within 25%.

[7]For example, if we assume that, after the population is inverted at $t=0$, the excited and ground state populations relax to equilibrium independently, $n_e=n_0\exp(-t/\tau_e)$ and $n_g=n_0[1-\exp(-t/\tau_g)]$, we would have $S(t)\sim n_e-n_g=n_0[\exp(-t/\tau_e)+\exp(-t/\tau_g)-1]$. In general, if τ_g and τ_e are different, the value of τ_{IR} should be between the two.

[8]C. H. Townes and A. L. Schawlow, *Microwave Spectroscopy* (McGraw-Hill, New York, 1955). Strictly speaking, the microwave pressure-broadening parameters give the phase memory time, τ_{g2}, while we are interested in the population lifetime, τ_{g1}. However, because of the small energy differences in microwave transitions, any phase-changing collision will also induce transition between states. Hence $\tau_{g1}=\tau_{g2}=\tau_g$. We note that τ_g of NH_3, ($J=8, K=7$) line has also been directly measured by microwave transient nutation measurements. See J. H. S. Wang, J. M. Levy, S. G. Kukolich, and J. I. Steinfeld, Chem. Phys. 1, 141 (1973), and references therein.

[9]T. Oka, J. Chem. Phys. 48, 5919 (1968); T. Shimizu and T. Oka, Phys. Rev. A 2, 1177 (1970).

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Decay of a Plasmon into Two Electromagnetic Waves

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(Received 13 February 1974)

In a plasma with a magnetic field, a plasmon can decay into two electromagnetic waves: a left-hand–polarized electromagnetic wave and right-hand–polarized whistler wave, both propagating in the direction of the magnetic field. Use of this process for a plasma laser is discussed.

Because an electromagnetic wave is cut off at a frequency below the plasma frequency, the de- of a plasmon into electromagnetic waves has been considered to be prohibited. I shall show here, however, that a plasma in a magnetic field is amenable to such a process.

In the presence of a magnetic field, electromagnetic waves are polarized. In particular, for the case of propagation parallel to the magnetic field, the left-hand polarized wave, whose dispersion relation is given by[1]

$$\epsilon_1(\omega,k)=1-\frac{\omega^2}{c^2k^2}+\frac{\omega_p^2\omega}{c^2k^2(\omega+i\nu+\omega_c)}=0, \quad (1)$$

has a cutoff frequency

$$\omega_{\rm cut}=\omega_p(1+\omega_c^2/4\omega_p^2)^{1/2}-\tfrac{1}{2}\omega_c, \quad (2)$$

which is smaller than ω_p. Hence the wave propagates at a frequency below the plasma frequency. Here, ω_p, ω_c, and ν are electron plasma, cyclotron, and collision (angular) frequencies, respectively.

The decay of a plasmon (Langmuir wave at $k\sim 0$) to this wave will produce a low-frequency, right-hand–polarized electromagnetic beat wave at $\omega<\omega_c/2$. First I show that the beat wave satisfies the dispersion relation of the whistler wave, given by[1]

$$\epsilon_2(\omega, k) = 1 - \frac{\omega^2}{c^2k^2} + \frac{\omega_p{}^2\omega}{c^2k^2(\omega + i\nu - \omega_c)} = 0. \quad (3)$$

With the use of subscripts 0, 1, and 2 for the plasmon, the left-hand-polarized electromagnetic wave, and the whistler, respectively, the resonant condition requires $\vec{k}_0 = \vec{k}_1 + \vec{k}_2$, $\omega_0 = \omega_1 + \omega_2$. Assume that the plasmon is at the condensed state so that $k_0 \ll k_1, k_2$. Then $\vec{k}_1 \simeq -\vec{k}_2 \equiv \vec{k}$ and Eqs. (1) and (3) give

$$\omega_2/\omega_c \sim \omega_c/\omega_p \sim c^2k^2/\omega_p{}^2, \quad (4)$$

where $\omega_c \ll \omega_p$ is assumed. The above expression shows that in fact the resonant condition can be satisfied by the Stokes and the low-frequency waves at $\omega_1 \simeq \omega_p - \omega_c{}^2/\omega_p$ and $\omega_2 \simeq \omega_c{}^2/\omega_p$. The matching condition is schematically shown in Fig. 1.

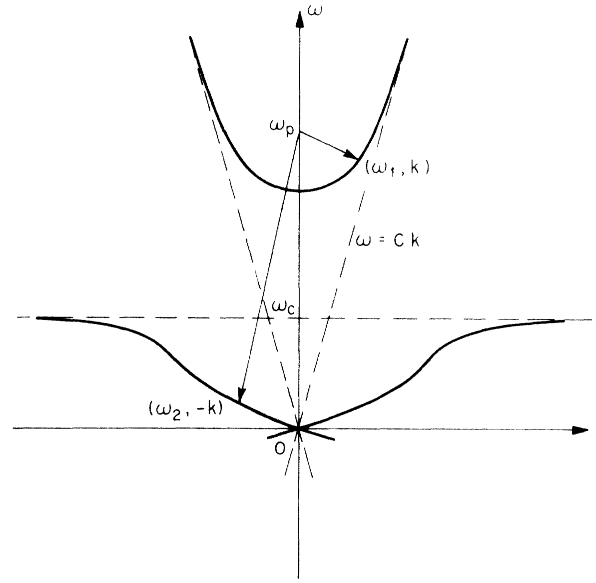

FIG. 1. Dispersion relation of the left-hand-polarized wave and the right-hand-polarized whistler wave. The resonant three-wave decay of the plasmon is indicated by the arrows.

Let us now construct the coupled equations between the two electromagnetic waves. Because $\omega_2 \ll \omega_c$, the wave-particle interaction may be negligible for most plasmas; hence I use the cold-fluid equations to express the electron dynamics:

$$i[\omega\vec{v}_{\vec{k}} + i\nu - \vec{v}_{k_0}\cdot(\vec{k}-\vec{k}_0)\vec{v}_{\vec{k}-\vec{k}_0}] = (e/m)[\vec{E}_{\vec{k}} + \vec{v}_{\vec{k}}\times\vec{B}_0 - \vec{v}_{\vec{k}_0}\cdot(\vec{k}-\vec{k}_0)\vec{E}_{\vec{k}-\vec{k}_0}/(\omega-\omega_0)], \quad (5)$$

$$\omega n_{\vec{k}} - \vec{k}\cdot(\vec{v}_{\vec{k}_0} n_{\vec{k}-\vec{k}_0}) = 0, \quad (6)$$

$$\vec{J}_{\vec{k}} = -e\textstyle\sum_{\vec{k}'} n_{\vec{k}-\vec{k}'} v_{\vec{k}'}. \quad (7)$$

A Maxwell's equation gives

$$[k^2 - (\omega^2/c^2)]\vec{E}_{\vec{k}} = i\omega\mu_0\vec{J}_{\vec{k}}. \quad (8)$$

Because of the circular polarization of the electromagnetic waves, it is convenient to introduce the left- and right-hand-polarized vector fields for $\vec{k}_1$ and $\vec{k}_2$ waves,

$$\vec{E}_{\vec{k}_1}{}^L \equiv E_{\vec{k}_1}\hat{x} + iE_{\vec{k}_1}\hat{y}, \quad \vec{E}_{\vec{k}_2}{}^R \equiv E_{\vec{k}_2}\hat{x} - iE_{\vec{k}_2}\hat{y}, \quad (9)$$

where z is the direction of the ambient magnetic field. Note here that $E_{-\vec{k}_2}\hat{x} + iE_{-\vec{k}_2}\hat{y} = \vec{E}_{\vec{k}_2}{}^{R*}$.

The coupling equation between $\vec{k}_1$ and $\vec{k}_2$ modes can be obtained by iterating Eqs. (5) and (6) and by constructing the nonlinear current $\vec{J}_{\vec{k}}{}^{(2)}$ from Eq. (7). Because $\vec{k}$ and also $\vec{v}_{\vec{k}_0}$ are parallel to the magnetic field, while $\vec{v}_{\vec{k}_1}$ and $\vec{v}_{\vec{k}_2}$ are perpendicular to $\vec{k}$, it follows that $n_{\vec{k}_1} = n_{\vec{k}_2} = 0$; hence the nonlinear term in Eq. (6) does not contribute. Consequently, $\vec{J}_{\vec{k}}{}^{(2)}$ is given by

$$\vec{J}_{\vec{k}}{}^{(2)} = -e(n_0\vec{v}_{\vec{k}}{}^{(2)} + n_{\vec{k}_0}{}^{(1)}\vec{v}_{\vec{k}-\vec{k}_0}{}^{(1)}), \quad (7')$$

where $\vec{v}_{\vec{k}}{}^{(2)}$ is given by the nonlinear term in Eq. (5). Substituting Eqs. (5) and (7′) into (8), we have the following coupled equations for $\vec{E}_{\vec{k}_1}{}^L$ and $\vec{E}_{\vec{k}_2}{}^{R*}$:

$$\epsilon_1(\omega_1,k_1)\vec{E}_{\vec{k}_1}{}^L - \frac{\omega_p{}^2}{k_1{}^2c^2}\frac{\omega_1\mu}{\omega_0-\omega_1-\omega_c}\left[1 - \frac{k_2}{k_0}\frac{\omega_0\omega_c}{(\omega_1+\omega_c)(\omega_0-\omega_1)}\right]E_{\vec{k}_2}{}^{R*} = 0,$$

$$\epsilon_2{}^*(\omega_0-\omega_1,k_2)\vec{E}_{\vec{k}_2}{}^{R*} - \frac{\omega_p{}^2}{k_2{}^2c^2}\frac{(\omega_0-\omega_1)\mu}{\omega_1+\omega_c}\left[1 - \frac{k_1}{k_0}\frac{\omega_0\omega_c}{\omega_1(\omega_c+\omega_1-\omega_0)}\right]\vec{E}_{\vec{k}_1}{}^L = 0, \quad (10)$$

where

$$\mu = n_{\vec{k}_0}^{(1)}/n_0 \tag{11}$$

is the amount of the density modulation of the Langmuir wave and shows the strength of the pump. The threshold value of the pump strength is readily obtained from these expressions as

$$\nu/\omega_p < \mu|\vec{k}|/|\vec{k}_0|, \tag{12}$$

where assumptions $\omega_p \gg \omega_c > \nu$ and $\vec{k}_1 \sim -\vec{k}_2 \equiv \vec{k}$ are used. The above expression tends to indicate that the threshold condition is more easily satisfied for a smaller value of the pump wave number k_0. However, this is misleading because the density perturbation of the Langmuir oscillation decreases for a longer-wavelength perturbation; hence $\mu/|\vec{k}_0|$ remains roughly constant. To eliminate this difficulty, we express μ in terms of the number density of plasmons N_p $(=\epsilon_0 E_{\vec{k}_0}^2/2\hbar\omega_p)$ as

$$\frac{N_p}{n_0} = \frac{1}{2\hbar\omega_p}\frac{m\omega_p^2}{k_0^2}\mu^2. \tag{13}$$

N_p/n_0 can also be expressed in terms of the electron speed in the Langmuir oscillation, $v_e = eE_{\vec{k}_0}/m\omega_p$, by

$$\frac{N_p}{n_0} = \frac{1}{2}\left(\frac{v_e}{v_{e\,\text{th}}}\right)^2 \frac{T}{\hbar\omega_p}, \tag{13$'$}$$

and

$$\mu/k_0 = v_e/\omega_p, \tag{14}$$

where $v_{e\,\text{th}}$ is the electron thermal speed and T is the temperature.

I will now discuss the means of pumping plasmons so that the above process may be used as a laser. In a gaseous plasma, the pumping may be achieved by injection of an electron beam. If a beam of speed v_0 is used, it excites the Langmuir wave at a wave number $k_0 \sim \omega_p/v_0$. The excited mode will cascade down to a longer wavelength by decaying into the ion-acoustic wave and condensate at $\omega = \omega_p$ and $k_0 \sim 0$.[2] During this process N_p is approximately conserved. If we write the plasma density perturbation excited directly by the beam as $\mu_0 = n_{\omega_p/v_0}/n_0$, the conservation of N_p gives

$$\mu/k_0 = \mu_0 v_0/\omega_p \tag{15}$$

and the threshold condition (12) becomes

$$v_0\mu_0/c > \nu/(\omega_p\omega_c)^{1/2}. \tag{12$'$}$$

In a laboratory plasma, the whistler wave can propagate[3] if the plasma density is increased to 10^{12} cm^{-3}, when $\nu/(\omega_p\omega_c)^{1/2} \lesssim 10^{-4}$ (for an electron-ion collision frequency ν). Hence, the above expression shows, if a beam of $v_0 \sim 10^{-1}c$ is used, the fractional density perturbation generated by the beam should exceed 10^{-3}, which may be easily achieved.

In a semiconductor plasma, plasmons may be generated directly by injection of current across the magnetic field,[4] by injection of photons,[5] or by the plasma instability described by Wolff.[6] In these cases it is convenient to write the threshold condition (12) in terms of the plasmon number density N_p. By combining Eqs. (4), (12), and (13), we have

$$\frac{N_p}{n_0} > \frac{m^*c^2}{2\hbar\omega_c}\left(\frac{\nu}{\omega_p}\right)^2, \tag{16}$$

where m^* is the effective mass of the electrons. If we use the example of Wolff,[6] $n_0 \sim 10^{18}$ cm^{-3}, $\hbar\omega_c \sim 10^{-2}$ eV, $(\nu/\omega_p)^2 \sim 10^{-3}$, $m^*c^2 \sim 10^4$ eV, then the threshold plasmon density becomes $N_p \sim 10^{21}$ cm^{-3} which may be marginally achievable.[5]

In a space plasma, the decay condition is most easily satisfied both in the magnetosphere and in the interplanetary medium. This mechanism may explain escape of radiation from a star or excitation of whistlers in the bow shock region.

Finally I compare the threshold condition (12) with the Nishikawa instability[2] (oscillating two-stream instability) which may be competing with the present process. The threshold value of the oscillating electron speed $v_e^{(\text{N})}$ of the Nishikawa instability is given by $v_e^{(\text{N})} = 2(\nu/\omega_p)^{1/2}v_{e\,\text{th}}$, while that of the present process $v_e^{(EM)}$ is given by Eqs. (4), (12), and (14) as $v_e^{(EM)} > [\nu/\omega_p\omega_c)^{1/2}]c$. Hence,

$$\frac{v_e^{(\text{N})}}{v_e^{(EM)}} = \frac{2v_{e\,\text{th}}}{c}\left(\frac{\omega_c}{\nu}\right)^{1/2}. \tag{17}$$

This shows the threshold value for the present process can be made smaller for a larger value of the electron-cyclotron frequency and the thermal speed.

The author wishes to thank Professor P. A. Wolff for suggesting the author consider the conversion process of plasmons to photons.

[1]T. H. Stix, *The Theory of Plasma Waves* (McGraw-Hill, New York, 1962).

[2]For example, K. Nishikawa, J. Phys. Soc. Jap. 24, 1152 (1968). The decay process continues as long as the group velocity $\partial\omega/\partial \boldsymbol{k}$ of the Langmuir wave exceeds the phase velocity of the ion-acoustic wave. However, when $\boldsymbol{k}_0 \to 0$, $\partial\omega/\partial\boldsymbol{k} \to 0$; hence the condensation ceases at a finite value of $\boldsymbol{k}_0$ corresponding to $\partial\omega/\partial\boldsymbol{k} = c_s$

$=v_{e\,\mathrm{th}}(m_e/m_i)^{1/2}$. Because $\partial\omega/\partial k = 3k_0 v_{e\,\mathrm{th}}^2/\omega_p$, the maximum value of k_0 after the condensation is given by $(\omega_p/v_{e\,\mathrm{th}})(m_e/m_i)^{1/2}/3$. For the present mechanism to be possible this value should be smaller than ω_p/c. This condition gives $T_e > 50$ eV for an electron-proton plasma, but the critical electron temperature is reduced in proportion to the ion mass. When $k_0 > \omega_p/c$, the excitation of the left-hand–polarized electromagnetic wave is still possible by the nonlinear cyclotron damping of electrons (these considerations will be published elsewhere).

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[5]D. H. Auston and C. V. Shank, private communication, and to be published.
[6]P. A. Wolff, Phys. Rev. Lett. 24, 266 (1970).

Excitation of Pure Electron Plasma Waves by Modulated Electron Beams

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A new type of exciter consisting of electron beams for launching electron plasma waves is investigated experimentally and theoretically. When fast electron sheet beams, modulated at a prescribed frequency, are injected into plasmas, electron plasma waves are found to be excited nearly perpendicularly to the electron beams. Compared with a mesh excitation, we found that the beam excitation has several merits: it hardly excites ballistic modes, the direct coupling due to an electrostatic induction with another probe is small, and it can even be used in a very hot plasma.

There have been reported several types of exciters of electron plasma waves, i.e., a wire probe,[1] a gridded parallel coupler,[2] and parallel plate grids.[3] All of these are metal-type exciters, which easily excite the ballistic mode[4,5] via the sheath around the metal. In this Letter, we report on the excitation of electron plasma waves by electron sheet beams as an electron version of an excitation of ion waves by ion beams.[6]

Experiments were performed in two types of vacuum chambers. One is 160 cm in length and 32 cm in diameter at Tohoku University. Argon gas is used at a pressure $P=(4-6)\times10^{-4}$ Torr. Forward-diffusion-type plasmas are used for the experiments. The typical parameters are $N_0=(1-2)\times10^{7}$ cm^{-3} (density) and $T_e=3-6$ eV (electron temperature). Another is 120 cm in length and 60 cm in diameter at the Institute of Space and Aeronautical Science, University of Tokyo. Plasma sources of a back diffusion type are used for the experiments. The typical plasma parameters are $N_0=(0.5-2)\times10^{7}$ cm^{-3} and $T_e=3-4$ eV. The electron sheet beams are injected into the plasmas through a slit 5 cm in length and 0.5 cm in width. The electrons are produced by a hot cathode. For the wave excitation, the electron sheet beams, modulated at a prescribed frequency, are injected into plasmas across the chamber.

In Fig. 1(a), we show the electric circuit for the excitation and the detection of electron plasma waves. In Fig. 1(b), the spatial distribution of the electron current across the beam is shown with the beam velocity as a parameter. The effect of the existence of the electron beam is clearly shown for the various beam velocities. The excited waves are detected by a circular planar mesh shielded by meshes. The typical wave patterns excited by electron sheet beams are indicated in Fig. 1(c) as the modulation frequency of the beams is varied.

The dispersion relation and damping coefficients obtained by electron-sheet-beam excitation are shown in Fig. 2, in which the experimental results obtained by a mesh excitation are also plotted. The experimental results obtained by the beam excitation are nearly in accord with those obtained by mesh excitation. It must be noted that the free-streaming mode, easily excited by mesh excitation, is hardly excited by the beam excitation. When the modulation voltage of the electron beams is changed, the amplitude of the excited waves is found to be nearly proportional to the modulation voltage which is less than 1 V peak to peak.

In Fig. 3(b), the effect of a direct coupling due to an electrostatic induction is shown for both beam and mesh excitation by changing the phase