Measurement of the Coherent Neutron Scattering Length of ^{3}He by Reflection from a Quartz–Liquid-Helium Interface*

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The real part of the bound-atom coherent neutron scattering length of ^{3}He has been measured by comparing the reflectivities of quartz–liquid-^{3}He and quartz–liquid-^{4}He interfaces and has been found to have the value $(6.1\pm0.6)\times10^{-13}$ cm.

Among the known excited nuclear states of ^{4}He,[1] there is one, a 0^+ state at about 20.2 MeV, which appears to be identified with the extremely large thermal cross section of the reaction ^{3}He(n, p)^{3}H.[2,3] Since this state lies below the neutron binding energy of 20.6 MeV, any resonance contribution it may make to the scattering should interfere constructively with the potential scattering and thereby enhance the scattering cross section.[4] Consequently it has been conjectured that the ^{3}He nucleus is not only an extremely good absorber of thermal neutrons but may also scatter them effectively as well.

This question is of course of intrinsic interest to the theory of the four-nucleon system. But in addition, recent studies[5] of ^{3}He-^{4}He liquid mixtures have demonstrated that neutron spectroscopy can be successfully applied to condensed phases containing small amounts of ^{3}He. Thus there is added interest in understanding the scattering properties of the ^{3}He nucleus, expecially those aspects relating to the study of coherent processes. As a first step in this direction we have undertaken to compare the neutron reflecting properties of quartz–liquid-^{3}He and quartz–liquid-^{4}He interfaces.

The underlying basis for such comparisons is the well-known Fresnel formula which relates the reflectivity of an interface between two media to their respective refractive indices. According to this formula, when radiation in a medium with index of refraction n_A is incident at grazing angle φ on a medium with index n_B, the reflectivity is[4,6]

$$R_{AB}=\left|\frac{1-[1-(1-n_B^2/n_A^2)/\varphi^2]^{1/2}}{1+[1-(1-n_B^2/n_A^2)/\varphi^2]^{1/2}}\right|^2. \tag{1}$$

The indices of refraction, n_A and n_B, are related in a simple way to the coherent scattering lengths, b_A and b_B, of the media involved, i.e.,

$$\frac{n_B^2}{n_A^2}=\frac{1-\lambda^2N_Bb_B/\pi}{1-\lambda^2N_Ab_A/\pi}$$
$$\cong1-\frac{\lambda^2}{\pi}(N_Bb_B-N_Ab_A), \tag{2}$$

where N represents the number of scattering centers per unit volume and λ is the neutron wavelength in vacuum. In general, neutron scattering lengths are complex quantities, the real and imaginary components given by the relations[4]

$$b_{\rm real}=\textstyle\sum_J[\sigma_{SJ}/4\pi-(\sigma_{TJ}/2\lambda)^2]^{1/2}g_J, \tag{3a}$$

and

$$b_{\rm imag}=-\textstyle\sum_J(\sigma_{TJ}/2\lambda)g_J=-\sigma_T/2\lambda. \tag{3b}$$

In the above, g_J represents the statistical weight factor which, for s-wave neutrons, is either $(I+1)/(2I+1)$ or $I/(2I+1)$ depending on whether J, the spin of the compound system of neutron plus nucleus, is $I+\frac{1}{2}$ or $I-\frac{1}{2}$. σ_{SJ} and σ_{TJ} are respectively the scattering and total cross sections for channel spin J. It is evident that a comparison of the reflectivities of quartz–liquid-^{3}He and quartz–liquid-^{4}He interfaces involves a comparison of the indices of refraction of liquid ^{3}He and liquid ^{4}He and is thus a comparison of the coherent scattering lengths of the two nuclei.

If, in Eq. (2), medium A is quartz and B liquid helium, $N_Bb_B-N_Ab_A<0$ for both liquid ^{3}He and ^{4}He. In this case it is easily seen from Eq. (1) that critical reflection will not occur and that even an absorption cross section as large as that of ^{3}He will have almost no influence on the reflectivity. For this reason, and also to avoid difficulties with neutron absorption in ^{3}He liquid and vapor, we designed our apparatus so that the beam was transmitted through the quartz and re-

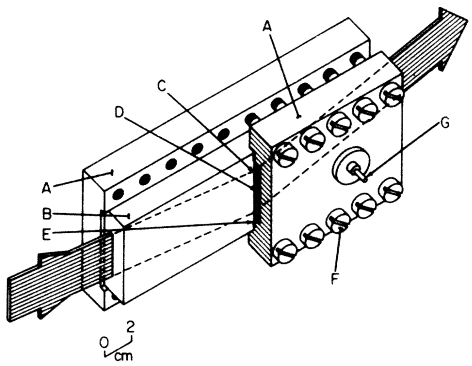

FIG. 1. Reflectivity cell. The shaded arrow represents the incident and reflected components of the beam.

flected internally at the liquid interface. Figure 1 shows the experimental arrangement. In essence it consisted of a $190\times45\times15$ mm^3 quartz plate (B), cut from a natural quartz crystal and polished optically flat on one face (C). This was clamped with brass bolts (F) between a pair of thick, rigid stainless-steel blocks (A) into one of which a liquid cell (D) of dimensions $177\times32\times2.5$ mm^3 was machined. Indium wire between the quartz and steel surfaces formed a superfluid-tight seal between cell and plate. The cell was filled through a capillary (G). Except for the mirror face, all of its interior surfaces were lined with roughened cadmium (E) as a precaution against neutron background from wall scattering or reflection. The entire assembly was mounted on the bottom flange of a ^{3}He refrigerator and held rigidly in position by fiberglass spacers.

All measurements were made with a beryllium-filtered cold neutron beam conducted from the reactor face to the apparatus by a reflecting guide tube and collimated to a horizontal angular divergence of 1 min arc (full width at half-maximum).

Initially, mixtures of heavy and light water were used to evaluate the performance of the mirror system. Water mixtures were chosen because their refractive index for neutrons can be shifted by simply varying the H:D ratio. Altogether six mixtures plus vacuum were studied, spanning the entire range from critical reflection, through nonreflecting conditions, to mixtures with refractive indices matching liquid ^{4}He. Originally, the observed reflected intensities were compared with reflectivities calculated from Eq. (1) by integration over the angular and wavelength distributions in the incident beam. These measurements indicated that although the system behaved qualitatively as expected, quantitatively both the magnitude of the reflectivity and its angular variation deviated systematically from the calculated values. This was most obvious when the refractive indices of the water mixtures were close to that of crystalline quartz, in which case the reflected intensity, although small, did not fall to the very low levels predicted by the Fresnel formula. In addition, there were indications that the reflectivity of the empty cell was determined to some extent by the refractive index of the last H_2O-D_2O mixture with which it had been filled.

The most plausible explanation for this behavior is that the refractive index of the quartz surface differs somewhat from that of the bulk material. This is not unexpected in view of the fact that polished surfaces, even those of the highest optical quality, are known to contain microscopic pits and fissures which act as preferred adsorption sites. In fact, other investigators have shown that such surfaces are never free of adsorbed material (primarily water vapor) unless outgassed by ion bombardment at the highest vacuum.[7] Indeed, we found that we could completely explain the observed deviations from Eq. (1) by introducing into our calculations a third medium (medium C) representing a transition region at the quartz surface with an index of refraction shifted somewhat from that of crystalline quartz by the presence of the liquid medium. In such a situation,[8] the reflectivity is properly expressed in terms of the reflectivities at each interface, i.e.,

$$R_{ACB}=(R_{AC}+R_{CB}-2R_{AC}R_{CB})/(1-R_{AC}R_{CB}), \quad (4)$$

provided two assumptions are made: (i) The thickness of medium C varies in such a way that macroscopic interference effects can be ignored and (ii) its composition varies slowly along the direction normal to the interface in which case the effective index of refraction is the average value. It should be emphasized that refraction at the A-C interface has to be taken into account in evaluating R_{CB}.

In comparing the observed reflectivities to those predicted by Eq. (4), we regarded the refractive indices of both the H_2O-D_2O mixtures and the quartz as known quantities while the refractive index of the transition layer (medium C) and the parameters of the neutron wavelength distribution $f(\lambda)$ were considered as quantities to be determined by fitting to the observed reflectivi-

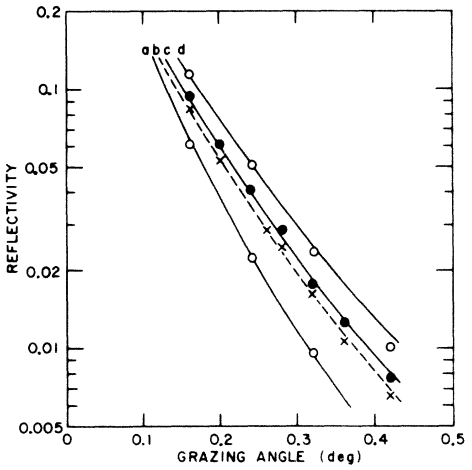

FIG. 2. Reflectivity versus angle for quartz interfaces with (a) a 0.40-H_2O–0.60-D_2O mixture, (b) liquid ^{3}He at 0.7 atm and 1.23°K, (c) liquid ^{4}He at 1.6 atm and 1.23°K, and (d) vacuum. Although the data are not shown, the reflectivities observed for a 0.805-H_2O–0.195-D_2O mixture (which has a refractive index matching liquid ^{4}He) were identical to those observed for liquid ^{4}He. The solid lines are computed from Eq. (4); the dashed line represents the best computed least-squares fit for liquid ^{3}He.

ties. To describe $f(\lambda)$ we used an expression of the form $f(\lambda) \sim \lambda^{-(m+p\lambda^2)}$. For a Maxwellian distribution, $m = 5$ and $p = 0$. However, the subthermal spectrum from a reactor is known to decrease more rapidly than λ^{-5} and in our case $f(\lambda)$ will also be modified because the beam was transmitted through the beryllium filter, reflecting guide tube, and quartz mirror. The best-fitting computed reflectivities were those obtained with $m = 6$ and $p = 0.189$. Integration extended over the wavelength range from 4.13 to 10.5 Å. Since the spectral parameters are overdetermined by the measurements, we believe the fit to be unique. It is not easy to understand why the effective short-wavelength cutoff, 4.13 Å, was shifted slightly from the beryllium-filter cutoff value of 4.03 Å, but this probably results from the reflecting properties of the guide tube. We should also note that the effective angular divergence of the beam transmitted through the quartz was found to be 1.5 min (full width at half-maximum); hence the angular integration in Eq. (4) was extended to cover this range.

In Fig. 2, the open circles represent the measured reflectivity for two of our seven calibration runs. The solid lines are reflectivities calculated from Eq. (4). The calculations for all seven runs were in excellent agreement with the observations but indicated, as conjectured, that the refractive index of the transition layer at the surface depended on the liquid in the cell. Clearly ^{3}He diffusing into this layer would present a problem since it would alter the reflecting and refracting properties of the system and invalidate our calibration procedure.

Fortunately, we found that the surface layer was not permeable to liquid helium, presumably because the surface pits and fissures were completely filled with adsorbed water. This was determined in the following way. After the cell was filled at room temperature with a 0.28-H_2O–0.72-D_2O mixture (chosen because its index of refraction matches that of crystalline quartz) it was evacuated and the reflectivity measured. Analysis of the data using Eq. (4) indicated $N_C b_C = 3.9 \times 10^{10}$ cm^{-2}, slightly lower than the value for crystalline quartz, 4.19×10^{10} cm^{-2}. The cell was than filled with liquid ^{4}He and cooled to 1.2°K. Again the reflectivity was measured. It was found to be that predicted from Eq. (4) assuming the properties of the surface layer unchanged. As a further check, 1% ^{3}He was added. Although the absorption cross section was thereby considerably increased, there was no change in the reflected intensity, proving both that neutrons were not reflected from the rear wall of the cell and that ^{3}He was not penetrating into the quartz. The reflectivities observed with this 1% mixture are plotted in Fig. 2 as closed circles.

Having confirmed that liquid helium was not diffusing into the surface layer to any measureable extent and having in effect calibrated the apparatus with liquid ^{4}He, we warmed the cell to 15°K and pumped it dry. The reflectivity quickly returned to its original vacuum value, indicating no detectable residue of ^{4}He or ^{3}He. Then, as a last step, the cell was refilled with pure liquid ^{3}He, cooled to 1.2°K, and the reflectivity measurements repeated. The results appear as the crosses plotted in Fig. 2. Also shown is a dashed line representing the computed best fit[9] by Eq. (4). From this we find for the real part of the bound coherent scattering length of ^{3}He the value

$$(6.1 \pm 0.6) \times 10^{-13} \text{ cm}.$$

This is large enough to encourage the view that neutron scattering can be extended to more concentrated ^{3}He systems, although difficulties with background from isotopic incoherent scattering must be anticipated as well as severe losses due to absorption.

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[9]The computation took into account the influence of ^{3}He absorption on the index of refraction. The value of the scattering lengths of H, D, O, and ^{4}He used in computing the calibration curves of Fig. 2 were $(-3.740 \pm 0.003, 6.672 \pm 0.007, 5.803 \pm 0.006$, and $3.0) \times 10^{-13}$ cm, respectively, as listed by C. G. Shull (private communication).

Prediction of Weak-Coupling Structure from a Shell-Model Basis*

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Mixed-configuration shell-model wave functions predict the sort of two-nucleon transfer phenomena which has been interpreted as evidence for weak coupling to excited 0^+ states.

We present in this note calculations with shell-model wave functions for $(0d, 1s)$-shell nuclei which indicate that an apparent weak-coupling phenomenon recently observed experimentally in the $0f, 1p$ shell is, indeed, attributable to weak coupling and, moreover, may be a general feature of the structure of light- and medium-mass nuclei.

The experimental observations we refer to have demonstrated that in several instances[1,2] the (p, t) reaction on odd-mass fp-shell nuclei populates both the ground state and an excited state of the residual nucleus with pure $L = 0$ angular distributions. This establishes, of course, that both the residual states have J^π equal to the J^π of the target state ($\frac{7}{2}^-$ in the examples studied), but the significance of the observations lies in the absence of any general selection rule which would limit the angular momentum transfer in such cases to *only* $L = 0$. The explanation of this phenomenon thus must arise from specific nuclear-structure properties of the states involved.

The purity of the $L = 0$ transitions to the ground states presumably originates from the dominance of $J = 0$ pairing in forming nuclear ground states. The last, "odd," nucleons of the odd-mass nuclei $A + 3$ and $A + 1$ are pictured as weakly coupled spectators to the 0^+ ground-state wave functions of the adjacent even-mass nuclei $A + 2$ and A, and the transition between the $A + 3$ and $A + 1$ $J^\pi = \frac{7}{2}^-$ ground states is viewed, essentially, as the pick-up of the same correlated $J = 0$ pair by which the $A + 2$ and A ground states are connected. This picture seems empirically verified by the approximate equality of the cross sections of the $A + 3 \rightarrow A + 1$ and $A + 2 \rightarrow A$ (p, t) ground-state transitions. Weak coupling in such a context is more or less implicitly assumed in all approaches to nuclear structure, just as it is in the very familiar analogous situation involving the "single-particle" states of a nucleus $A + 1$ and the 0^+ ground state of nucleus A.

The question of the extent to which a weak-coupling picture of nuclear structure can be gen-