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New Interpretation of the Pion-Nucleus Optical Potential for Pionic Atoms*

S. Barshay, G. E. Brown, and M. Rho†
Institute for Theoretical Physics and Department of Physics, State University of New York, Stony Brook, New York 11790
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We show that if the P-wave interaction volumes relevant for construction of the pion-nucleus optical potential used in pionic-atom studies turn out to be simply related empirically to the free-space pion-nucleon scattering volumes, then this would be indicative of both sets of quantities being dominated by the Δ(1236 MeV), which can be treated as an independent baryonic excitation in nuclei. Within this framework, we give a simple derivation of modifications to the optical potential coming from baryon-baryon short-range correlations.

In this note we consider the four P-wave interaction volumes for slow pions, first for scattering on free nucleons and then for interaction with nucleons in nuclei subject to the restrictions of the Pauli principle. The exclusion principle and associated many-body effects can, in general, modify the sum of diagrams usually calculated in the Chew-Low-type theory,[1] so that it is not clear that the usual assumption[2,3] that the free-space and in-medium volumes are the same is justified. Strong-interaction shifts in pionic atoms[4] should be especially sensitive to possible dynamical modifications. We show that if the free-space and in-medium volumes turn out to be simply related empirically, this could be interpreted in terms of the Δ(1236 MeV) behaving as an independent baryonic excitation in nuclei, and we then can give a very simple description of the effect of short-range baryon-baryon correlations.

Approximate solution of integral equations of the Chew-Low model[1] leads to the P-wave interaction volumes $\alpha_{2T,2J}$ for the scattering of slow pions on free nucleons:

$$\alpha_{33}=\frac{\frac{4}{3}f^2}{1-m_\pi/\omega^*}m_\pi^{-3}\cong 0.21m_\pi^{-3}\quad(0.215\pm0.005),$$
$$\alpha_{11}=\frac{-\frac{8}{3}f^2}{1+m_\pi/\omega^*}m_\pi^{-3}\cong -0.14m_\pi^{-3}\quad(-0.09\pm0.01),\tag{1}$$
$$\alpha_{13}=\alpha_{31}=-\tfrac{2}{3}f^2m_\pi^{-3}\cong -0.053m_\pi^{-3}\quad(-0.035\pm0.005).$$

Here m_π denotes the pion mass, $\omega^*\cong 2m_\pi$ denotes the pion total c.m. energy at resonance, and $f^2=0.08$. The numbers at the right are the empirical free-space scattering volumes.[5] The approximate solutions effectively sum series of graphs, a few of which are shown in Fig. 1.

Now consider the case of scattering from a nucleon in a nucleus with $N=Z$, viewed as a Fermi sea, for pion momentum going to zero. In pionic atoms, the pion momentum k will be small compared with typical nucleon momenta.[6] The Born graphs, Figs. 1(a) and 1(b), *in each eigenchannel*[7] will be suppressed by the Pauli principle, since the pion brings in no momentum, and therefore the nucleon cannot make a transition to an unoccupied state. Since the (3, 3) resonance is built up out of an infinite number of such graphs, one might expect that the resonance will still be present, but that a suppression of one graph would lead to a diminishing of the scattering volumes for $\omega \to m_\pi$. This might be roughly estimated from[8]

$$\alpha_{33} \to \alpha_{33}^{(N=Z)} \approx \left(\frac{\frac{4}{3} f^2}{1 - m_\pi/\omega^*} - \tfrac{4}{3} f^2\right) m_\pi^{-3} = \frac{\frac{4}{3} f^2 m_\pi/\omega^*}{1 - m_\pi/\omega^*} m_\pi^{-3},$$

$$\alpha_{11} \to \alpha_{11}^{(N=Z)} \approx \left[\frac{-\frac{8}{3} f^2}{1 + m_\pi/\omega^*} - (-\tfrac{8}{3} f^2)\right] m_\pi^{-3} = \frac{\frac{8}{3} f^2 m_\pi/\omega^*}{1 + m_\pi/\omega^*} m_\pi^{-3}, \qquad (2)$$

$$\alpha_{13} = \alpha_{31} \to \alpha_{13}^{(N=Z)} = \alpha_{31}^{(N=Z)} \approx 0.$$

Some contributions to the building up of the $1/\omega^*$ relevant *for the medium* will also be suppressed by the exclusion principle, and there will be additional many-body processes. In the important processes shown in Figs. 1(c) and 1(d), the nucleon can have large intermediate momenta due to the presence of a high-momentum virtual pion, so one would not expect the Pauli principle to have a very large effect on these higher-order graphs. The usual P-wave part of the optical potential[2,3] for a nucleus with $N=Z$ is

$$2\omega V_P(\omega, r) = 4\pi\nabla \cdot c_0 \rho(r) \nabla, \qquad (3a)$$

where the *coherent* parameter c_0 given by

$$c_0 = \tfrac{1}{3}(4\alpha_{33} + \alpha_{11} + 2\alpha_{31} + 2\alpha_{13}) \qquad (3b)$$

would be expected from Eq. (2) to be less (since $1/\omega^* \lesssim 1/2m_\pi$) than $0.16 m_\pi^{-3}$, whereas empirically[4] $c_0 \gtrsim (0.21 \pm 0.07) m_\pi^{-3}$ is indicated. Note, however, that since the Born terms do not contribute to c_0 in the static Chew-Low approximation, we *cannot* ascribe any difference between theoretical and empirical values to suppression of the *lowest-order* terms by the exclusion principle.

There is an alternative interpretation of the free-space pion-nucleon P-wave scattering volumes, compatible with the dispersion relation from which the dynamical model of Chew and Low may be constructed. This involves simply calculating these volumes from the four graphs shown in Fig. 2 where the Δ(1236 MeV) isobar is treated as a basic excitation of baryonic matter, characterized by a mass and coupling constant.[9] The results of this calculation are

$$\alpha_{33} = \left(\tfrac{4}{3} f^2 + \frac{G^2 m_\pi/\omega^*}{1 - m_\pi/\omega^*} + \frac{\frac{1}{9} G^2 m_\pi/\omega^*}{1 + m_\pi/\omega^*}\right) m_\pi^{-3} \cong 0.25 m_\pi^{-3},$$

$$\alpha_{11} = \left(-\tfrac{8}{3} f^2 + \frac{\frac{16}{9} G^2 m_\pi/\omega^*}{1 + m_\pi/\omega^*}\right) m_\pi^{-3} \cong -0.33 m_\pi^{-3}, \qquad (4)$$

$$\alpha_{31} = \alpha_{13} = \left(-\tfrac{2}{3} f^2 + \frac{\frac{4}{9} G^2 m_\pi/\omega^*}{1 + m_\pi/\omega^*}\right) m_\pi^{-3} \cong -0.033 m_\pi^{-3},$$

with

$$G^2/m_\pi^2 = (m^*/m)\tfrac{1}{2}\gamma^*/(k^*)^3 \cong 0.137 m_\pi^{-2},$$

for $m^* \cong 1236$ MeV, $\frac{1}{2}\gamma^* \cong 60$ MeV, and $k^* \cong 225$ MeV/c. These numbers are quite similar to the empirical ones of Eq. (1).[10]

The α_{33} represents $\tan\delta_{33}/k^3$. Thus if one computes the resonant scattering amplitude, setting $G^2 \cong \frac{4}{3} f^2$, one obtains the usual Chew-Low result,[1]

$$\frac{1}{k}\exp(i\delta_{33})\sin\delta_{33} = \frac{\frac{4}{3} f^2 k^2/\omega m_\pi^2}{1 - \omega/\omega^* - i\frac{4}{3} f^2 k^3/\omega m_\pi^2}. \qquad (5)$$

One might be tempted to ascribe the ω^{-1} dependence in the numerator to a π-N vertex function

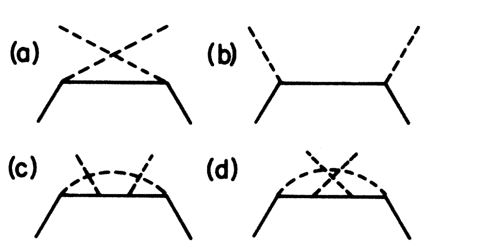

FIG. 1. Graphs (a) and (b) represent the Born terms, and (c) and (d) some contributions of $O(f^4)$ for P-wave π-N scattering.

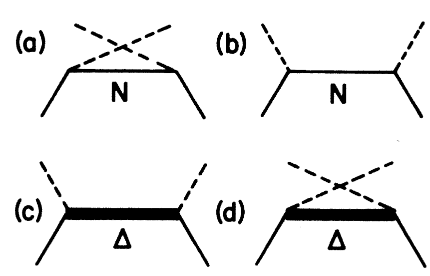

FIG. 2. Graphical representation of P-wave π-N scattering via nucleon exchange and isobar excitation.

of the form[11] [we take the Chew-Low $v(k) \cong 1$]

$$\Gamma(k) = \omega^*/\omega_k \cong Z(1 + k^2/m_\pi^2)^{-1/2}, \tag{6}$$

but this model shows that such an interpretation may be unjustified. There are no vertex functions in the elementary processes in Fig. 2; the pseudo vertex function (6) arises from the sum of the two elementary processes.

Inside a nucleus with $N=Z$, viewed as a Fermi sea, the Pauli principle will suppress the nucleon-exchange terms, so that one is left with only the terms from the intermediate isobar in Eq. (4), giving

$$\begin{aligned} \alpha_{33}^{(N=Z)} &\cong 0.14 m_n^{-3}, \\ \alpha_{11}^{(N=Z)} &\cong 0.08 m_\pi^{-3}, \\ \alpha_{31}^{(N=Z)} &\cong \alpha_{13}^{(N=Z)} \cong 0.02 m_\pi^{-3}; \end{aligned} \tag{7}$$

and a corresponding optical potential parameter

$$c_0 \cong 0.24 m_\pi^{-3}, \tag{8}$$

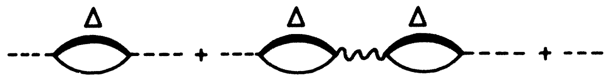

FIG. 3. Series of isobar-hole excitation graphs leading to the P-wave part of the pion-nucleus optical potential. The wiggly line connecting two bubbles represents the effective isobar-nucleon force with the one-pion–exchange piece subtracted out.

which is close to the number which would be obtained by substituting (without justification) the *empirical* free-space volumes into Eq. (3b), namely, $\sim 0.21 m_\pi^{-3}$.

Within our model for the origin of the P-wave scattering volumes in nuclei, a simple derivation of the effect of a short-range baryon-baryon correlation upon the optical potential can be given. This potential is represented by the sum of isobar-hole excitation bubbles (Fig. 3) corrected for the short-range correlations between successive bubbles. In terms of the basic "self-energy" bubble[12] $\Pi(k,\omega) = k^2 U(k,\omega)$, it is given by

$$2\omega V(k,\omega) = \frac{-4\pi k^2 U(k,\omega)}{1 + 4\pi\{g(k,\omega) - [-k^2/(k^2 + m_\pi^2 - \omega^2)]\} U(k,\omega)}, \tag{9}$$

where $g(k,\omega)$ represents the full nonstatic ($\omega \neq 0$) isobar-hole effective interaction analogous to the effective particle-hole interaction $g^-(k,\omega)$.[13] For a pure single-pion exchange potential, we would have

$$g(k,\omega) = -k^2/(k^2 + m_\pi^2 - \omega^2), \tag{10}$$

for which case the denominator in Eq. (9) would become unity; i.e., there would be no short-range correlation. However, this potential contains a δ-function attraction in configuration space, of the form[13]

$$G^2 \vec{S}_1 \cdot \vec{S}_2 [-\tfrac{4}{3}\delta(\vec{r})], \tag{11}$$

where $G^2 \vec{S}_1 \cdot \vec{S}_2$ replaces the usual $f^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2$ when one refers to isobar-hole excitation, with $\vec{S}$ a transition spin operator.[14] It is *likely* to be removed by a hard core in the isobar-nucleon force, just as it *is* removed by a hard core in the nucleon-nucleon force.[13] Thus removing the δ-function piece, we have

$$\begin{aligned} g(k,\omega) &= -k^2/(k^2 + m_\pi^2 - \omega^2) + \tfrac{1}{3}, \\ 2\omega V(k,\omega) &= -4\pi k^2 U(k,\omega)/[1 + \tfrac{4}{3}\pi U(k,\omega)]. \end{aligned} \tag{12}$$

In a simple approximation,[2] $U(k,\omega) \to c_0 \rho(r)$ and $-k^2 U(k,\omega) \to \nabla \cdot U \nabla$ in configuration space, Eq. (12) becomes identical to the Ericson-Ericson potential for the P-wave pion with the Lorentz-Lorenz correction taken into account.[3]

There are two points worth noticing in the derivation of Eq. (12). The first is that the baryon-baryon short-range correlation manifests itself in a simple and crucial way. The single-scattering approximation contains among others the situation where two nucleons are on top of each other. If the nucleons are kept apart by small hard cores, these contributions must be subtracted. This is just what the Lorentz-Lorenz correction does. Secondly, as noted, our model shows that the $1/\sqrt{\omega}$ factor which multiplies $v(k)$ in the Chew-Low amplitude, and also appears in the off-shell amplitude constructed from a separable potential,[11] must be removed in any analysis which seeks empirical evidence for a *bona fide* π-N-N vertex structure. The most recent empirical analysis[15] limits such a hypothetical vertex structure to a shorter range (<0.4 fm) than that affected by the N-N hard-core size, and hence there is no drastic reduction in the Lorentz-Lorenz effect of the sort discussed by Eisenberg, Hüfner, and Moniz.[16]

Our simple derivation has been carried out for a pion following a straight-line path with momentum k through infinite nuclear matter and making successive isobar-hole excitations so long as the two successive nucleons are outside of the hard core.[17] However, the connection with the multiple-scattering picture in a finite nucleus is clear: Between successive scatterings, a *virtual* pion can propagate with high momentum q with an effective propagator $q^2[i\pi\delta(q^2-k^2)+\mathrm{P}(q^2-k^2)^{-1}]$, where P denotes the principle value in the integration over d^3q. The effect of the hard core is again to remove the δ function in configuration space,

$$\mathrm{P}\left(\frac{q^2}{q^2-k^2}\right)\to\mathrm{P}\left(-\tfrac{1}{3}+\frac{q^2}{q^2-k^2}\right)=\tfrac{1}{3}\mathrm{P}\left(\frac{2q^2+k^2}{q^2-k^2}\right), \qquad (13)$$

which, as expected, simply weakens the high-momentum components of the virtual pion. Thus the correction for hard-core interactions cuts down the off-shell part of the propagation and would presumably account for the success of the approximations (e.g., Glauber theory) which neglect off-shell propagations. (The hard cores make the pions go a certain minimum distance between interactions; it is easier for them to go a long distance on-shell than off-shell.)

Our general conclusion is that the optical potential built with free-space π-N amplitudes can be successful in pionic atoms mainly because of the prominent role played by the *independent* isobar excitation. The Lorentz-Lorenz correction appears in this picture simply as a consequence of a short-range isobar-nucleon correlation. Furthermore the observation of the behavior of baryonic excitations induced by pion probes in nuclei yields basic information toward an understanding of the nature of these excitations, complementing the information obtainable from exciting free nucleons.

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†Permanent address: Service de Physique Theorique, Centre d'Etudes Nucléaires de Saclay, B.P. 2, Gif-sur-Yvette, France.

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[6]When account is taken of the fact that it is the wave number *in the medium* that is relevant, momenta $< p_F/2$ are involved. To make our physical point clearly we discuss the *idealized* situation for a vanishing momentum.

[7]In calculating the scattering from the entire many-body system in *Born* approximation, one obtains identical answers with or without the Pauli principle, differences in scattering from individual nucleons in the two cases summing to zero.

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[9]The excitation is no more "elementary" than the nucleon itself, namely, it is an excited distribution of matter. The effective complex mass of the isobar can be modified in the medium. We assume here little modification for slow pions in peripheral nuclear densities. Cf., for example, S. Barshay, V. Rostokin, and G. Vagradov, Nucl. Phys. B59, 189 (1973), and Phys. Lett. 43B, 271 (1973).

[10]The theoretical α_{11}, which deviates the most from the empirical value, would receive a significant contribution from an intermediate state with the quantum numbers of the nucleon, such as the $N'(1470)$. Such an interaction is attractive and would indeed tend to reduce $|\alpha_{11}|$.

[11]See R. H. Landau and F. Tabakin, Phys. Rev. D 5, 2746 (1972), in particular their Eq. (33).

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