[4]For an account of the fluctuation-dissipation theorem, see, e.g., P. C. Martin, in *Many Body Physics*, edited by C. DeWitt and R. Belian (Gordon and Breach, New York, 1968), p. 37.

[5]R. Bullough and P. J. Caudrey, J. Phys. A: Proc. Phys. Soc., London 4, L41 (1971).

[6]It should be noted that if the two-level atom were treated as a harmonic oscillator, then (2), (4), and (8) should be replaced by

$$\gamma^{(-)}(\vec{a},\omega)+i\Omega^{(-)}(\vec{a},\omega)=\int_0^\infty d\tau\, D(\vec{a},\tau)e^{i\omega t}, \tag{2a}$$

$$\Omega^{(-)}(\vec{a},\omega)=-\pi^{-1}\int_0^\infty d\omega_0\,\gamma^{(-)}(\vec{a},\omega_0)[(\omega_0+\omega)^{-1}+(\omega_0-\omega)^{-1}], \tag{4a}$$

$$\gamma^{(-)}(\vec{a},\omega)=\mathrm{Im}\sum\nolimits_{ij}d_i d_j\chi_{ij}(\vec{a},\vec{a},\omega). \tag{8a}$$

Thus the damping is identical to that for a two-level atom, but the frequency shift is very different. Note further that (2a), (4a), and (8a) can also be obtained from classical considerations. I would also like to point out that $\gamma(\vec{a},\omega)=\gamma^{(-)}(\vec{a},\omega)$ only for the case when initially the field is in a vacuum state.

[7]K. H. Drexhage, Sci. Amer. 222, No. 3, 108 (1970), and to be published.

[8]In quantum electrodynamics a perfect conductor seems to have been used in this sense [cf. H. B. Casimir and D. Polder, Phys. Rev. 73, 360 (1948)]. Strictly speaking, one should include the dispersion of the dielectric function even for a conductor. At any rate, formulas (14) and (15) are quite instructive and that is why I have presented them.

[9]*Handbook of Mathematical Functions*, edited by M. Abramowitz and I. Stegun (Dover, New York, 1964), p. 231.

[10]H. Morawitz, Phys. Rev. 187, 1792 (1969).

Propagation of Sound in Two-Dimensional He^3†

J. R. Eckardt,* D. O. Edwards, P. P. Fatouros, F. M. Gasparini,‡ and S. Y. Shen
Department of Physics, Ohio State University, Columbus, Ohio 43210
(Received 4 February 1974)

"Surface sound" has been observed for the first time, in He^3 adsorbed on the free surface of liquid He^4. Measurements of the surface-sound velocity and surface tension are used to obtain new values for the effective mass $M=(1.3\pm0.1)m_3$ and binding energy $\epsilon_0/k_B=2.28\pm0.03$ K of the adsorbed He^3.

This Letter reports measurements of the velocity of surface sound on liquid He^4, cooled below 100 mK, and covered with about 0.2 atomic layer of adsorbed He^3. Earlier surface-tension studies[1-3] have shown that in such low concentrations the adsorbed He^3 can be treated as a two-dimensional Fermi gas of weakly interacting quasiparticles. Surface sound[4] is a compressional, adiabatic, longitudinal wave in this almost ideal, two-dimensional system. The measurements allow one to check the theory governing the surface-sound velocity u_s, and to derive some of the properties of the adsorbed He^3 at low number density. These are the quasiparticle effective mass M, the binding energy to the surface relative to the bulk, ϵ_0, and the effective interaction between the quasiparticles. The binding energy ϵ_0 has been calculated by a number of theorists[5-7] but no theoretical estimates for the effective mass of interaction have been published yet. The sign and magnitude of the interaction is of particular interest because of the possibility of observing two-dimensional superfluidity in adsorbed He^3.

The existence of surface sound was predicted by Andreev and Kompaneets.[4] In their theory, because of the existence of surface excitations—quantized capillary waves ("ripplons")[8] and adsorbed He^3 quasiparticles—the free surface of superfluid helium can transport mass, entropy, momentum, etc., and it obeys a set of hydrodynamic equations which are analogous to the two-fluid bulk equations. At low temperatures in pure He^4, or in very dilute solutions of He^3 in He^4, the influence of the normal fluid in the bulk becomes negligible compared to that at the surface, and there are then two forms of small oscillations of the surface: capillary waves and what Andreev and Kompaneets called "surface second sound" (which we abbreviate to "surface sound"). The surface sound has velocity u_s given by

$$\nu_n u_s^2=-(\partial\sigma/\partial\ln N_s)_s \tag{1}$$

where σ is the surface tension, $N_s=-(\partial\sigma/\partial\mu_3)_T$

is the number of adsorbed He^3 per unit area of the surface, $s = S/N_s$ with $S = -(\partial\sigma/\partial T)_{\mu_3}$ the surface entropy per unit area, and μ_3 is the He^3 chemical potential. Clearly $-(\partial\sigma/\partial \ln N_s)_s$ is the two-dimensional analog of an adiabatic bulk modulus, since s is the entropy per He^3 atom. The quantity ν_n is the mass per unit area of the normal fluid on the surface. It is the two-dimensional analog of the normal density ρ_n. In our experimental situation ν_n and S are mainly due to adsorbed He^3 but, in principle, there are small contributions from thermally excited ripplons which become negligible below ~100 mK.

Our original observation of surface sound[9] (unpublished except for this reference) was made on a helium surface with an unknown number of He^3 per unit area; the present experiments were made in conjunction with measurements of the surface tension σ from which, in principle, the surface number density N_s can be determined. The experiments were made on two 0.337-liter samples of liquid helium, the first of commercial He^4 containing about 0.13 ppm of He^3, the second formed from the first by adding 0.0575 ppm of He^3. The effective "free surface" of the samples was large—about 10^4 cm^2. This was due to some fine copper wire and some fibrous materials in the upper part of the experimental cell which, at low temperatures, were covered with saturated helium film. The concentration and area of the samples was such that, below ~90 mK, all of the He^3 was adsorbed on the surface (including that of the saturated film) and none was dissolved in the bulk of the liquid.

The experimental cell contained two "transducers," which are small, vertical squares of carbon resistor board, which could be raised or lowered with respect to the liquid surface by superconducting motors.[10] In addition, there were a number of other fixed transducers at different distances stationed so that they intersected the liquid level. Each transducer could be used in a time-of-flight measurement as the transmitter (by applying a brief heating pulse) or as the receiver. For the receiver, a feedback circuit maintained the transducer temperature constant at about 100 mK. Signals incident on the receiver were measured as a decrease in the power supplied by the circuit. To detect surface sound we averaged 10^3 to 10^5 signals to raise the signal-to-noise ratio. The cell also contained a parallel-plate capacitor to obtain σ from the capillary rise of the liquid between the plates. Further experimental details are given in Edwards *et al.*[10] and Gasparini *et al.*[11]

A typical surface-sound signal is shown in Fig. 1. In this case the transducers intersected the surface at a distance of 84 mm. If the input heat pulse is increased it is possible to observe[10] the transmission of phonons through the interior of the liquid and evaporated atoms through the vacuum above the liquid, as well as the surface sound. One can also observe surface sound generated by evaporated atoms striking the surface. For small input pulses, however, when there are no evaporated atoms, the surface sound disappears as soon as the transmitter is lifted out of or submerged below the surface. In addition, the signal strengthens as one increases the area of the transmitter or receiver exposed to the vacuum, indicating that the saturated film on the surface of the transducer plays a role in generating and detecting the surface sound. A plausible explanation for this will be discussed in a later paper. It was verified that the velocity of propagation u_s was independent of the size of the transmitter pulse, the path length, and the receiver power.

Figure 2 shows the measured values of u_s versus the temperature while Fig. 3 shows the surface tension $\sigma(T)$ for the same two samples of helium. The dashed curve in Fig. 3 represents σ for pure He^4, including the ripplon contribution given by Atkins.[8] We have found from measurements above 0.3 K that the ripplon theory is asymptotically correct for pure He^4 as one approaches low temperatures. The difference between the dashed curve and the data, $\Delta\sigma \equiv \sigma_4(T) - \sigma(T)$, can be thought of as the "spreading pressure" of the adsorbed He^3.

In Fig. 3, as the temperature is increased from 0 K, the number density N_s of He^3 on the surface remains constant at first and both u_s and the two-

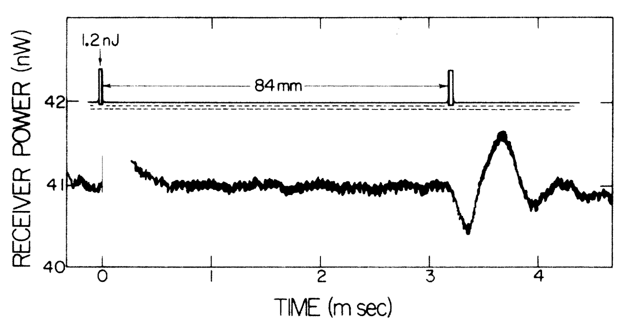

FIG. 1. Measurement of the velocity of surface sound at $T = 28$ mK. The surface-sound signal, which begins at $t \cong 3.2$ msec, was produced by a transmitter pulse of 1.2 nJ at $t = 0$.

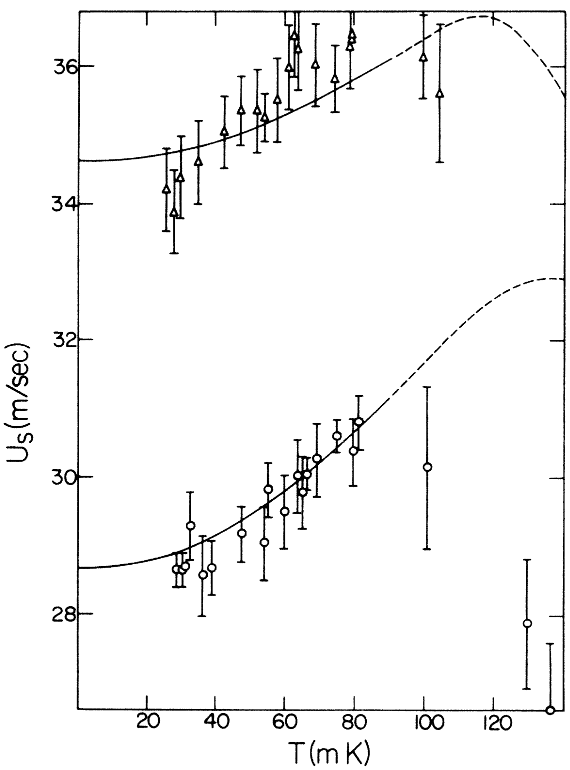

FIG. 2. Surface-sound velocity u_s versus temperature for the first sample (circles) and the second sample (triangles). The curves are theoretical (see text).

dimensional pressure $\Delta\sigma$ increase by an amount which varies approximately as T^2. This is typical of a degenerate Fermi system. At about 90 mK the adsorbed He^3 begins to dissolve in the interior of the liquid and $\Delta\sigma$ decreases quite rapidly. At the same temperature u_s begins to decrease below the theoretical curve, followed by the extinction of the surface sound, presumably because of damping by the He^3 below the surface.

Figures 2 and 3 show theoretical curves fitted to our u_s and σ data, as well as to some unpublished σ measurements by Crum,[12] and some of the earlier σ measurements of Guo *et al.*,[3] those with $N_s \leq 5\times10^{14}$ cm^{-2}. The curves were calculated by treating the adsorbed He^3 as an *ideal* Fermi gas with mass $M = 1.28m_3$ and binding energy $\epsilon_0/k_B = 2.28$ K. The surface normal mass ν_n in Eq. (1) was assumed to be N_sM plus the small ripplon contribution. In fitting the data, the effective area of the system and the He^3 concentration of the first sample were adjusted, as well as the values of ϵ_0 and M. The values of N_s at 0 K for the two samples were found to be 0.97×10^{14} cm^{-2} and 1.42×10^{14} cm^{-2}. The fit to the data is excellent and in our opinion it confirms the theory underlying Eq. (1) very effectively.[13]

To estimate the uncertainties in ϵ_0 and M we have also tried a more complicated version of the model by introducing a weak He^3-He^3 interaction V_0^s, as in Ref. 5. This is equivalent to a simple δ-function potential. The ranges in the possible values of ϵ_0, M, and V_0^s are $\epsilon_0/k_B = 2.28 \pm 0.03$ K, $M = (1.3\pm0.1)m_3$, and $V_0^s = (0.4\pm1.4)\times10^{-31}$ erg cm^2. Although the magnitude and sign of the interaction are not well determined, we note that a positive V_0^s corresponds to a repulsion, implying that surface He^3 will not become superfluid, at least at low densities. Since the

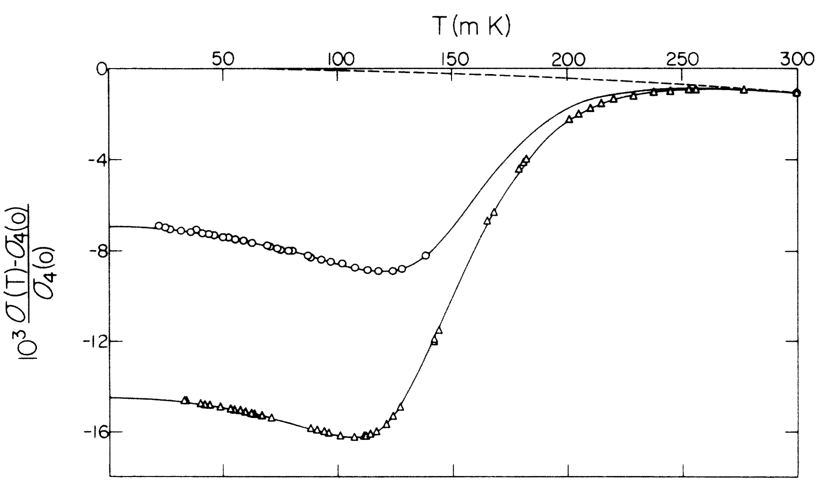

FIG. 3. The fractional difference between $\sigma_4(0)$, the surface tension of pure He^4 at 0 K, and the measured surface tension $\sigma(T)$ for the same two samples as in Fig. 2. The dashed line is the Atkins theory (Ref. 2) for pure He^4. The solid curves are theoretical, fitted to the data (see text).

present fit includes data at low densities from Guo *et al.*,[3] the new results for ϵ_0, M, and V_0^s supersede the less accurate values given in their paper. The experimental value of ϵ_0 is in rather good agreement with the latest theoretical value of Chang and Cohen,[7] $\epsilon_0/k_B = 2.35$ K.

†Work supported by the National Science Foundation under Grants No. GH-31560A#1 and No. GP-27242.

*Now at Logicon Inc., 255 West 5th St., San Pedro, Calif. 90731.

‡Now at Department of Physics, State University of New York at Buffalo, Buffalo, N. Y. 14214.

[1]A. F. Andreev, Zh. Eksp. Teor. Fiz. 50, 1415 (1966) [Sov. Phys. JETP 23, 939 (1966)].

[2]K. N. Zinov'eva and S. T. Boldarev, Zh. Eksp. Teor. Fiz. 56, 1089 (1969) [Sov. Phys. JETP 29, 585 (1969)].

[3]H. M. Guo, D. O. Edwards, R. E. Sarwinski, and J. T. Tough, Phys. Rev. Lett. 27, 1259 (1971).

[4]A. F. Andreev and D. A. Kompaneets, Zh. Eksp. Teor. Fiz. 61, 2459 (1971) [Sov. Phys. JETP 34, 1316 (1972)].

[5]W. F. Saam, Phys. Rev. 4, 1278 (1971).

[6]Y. M. Shih and C.-W. Woo, Phys. Rev. Lett. 30, 478 (1973).

[7]C. C. Chang and M. Cohen, Phys. Rev. A 8, 3131 (1973).

[8]K. R. Atkins, Can. J. Phys. 31, 1165 (1953).

[9]D. O. Edwards, in *Low Temperature Physics, LT-13*, edited by W. J. O'Sullivan, K. D. Timmerhaus, and E. F. Hammel (Plenum, New York, 1973).

[10]J. R. Eckardt, D. O. Edwards, F. M. Gasparini, and S. Y. Shen, in *Low Temperature Physics, LT-13*, edited by W. J. O'Sullivan, K. D. Timmerhaus, and E. F. Hammel (Plenum, New York, 1973).

[11]F. M. Gasparini, J. R. Eckardt, D. O. Edwards, and S. Y. Shen, J. Low Temp. Phys. 13, 437 (1973).

[12]D. Crum, Ph. D. dissertation, The Ohio State University, 1973 (unpublished).

[13]A. F. Andreev and D. A. Kompaneets, Pis'ma Zh. Eksp. Teor. Fiz. 17, 379 (1973) [JETP Lett. 17, 268 (1973)], have recently proposed a two-dimensional *liquid* phase in surface He^3. There is no sign of this in the present data; nor is there any evidence that we are observing a form of collisionless or zero sound.

Effects of Superconducting Fluctuations on the Ultrasonic Attenuation in a Thin Aluminum Film*

D. A. Robinson, K. Maki,† and M. Levy

Department of Physics, University of Wisconsin-Milwaukee, Milwaukee, Wisconsin 53201

(Received 5 December 1973)

The attenuation of 2-GHz acoustic surface waves in a 300-Å aluminum film has been measured in the temperature region around the superconducting transition temperature. The simple BCS theory yields a gap parameter of 3.60. While microwave electromagnetic-attenuation measurements in the film indicate critical fluctuation effects near T_c, there appear to be no such fluctuation effects in the ultrasonic-attenuation data of this short-mean-free-path Al film.

We report here observations on the attenuation of 2-GHz acoustic surface waves on lithium niobate by a thin, granular aluminum film as a function of temperature near the superconducting transition temperature T_c. This film was chosen to investigate the interaction of surface phonons with thermodynamic fluctuations near T_c since there have been several experimental and theoretical studies of their effect on the conductivity of such superconductors with a short mean free path.[1]

Previously it was pointed out by Aslamazov and Larkin (AL)[2] that the sound attenuation coefficient would have a sharp peak which diverges like $(T-T_c)^{-3/2}$ in a bulk system and as $(T-T_c)^{-2}$ in a two-dimensional system. However, their expression applies only in the limit $ql \gg 1$, where q is the sound wave vector and l is the electron mean free path. Furthermore the numerical factor of the fluctuation component of the ultrasonic attenuation coefficient, α_{AL}, is so small $\{$i.e., $\alpha_{AL}/\alpha_n \propto (\xi p_0)^{-2}[T_c/(T-T_c)]\sigma_{AL}/\sigma_n$ in the dirty limit, where p_0 is the Fermi momentum and $\xi = (\xi_0 l)^{1/2}$ is the coherence distance for a dirty superconductor$\}$ that an experimental observation of this anomaly appears impossible. The AL attenuation coefficient is concerned with diagrams (d) and (e) in Fig. 1. In these, fluctuations carry the superconducting current; each of the two vertices accounts for a factor $k_BT_c/E_F \rightarrow (\xi p_0)^{-1}$.