ly more complicated than for $l_n = 0$.

[13]This assumption is valid because q, which is greater than 10 fm for the cases considered here, is much larger than the range of V_{np}.

[14]J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962), p. 541.

[15]R. C. Johnson, Nucl. Phys. A90, 289 (1967).

[16]In the calculation the functions u_2 and u_0 were evaluated using the same approximations and parameters (see Ref. 6) which were used in the DWBA calculations. The values of q were calculated using Eq. (6) of Ref. 4. The kinetic energy was taken to be the average of the center-of-mass energies in the proton and deuteron channels.

Asymptotic Limit of $\sigma(e^+ + e^- \to \text{hadrons})/\sigma(e^+ + e^- \to \mu^+ + \mu^-)$†

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Combining the low-energy theorem by Crewther and by Chanowitz and Ellis on the anomalous vertex function for partially conserved dilation current and a new result from the most conventional vector-meson dominance, we predict the asymptotic limit of the ratio $\sigma(e^+ + e^- \to \text{hadrons})/\sigma(e^+ + e^- \to \mu^+ + \mu^-)$ to be $R \cong 16\pi^2/f_0{}^2 = 5.7 \pm 0.9$, which is in remarkable agreement with the preliminary data from the Cambridge Electron Accelerator e^+-e^- colliding-beam experiment but in strong contradiction with the other predictions $R = \frac{2}{3}$ and $R = 2$ in the single- and three-triplet fractionally charged quark models.

One of the most intriguing experimental discoveries recently reported is the surprisingly large total cross section for hadron production which has been observed in the e^+-e^- colliding-beam experiment at Cambridge Electron Accelerator (CEA).[1] The data have been analyzed in such a way that the ratio $R(q^2)$ of the observed total cross section for $e^+ + e^- \to$ hadrons to the theoretical one ($\simeq 4\pi\alpha^2/3q^2$) for $e^+ + e^- \to \mu^+ + \mu^-$ is calculated to be 4.7 ± 1.1 at $q^2 = 16$ GeV^2 and to be about 6.3 at $q^2 = 25$ GeV^2, where q^2 is the square of the total c.m. energy of the colliding beams. A naive comparison of the CEA data with the Frascati data[2] leads to the conclusion that $R(q^2)$ continues to rise as q^2 increases from above 1 up to 25 GeV^2. On the other hand, many authors[3] expect that $R(q^2)$ approaches a constant when q^2 goes up. It is now extremely interesting to ask what the asymptotic value for $R(q^2)$ is. The purpose of this short note is to report our new prediction for $R = \lim_{q^2 \to \infty} R(q^2)$, if it exists, based on the anomaly in partially conserved dilation current (PCDC) and the usual vector-meson dominance.

Let us start with the definition of the photon form factor for the trace of the stress-energy tensor,

$$\langle \gamma(\epsilon_1, k_1) | \theta_\lambda{}^\lambda(0) | \gamma(\epsilon_2, k_2)\rangle = -(\epsilon_1 \cdot \epsilon_2 k_1 \cdot k_2 - \epsilon_1 \cdot k_2 \epsilon_2 \cdot k_1) F((k_1 - k_2)^2). \tag{1}$$

Recently, Crewther[4] and, independently, Chanowitz and Ellis[5] have pointed out, on the canonical anomaly existing in the PCDC,[6] that the coefficient of the anomalous term is completely determined by R. Furthermore, they have found that, because of the anomaly, $F(0)$ does not vanish and is given by

$$F(0) = -(e^2/12)\int dx\, dy\, x \cdot y \langle 0 | T(J_\mu(x) J^\mu(0) \theta_\lambda{}^\lambda(y)) | 0\rangle = (e^2/6\pi^2)R. \tag{2}$$

If we assume vector-meson dominance, the left-hand side of Eq. (1) can be approximated by

$$(e^2/f_\rho{}^2)\langle \rho(\epsilon_1, k_1) | \theta_\lambda{}^\lambda(0) | \rho(\epsilon_2, k_2)\rangle + (\text{isoscalar terms}), \tag{3}$$

where f_ρ is the γ-ρ coupling constant ($f_\rho{}^2/4\pi = 2.2 \pm 0.3$). Now, the point is that we can evaluate the matrix element in (3) by the definition of the stress-energy tensor when $k_1 = k_2$, namely,

$$\langle \rho(\epsilon_1, k) | \theta_{00}(0) | \rho(\epsilon_2, k)\rangle = -2k_0 k_0 (\epsilon_1 \cdot \epsilon_2 - \epsilon_1 \cdot k \epsilon_2 \cdot k / k^2). \tag{4}$$

Thus we obtain the following approximate result:

$$F(0) \cong 2e^2/f_\rho^2 + (\text{isoscalar terms}). \quad (5)$$

If the electromagnetic current of hadrons $J_\mu(x)$ is a pure SU(3) octet operator,[7] the isoscalar contribution is roughly $\frac{1}{3}$ the isovector (ρ) contribution. Therefore,

$$F(0) \cong \tfrac{4}{3}(2e^2/f_\rho^2). \quad (6)$$

Comparing these two results, (2) and (6), we finally predict the relation

$$R \cong 16\pi^2/f_\rho^2 = 5.7 \pm 0.9. \quad (7)$$

It is remarkable that this prediction is perfectly consistent with the CEA preliminary data for $q^2 = 25\ \text{GeV}^2$ as well as the rough estimate ($R \sim 6.5$)[8] from the first evidence found at Frascati[9] for hadron production by the two-photon process,[10] but incompatible with the other predictions in the single- and three-triplet fractionally charged quark models. Notice that this prediction is independent of and different from those made by Bramon, Etim, and Greco[11] and by Sakurai,[12] based on an infinite series of vector-meson resonances or on the "new duality", since we have assumed the most conventional form of vector-meson dominance for *real photons*. It can not be stressed too strongly that *our application of the vector-meson dominance to the "softer" operator* $T(J_\mu(x)J^\mu(0) \times \theta_\lambda{}^\lambda(y))$ *for real photons is perfectly consistent with the PCDC anomaly and the nonvanishing constant* R.[13] We have never applied vector-meson dominance directly to the "hard" operator $T(J_\mu(x)J_\nu(0))$. If one were to do this, one would reach the contradictory result $R=0$ unless one adopts the "new duality".[11,12] Notice also that the relation needs two corrections: One is due to the extrapolation of the matrix element from $k^2 = m_\rho^2$ to $k^2=0$, which has been estimated to be a few percent, and the other is due to vector states with negative charge conjugation other than ρ, ω, and φ, such as ρ'. The latter correction is supposed to be less than 10%. More detailed discussions on our result will be published elsewhere.[8]

Future e^+-e^- colliding-beam experiments at Stanford Linear Accelerator Center and DESY will decide whether our prediction based on the PCDC anomaly and vector-meson dominance agrees with the data.

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[7]If not, $F(0) \geq \frac{4}{3}(2e^2/f_\rho^2)$, which results in $R \geq 16\pi^2/f_\rho^2$. Therefore, this possibility yields an even stronger result against the other predictions $R=\frac{2}{3}$ and $R=2$.

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[13]It is worth mentioning here that the old application of vector-meson dominance to the decay of π^0 into 2γ is still consistent with the present data, independent of the PCAC anomaly. See M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Lett. 8, 261 (1962). By "softer" we mean that the dimension of $\theta_\lambda{}^\lambda$ is less than four. It may be more relevant here to state that, according to Weinberg's theorem [S. Weinberg, Phys. Rev. 118, 838 (1960)], the Fourier transform of $T(J_\mu(x)J^\mu(0)\theta_\lambda{}^\lambda(y))$ increases more slowly (or decreases more rapidly) than that of $T(J_\mu(x)J_\nu(0))$ as k^2, the square of the photon mass, goes up.