Shape Coexistence and Its Cause in ^{151}Gd†

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The $(\alpha, 3n\gamma)$ reaction has been used to study the level structure of ^{151}Gd. Energy spacings and other information indicate a small deformation for the observed $i_{13/2}$ decoupled band; whereas, a large prolate deformation is indicated by the normal rotational band associated with the $h_{11/2}$ hole excitation. A simple explanation for these two shapes in ^{151}Gd is given, based on the Nilsson diagram, and the behavior is shown to be related to the well-known shape change occurring between 88 and 90 neutrons.

In order to obtain information about the moment of inertia of a nucleus from energy-level spacings, the structure of the levels must be clear and reasonably simple. It has been pointed out that under certain conditions a system consisting of a single particle (or hole) with pure j coupled to an axially symmetric rotor gives rise to a particularly simple spectrum.[1-3] Specifically, for prolate deformations of intermediate size, a single-particle-plus-rotor system develops a band with spin sequence $j, j+2, j+4, \ldots$ and energy spacings equal to those in the ground-state band of the even-even core; whereas a single hole coupled to such a rotor gives rise to a normal rotational band built on the state with $I=j$, having members with $\Delta I=1$ and energy spacings approaching $I(I+1)$ at moderately large deformations. An odd-A nucleus with the Fermi surface below (or above) the entire group of Nilsson orbitals that originate in a high-j unique-parity spherical state very nearly constitutes such a particle- (or hole-) plus-rotor system, and thus the energy spacings of the lowest-lying band associated with the particular high-j shell can provide rather unambiguous information on the moment of inertia of that odd-A nucleus.

In ^{151}Gd, which has five neutrons beyond the $N=82$ closed shell, the Fermi surface lies well outside both the $i_{13/2}$ and $h_{11/2}$ unique-parity shells, which makes the interpretation of the bands associated with these excitations particularly unambiguous. From a study of these bands we determined the moments of inertia of ^{151}Gd in these two configurations, and the results suggest a marked difference in deformation between the two bands.

The experiments consisted of in-beam γ-ray measurements following the reaction ^{150}Sm$(\alpha, 3n)$. An enriched metallic ^{150}Sm target (≈ 10 mg/cm^2 thick) was bombarded with 35–45-MeV α particles provided by the Berkeley 88-in. cyclotron. The γ-ray measurements included excitation function studies and two-point angular distributions, as well as $\gamma\gamma$ coincidence measurements. The angular anisotropies and coincidence data identify a strongly populated cascade of stretched $E2$ transitions based on a level at 851 keV, establishing a band with states having $\Delta I=2$. About 25% of the γ decay proceeds through this band. A second, weakly populated band (5% of the γ decay) based on a level at 1210 keV was also identified. This latter band has very recently been seen in an independent $(\alpha, xn\gamma)$ experiment.[4] Observation of $E2$ crossover as well as $M1+E2$ cascade transitions clearly establishes the $\Delta I=1$ sequence for these band members, and their approximate $I(I+1)$ energy spacings suggest a normal rotational character. The remaining de-excitation intensity proceeds through a series of negative-parity levels probably associated with the $f_{7/2}$ and $h_{9/2}$ shells. A partial level scheme for ^{151}Gd showing the bands and their prominent modes of decay is given in Fig. 1.

The ground-state spin of ^{151}Gd has recently been measured[5] to be $\frac{7}{2}$, and the band-head spins assigned in Fig. 1 are in accord with the γ-ray data. They are independently supported by the results from an earlier ^{152}Gd$(d,t)^{151}$Gd experiment.[6] In that study high-l transfer cross sections were observed with 7 and 29 μb/sr (at 90°) going to levels at 847 and 1204 keV, respectively, which we attribute to the two band heads estab-

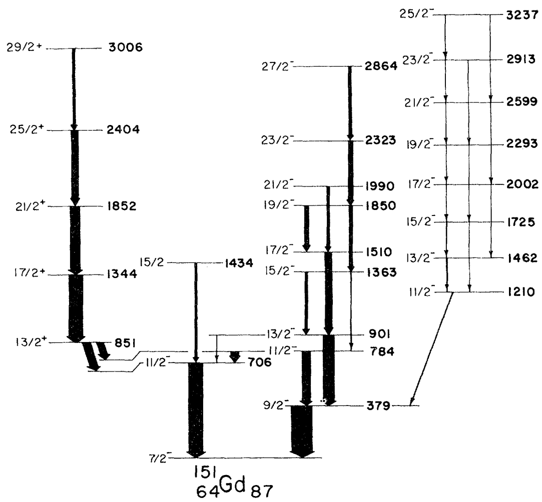

FIG. 1. The level scheme of ^{151}Gd.

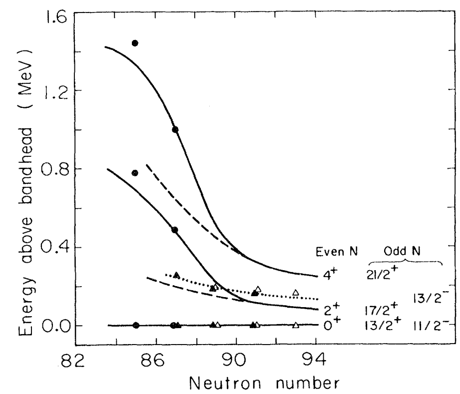

FIG. 2. Energy systematics of selected rotational states in the $N=88$ region. Solid and dashed lines, energy separations in the ground and excited 0^+ bands in the even-even Gd and Sm nuclei (see Ref. 8). Circles, analogous separations in the $i_{13/2}$ decoupled bands in ^{149}Gd and ^{151}Gd. This band in ^{149}Gd is reported in Ref. 8, and has members I^π (E in keV): $\frac{13}{2}^+$ (955); $\frac{17}{2}^+$ (1739); $\frac{21}{2}^+$ (2400); and $\frac{25}{2}^+$ (3292). Triangles and dotted line, $\frac{13}{2}$ to $\frac{11}{2}$ separation in the $\frac{11}{2}^-$ |505| bands in odd-A Gd and Dy nuclei.

lished in the present experiment. The large cross section at 1204-keV excitation strongly suggests that it is associated with the $h_{11/2}$ hole state. The much smaller transfer cross section to the 847-keV level is compatible with an $i_{13/2}$ particle character for that excitation. These cross sections are in agreement with the predictions of the particle-rotor model as discussed below.

The band based on the level at 851 keV is analogous to similar decoupled bands recently identified in the odd-A lanthanum nuclei,[7] in that the level spacings very closely resemble those of the neighboring even isotopes (Fig. 2). It was already indicated in the heavier lanthanum isotopes that the decoupled-band energy spacings follow those of the even-even ground band even if the latter deviate significantly from the $I(I+1)$ spacings of a perfect rotor. This is also demonstrated by the $\frac{13}{2}^+$ band in ^{151}Gd, and by a similar decoupled $\frac{13}{2}^+$ band in ^{149}Gd identified in a parallel experiment (cf. Fig. 2 and Kleinheinz *et al.*[8]). Using the very general empirical relationship[9] between the 2^+ energies and the $E2$ transition lifetimes, which essentially all even-even nuclei follow, we can estimate[1] the deformation β as

$$\beta \approx \left(\frac{1225}{6(\hbar^2/2\mathscr{G})A^{7/3}}\right)^{1/2}, \qquad (1)$$

where $\hbar^2/2\mathscr{G}$ is in MeV. For the $\frac{13}{2}^+$ decoupled band in ^{151}Gd we have $E_{17/2\to 13/2} = 6\hbar^2/2\mathscr{G} = 0.493$ MeV, which gives $\beta_{13/2} = +0.14$.

The negative-parity band based on the 1210-keV $\frac{11}{2}^-$ hole state is analogous to similar bands observed in several odd-N rare-earth nuclei around $N=91$. The level energies for this band, based on a single hole in the high-j shell, are predicted to approach $\hbar^2/2\mathscr{G}$ times $I(I+1)$, as mentioned above. The absence in the observed energies of an oscillating term suggests that the $\frac{11}{2}^-$ band in ^{151}Gd is rather close to that limit. From the first level spacing $E_{13/2\to 11/2} = 13\hbar^2/2\mathscr{G} = 0.252$ MeV we calculate, with Eq. (1), $\beta_{11/2} = +0.29$. This value would be lowered by 5–10% if the Coriolis mixing were taken into account.

From the remaining negative-parity states no definite conclusions on the moment of inertia can be drawn at present. These levels also exhibit a bandlike pattern, but the occurrence of a very fast $\frac{11}{2}^- \to \frac{11}{2}^-$ transition clearly indicates that the $h_{9/2}$ and $f_{7/2}$ shells both contribute heavily to the level wave functions, which greatly complicates the analysis. Such strongly mixed wave functions are indeed expected; Nilsson-model calculations give rather complete mixing of these two j values at deformations as low as $\beta \approx 0.1$.

The occurrence of the two different deformations derived from the $\frac{11}{2}^-$ and $\frac{13}{2}^+$ bands is a very unusual result. Our conclusions so far are based on the moment-of-inertia parameters extracted from rotational-energy spacings, which we expect to be especially reliable in these particular bands, as discussed above. Independent support for these deformations comes from the single-neutron pickup cross sections[6] for the two band heads, and from energy systematics in this region.

From the data in Ref. 6 we deduce the spectroscopic strengths: $S_{11/2} = 0.18 \pm 0.06$ $(2j+1)$ or $C_{11/2}{}^{\mathrm{eff}} = 1.1 \pm 0.3$; and $S_{13/2} \geq 0.61$ $(j+1)$ or $C_{13/2}{}^{\mathrm{eff}} \geq 4.3$. For this estimate we have used $\Delta(^{152}\mathrm{Gd}) = 1.08$ MeV and assumed a 30% uncertainty in the experimental cross sections. As the deformation of ^{152}Gd is between those of the $\frac{11}{2}^-$ and $\frac{13}{2}^+$ configurations in ^{151}Gd we have assumed a unity overlap integral between the target and final nuclear wave functions. In the particle-rotor model these values give the deformations $\beta_{11/2} \geq 0.23$ and $-0.10 < \beta_{13/2} < +0.15$, in agreement with the results obtained from the energy spacings in the bands.

A comparison with the energy systematics of particular 0^+ bands in the even-even nuclei, which is shown in Fig. 2, gives further support for our interpretation. First, it can be seen that the $\frac{13}{2}^+$ decoupled-band energy spacings in ^{151,149}Gd ($N = 87, 85$) follow rather closely those of the even-even ground bands. Second, Fig. 2 also shows the energy spacings of bands built on excited deformed 0^+ states. Such deformed states have been identified[10,11] in the $N = 86$ and 88 isotopes of Gd and Sm by two-neutron pickup studies, and in some cases the associated 2^+ and 4^+ rotational states are also known. The correlation of the $2^+ \to 0^+$ separation in this band with the $\frac{13}{2}^- \to \frac{11}{2}^-$ separation in the $\frac{11}{2}$ |505| band seems likely to be significant. We note, however, that the moments of inertia for the $\frac{11}{2}$ |505| bands are distinctly larger than those of the deformed 0^+ bands, particularly for the lower-mass isotopes.

The very different deformations in ^{151}Gd resulting from a single neutron in the $i_{13/2}$ shell or a single hole in the $h_{11/2}$ shell may be qualitatively understood in terms of the Nilsson diagram. Figure 3 shows the portion of the diagram around the $N = 82$ closed shell. Occupation of the $\frac{11}{2}$ |505| orbital with its strong upward slope will favor a spherical nuclear equilibrium shape, whereas the orbitals immediately above $N = 82$ generally slope downwards and thus favor deformation. In its ground state ^{151}Gd has two pairs and the odd particle occupying the down-sloping orbitals above $N = 82$. Promotion of the odd neutron from the ground-state orbital to the lowest $i_{13/2}$ orbital represents a promotion from a moderately down-sloping orbital to a similar down-sloping orbital. Such a change in occupation should leave the nuclear deformation relatively unaffected. In the $\frac{11}{2}^-$ configuration, however, a neutron has been lifted above the $N = 82$ gap. The nucleus now has three pairs in the down-sloping orbitals and the $\frac{11}{2}$ |505| orbital has become 50% unoccupied. Both consequences of this rearrangement produce a very strong tendency towards deformation.

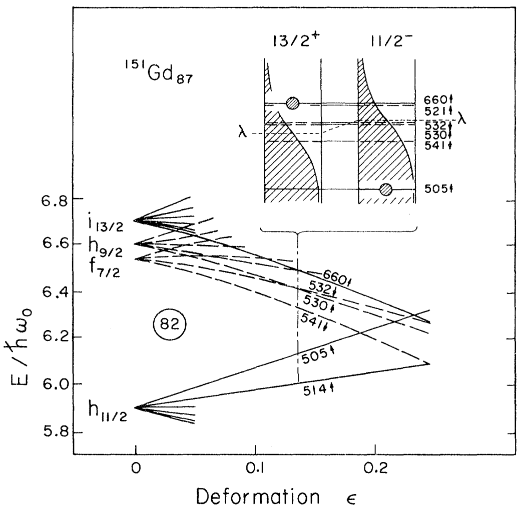

FIG. 3. Nilsson diagram for the $N = 82$ region. Insets, intrinsic configurations of the $\frac{13}{2}^+$ and $\frac{11}{2}^-$ band heads at $\beta \approx 0.14$, where the Fermi levels λ are estimated to be those calculated for $N = 86$ and $N = 88$, respectively.

The considerable difference in ground-state deformation of even-even nuclei observed as one goes from 88 to 90 neutrons was first theoretically interpreted by Mottelson and Nilsson[12] as a consequence of the similar transfer of a pair of neutrons from the $\frac{11}{2}$ |505| orbital to the down-sloping orbitals above the 82 shell. We would assume that the excited deformed 0^+ states, in the $N < 90$ nuclei, similarly involve, to a considerable extent, the removal of a neutron pair from the $\frac{11}{2}$ |505| Nilsson orbital. However, this mechanism is complicated in the even-even nuclei by the pairing correlations which smear out the occupation probabilities. The uncertainty is reduced in an odd-A nucleus, where the presence of an odd particle unambiguously determines a 50% occupation probability. Thus, whereas it is difficult to specify the configuration of the deformed 0^+ states in these even-even nuclei, the connection of a specific orbital to the deformed shapes is clear in an odd-A nucleus like ^{151}Gd.

This more detailed picture of the origin of shape coexistence in an odd-A nucleus might provide the means to identify similar phenomena in other regions of the nuclear chart. Bands similar to those in ^{151}Gd should generally appear whenever the nuclear potential is soft towards β deformation and the Fermi surface lies between two spherical j shells, and recently identified[13,14] bands in ^{45}Sc and ^{75}Se might be interpreted in this way.

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Finite-Range Full-Recoil Calculation for Two-Nucleon Transfer between Heavy Ions

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A distorted-wave Born-approximation (DWBA) calculation has been done for the reaction ^{62}Ni(^{18}O, ^{16}O)^{64}Ni, in which full account is taken of finite-range and recoil effects. It is found that finite-range no-recoil calculations for this reaction overestimate the DWBA cross-section by about a factor of 5. The no-recoil result concerning the strong dependence of the cross section on the single-particle states involved is confirmed by this full-recoil calculation.

Several calculations[1-4] have been published in which the distorted-wave Born approximation (DWBA) has been applied to the transfer of two nucleons in a heavy-ion reaction. These calculations all involve the "no-recoil approximation."[5] Let $\vec{r}_i$ locate the c.m. of the incident projectile relative to the c.m. of the target nucleus. This vector is the argument of the incident-channel optical wave function. Similarly, let $\vec{r}_f$ locate the c.m. of the outgoing projectile relative to the c.m. of the residual nucleus, so that $\vec{r}_f$ is the argument of the outgoing-channel optical wave function. The no-recoil approximation is used in the form

$$\vec{r}_i = \alpha \vec{r}_f, \tag{1}$$

where α is kept constant throughout the calculation. Sometimes α is set equal to the ratio of the target and residual masses; sometimes it is varied about this value by a few percent. In all cases, the directions of $\vec{r}_i$ and $\vec{r}_f$ are assumed to be the same. These approximations become exact in the limit in which the mass of the transferred nucleons is negligible compared to the masses of the cores between which they transfer. However, for realistic nuclei, in the energy ranges in which heavy-ion experiments are often done, the no-recoil approximation can lead to serious errors. For example, in the reaction ^{62}Ni(^{18}O, ^{16}O)^{64}Ni, with 65-MeV incident ^{18}O, important regions of configuration space have r_i and r_f differing by as much as 0.5 fm, in either direction. Since the wavelengths of the optical wave functions are about 1 fm, the strict proportionality implied by (1) cannot be trusted to yield accurate values of the optical wave functions. Moreover, for the high partial waves ($l \sim 40$) important in this reaction, the slight differences between the directions