Phys. A133, 481 (1969).
[5]M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1965), p. 297, Eq. (348).
[6]N. Auerbach, J. Hüfner, A. K. Kerman, and C. M. Shakin, Rev. Mod. Phys. 44, 48 (1972).
[7]F. Tabakin, in *The Two-Body Force in Nuclei*, edited by S. M. Austin and G. M. Crawley (Plenum, New York, 1972), p. 101, and references there.

Asymmetry in Peripheral Production Processes

Richard M. Weiner
*Deutsches Elektronen-Synchrotron, Hamburg, Germany, and Fachbereich Physik, Philipps-Universität, 355 Marburg, Germany**
(Received 11 December 1973)

As a consequence of local excitation, an asymmetry with respect to the leading particle in peripheral production processes is predicted. It is interpreted in terms of heat propagation in hadronic matter.

Consider a peripheral collision of two hadrons accompanied by particle production. The peripheral nature of this collision manifests itself in the fact that there exists a leading particle which retains a high proportion of the incoming momentum and which is scattered through a small angle. Classically such a process can be viewed as a grazing collision in which the projectile (leading particle) touches only the surface of the target, which for reasons of simplicity is supposed to be spherical (Fig. 1).

Quantum mechanically the localization occurs if $q \gg R^{-1} \sim m_\pi$, where q is the exchanged momentum and R the radius of the target; peripheralism is assured by the condition $q \ll p_i$, where p_i is the incoming momentum. In the following we shall assume that these two conditions are fulfilled. Radiation (particle emission), however, will take place not from the hot spot but from a region of dimensions comparable with the range of strong interaction forces $\sim R$, because only in this case does the number of particles become defined.[1] Hence it follows that there will be a time delay, $\tau_0 \sim R/c$, before particle emission starts (this follows also from the indeterminacy principle since at a given energy transfer q_0 it takes a certain time $\tau_0 > q_0^{-1}$ after which this energy can be measured), since the excitation concentrated initially in a small region has to propagate with *finite* (sound) velocity c until it extends over a region $\sim R$. In the half-space below the tangent plane at N in Fig. 1, there is the body of the target, characterized by a thermal heat conductivity K and a radiation constant λ, and along the line NS (Fig. 1) a temperature gradient develops so that at the moment τ_0 when radiation starts, the temperature at the south pole S is lower than that at the north pole N.

In the half-space above NM there is "vacuum" and no such gradient of temperature is expected to occur, i.e., no diffusion should take place. This leads us to expect a yet unobserved asymmetry in peripheral collisions for particles which are produced in the target, i.e., for target "fragments." (The same is obviously true for projectile fragments.) The asymmetry has two experimental consequences: (i) Particles produced in the hemisphere $\vec{q}_\perp$ will have, on the average, smaller momenta than those in the hemisphere $-\vec{q}_\perp$. (ii) Heavier particles (e.g., kaons or antinucleons) will preferably be produced in the hemisphere $-\vec{q}_\perp$. This should be especially true if, because of quantum-number conservation, these particles can be produced only in pairs.

In general the smaller the angle φ at which particles are emitted, the smaller the corresponding momentum and mass.[2] The size of this

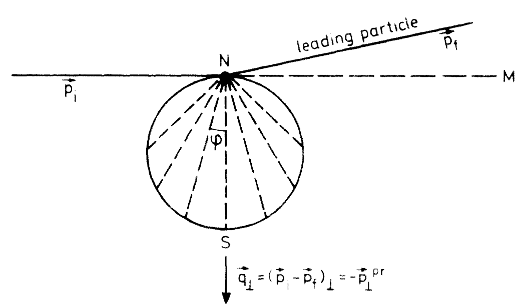

FIG. 1. Local excitation of a spherical target in a peripheral collision.

asymmetry depends essentially on $K\tau_0$. It will be shown below by solving the corresponding diffusion equation that for "reasonable" values of this parameter the asymmetry could be appreciable. On the other hand the absence of such an asymmetry in experimental data would provide a limit on K. In both cases this would be important new information, especially for the thermodynamical[3] and hydrodynamical[4] models of strong interactions. It is challenging to realize that the data necessary to investigate this problem exist but have not been analyzed along these lines. We hope that the present paper will stimulate such an effort.

In the following we shall study quantitatively the asymmetry within the frame of a simple model and then try to outline how this asymmetry could be detected.

The diffusion equation and its solution.—We assume that propagation of temperature T in hadronic matter is described by the relativistic diffusion equation[5]

$$c^{-2}(\partial^2 T/\partial\tau^2)+K^{-1}(\partial T/\partial\tau)=\Delta T. \tag{1}$$

Here τ is the time variable. We want to solve Eq. (1) with the following initial and boundary conditions. For initial conditions, i.e., at $\tau=\tau_i=0$, we take

$$T=T_i\delta(\vec{r}-\vec{a}). \tag{2}$$

T_i is the initial temperature which develops at the collision center. It is related to the energy loss of the projectile. We assume for the sake of simplicity that T_i is concentrated at the north pole (Fig. 1) of spherical coordinates $|\vec{a}|=R$, 0, 0 in a coordinate system with the origin O in the center of the target and the Z axis along ON. For boundary conditions, we take

$$\partial T/\partial r=-\lambda T \text{ at } r=R;\quad \lim_{r\to 0} T\neq\infty. \tag{3}$$

The radiation constant λ determines the heat flow from our medium (the target) into vacuum, which is supposed to be at zero temperature. These boundary conditions define our model of particle production: Emission of particles takes place through "radiation" from the surface of the target, the temperature of which is determined by Eq. (1) (cf. also below).

It is remarkable that Eq. (1) with conditions (2) and (3) can be solved exactly.[6] Here we shall be interested only in the angular dependence of the solution T at $r=R$, i.e., in $T=T(R,\theta;\tau_0)$ (the problem has obviously azimuthal symmetry). τ_0 is the characteristic time delay introduced above. One finds

$$T(R,\theta;\tau_0)=\frac{T_i}{2\pi}\sum_{n=0}^{\infty}(2n+1)P_n(\cos\theta)\times\sum_{\alpha_n}\frac{(R\alpha_n)^2\exp(-A\tau_0)}{(R\lambda-\frac{1}{2})^2+(R\alpha_n)^2-(n+\frac{1}{2})^2}, \tag{4}$$

where

$$A=\tfrac{1}{2}[c^2/K\pm(c^4/K^2-4\alpha_n^2c^2)^{1/2}], \tag{5}$$

P_n are Legendre polynomials, and α_n are the roots of the transcendental equation

$$(R\lambda-\tfrac{1}{2})J_{n+1/2}(R\alpha_n)+R\alpha_n J_{n+1/2}{}'R(\alpha_n)=0, \tag{6}$$

where J is the Bessel function of the first kind and n is an integer.

The asymmetry can be defined as follows:

$$\eta=\{[T(\theta=0)-T(\theta=\pi)]/T(\theta=\pi)\}|_{R,\tau_0}. \tag{7}$$

In order to estimate η from the relations given above, we have to make some assumptions about the constants of the problem. We shall consider the physically interesting case[3,4] $c/K\gg 2\alpha_n$, $K\neq 0$; as a matter of fact, this coincides with the nonrelativistic approximation to the diffusion equation. η depends on two constants $A\tau_0\simeq K\alpha_n^2\tau_0$ and $R\lambda$ [α_n is determined from Eq. (6) as a function of λ]. Dimensionally in a unit system $c=1$, all constants can be expressed in terms of a length L. If the phenomenological description for the asymmetry suggested above makes sense, one would expect L to be of the order of the range of forces of strong interactions, i.e., the radius of the target (projectile): $L\simeq R$. That is what we called before the "reasonable" values of the parameters. In the table, which shows the asymmetry coefficient η as a function of $K\tau_0$ for $\lambda=1$ in units $R=1$, we give some numerical results.

$K\tau_0$	η (%)
0.5	83
1	5
1.5	0.2
2	0.02

It is seen that in the range $R\lambda\simeq 1$, η is a very sensitive function of $K\tau_0$: The smaller $K\tau_0$, the larger η. For $K\tau_0<1$ the asymmetry is significant (for $K\tau_0=0.5$ it exceeds 80%), while for $K\tau_0>1$ it becomes negligibly small. This is intuitively understandable since large values of K as well as large values of τ_0 correspond to a thermodynamical equilibrium situation when emission of particles should be isotropic.

Experimental implications.—In order to relate temperature to measurable quantities like momenta, multiplicities, etc., one could use the conventional phenomenological prescription in which the momentum distribution density n of produced particles is assumed to be of Bose-Einstein or Fermi-Dirac form:

$$n = \{\exp[(\mu^2 + \vec{p}^2)^{1/2}/T] \pm 1\}^{-1}, \tag{8}$$

where μ is the mass of the particle. The asymmetry along $q_\perp$ would manifest itself in (i) an asymmetry of $\langle p\rangle$ according to Eqs. (7) and (8) [in a first approximation for the target (projectile) fragments $\langle p\rangle \sim \langle p_\perp\rangle \sim T$ and hence what was said about the temperature applies directly to $\langle p\rangle$], and (ii) an asymmetry of, e.g., the K/π ratio. In order to estimate the latter, an assumption about the equation of state has to be made. A rough (and conventional) guess is that $E = (\mu^2 + \vec{p}^2)^{1/2} \sim T^4$ and hence the increase of the K/π ratio with T follows immediately.

The asymmetry discussed above is defined in terms of the leading particle and it is obvious that it has no meaning but in this context. Furthermore an essential element is the identification of the particles emitted from the target (projectile) and their separation from the particles emitted from the projectile (target). It is evident that the "up-down" asymmetry refers only to those particles which are emitted from the target (projectile), i.e., those particles which are slow in the respective frame of reference.

The question arises as to what are the optimal conditions for the search for this effect. Provided the cross sections are large enough, it would seem that the higher the multiplicities the better. One should also restrict oneself to the condition $m_\pi \ll q \ll p_l$.[7] Finally, one would expect that this effect should be enhanced if nuclei were used as targets, since the finite dimensions play here a more important part.[8]

A byproduct of the local excitation mechanism suggested above is a possible simple explanation of the recent observation[9] of the increase of the average multiplicity of produced particles $\bar{n}$ with the transverse momentum of the projectile $p_\perp^{\mathrm{pr}}$ in p-p collisions at 30 GeV/c. Indeed the temperature T_i which develops at the collision center in a peripheral reaction is an increasing function of the momentum transfer $q_\perp = -p_\perp^{\mathrm{pr}}$. On the other hand $\bar{n}$ is an increasing function of the temperature of the target and this might explain the above-mentioned experimental finding.

The discussion given above was based on the assumption that hadronic matter can be characterized by a *constant* thermal conductivity K. This is of course an idealization and there are some theoretical indications[4] that K should increase with T. In this connection it is interesting to mention the possibility that hadronic matter might have superfluid properties.[10] In this case, propagation of heat would be accompanied by propagation of second sound and the heat wave might present at low temperatures a stationary (oscillatory) character as happens in He II. The diffusion (and radiation) of heat (propagation of first sound) would then start only after a certain critical temperature is reached. This would possibly also explain why the increase of $\bar{n}$ with $p_\perp^{\mathrm{pr}}$ becomes significant at large $p_\perp^{\mathrm{pr}}$. On the other hand, this might also suggest that the increase of multiplicity is connected with an "increase" of localization.

Although the use of the diffusion equation in particle physics might be debatable and more conventional approaches might be possible, we feel that the arguments put forward in order to predict the *existence* of the asymmetry are sufficiently convincing to make the search for this effect a worthwhile enterprise.[6]

This investigation was performed during the author's stay at Imperial College, London, and had the support of the British Science Research Council. I am indebted to Professor E. Feinberg, Professor R. Hagedorn, Professor T. Kibble, Professor Ch. Kuper, Professor E. Leader, and Professor D. Lichtenberg for stimulating discussions and correspondence, and to Dr. D. Schildknecht and Dr. T. Walsh for instructive comments on the manuscript.

*Permanent address.

[1] I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR 78, 889 (1951).

[2] One might perhaps also expect a difference in the multiplicities of particles in the upper and lower half-spaces because one might argue that according to Fig. 1 in the lower half-space multiple scattering processes occur which lead to the production of more and slower particles. No such multiple scattering should occur in the upper half-space where few and fast particles should be observed.

[3] R. Hagedorn, CERN Report No. 71-12, 1971 (unpublished). In this model it is assumed that "local" thermal equilibrium is established instantaneously and no explicit discussion of the role of heat conductivity is made. However, in the "local" region where this happens, K can be considered extremely large ($\sim\infty$). On the other hand, the fact that this equilibrium is only

"local" means that there is no heat conduction between different local regions, i.e., on a larger scale $K=0$. The importance for the thermodynamical model of the finite rate at which thermodynamical equilibrium is reached was stressed in a different context by K. Imaeda [Lett. Nuovo Cimento 1, 290 (1971)].

[4]L. Landau, Izv. Akad. Nauk SSSR, Ser. Fiz. 17, 51 (1953); cf. also P. M. Carruthers and Minh D.-V., Phys. Lett. 42B, 597 (1972), and 44B, 507 (1973); F. Cooper and B. Schönberg, Phys. Rev. Lett. 30, 880 (1973). In this model it is explicitly assumed that $K=0$ (ideal fluid approximation). The importance of a more realistic treatment ($K\neq 0$) has been recognized. A review along these lines can be found in E. L. Feinberg, *Quantum Field Theory and Hydrodynamics, Lebedev Physics Institute,* edited by D. V. Skobel'tsyn (Consultant Bureau, New York, 1967), Vol. 29, p. 151. It is to be emphasized that Landau's model applies only to central collisions.

[5]Ph. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill, New York, 1953), Vol. 1, p. 865.

[6]A more detailed account along these lines will be published elsewhere.

[7]It has been suggested by E. Leader in a discussion with the author that in a semiclassical picture like ours the possibility arises that in peripheral collisions the target begins to rotate along an axis normal to the recoil direction and in this way the asymmetry might be washed out. As long as $q_{\parallel}R \ll 1$, this effect is negligible. Furthermore, if we consider the target as a composite system, then, as in nuclear physics, the existence of such an effect would be possible only if the target is deformed. There is no evidence (so far) for nonspherical shape of particles.

[8]As regards the projectile, it would be very interesting to compare this effect using different projectiles (mesons, nucleons). In principle, these considerations should apply also to e-p inelastic scattering, although the effect there might be less pronounced because this process is probably less peripheral.

[9]A. Ramanauskas *et al.*, Phys. Rev. Lett. 31, 1371 (1973).

[10]A. Mann and R. Weiner, Nuovo Cimento 10A, 625 (1972), and Phys. Lett. 40B, 383 (1972); S. Eliezer and R. Weiner, to be published.

ERRATA

SMALL-MOMENTUM-TRANSFER p-p INELASTIC SCATTERING AT 300 GeV/c. S. Childress, P. Franzini, J. Lee-Franzini, R. McCarthy, and R. D. Schamberger, Jr. [Phys. Rev. Lett. 32, 389 (1974)].

On page 389, right-hand column, line 10 should read 10^{12} instead of 10^2. On page 390, left-hand column, line 19 should read 0.6 in place of 0.06.

NONLINEAR ABSORPTION AND ULTRASHORT CARRIER RELAXATION TIMES IN GERMANIUM UNDER IRRADIATION BY PICOSECOND PULSES. Chandler J. Kennedy, John C. Matter, Arthur L. Smirl, Hugo Weichel, Frederic A. Hopf, Sastry V. Pappu, and Marlan O. Scully [Phys. Rev. Lett. 32, 419 (1974)].

The fourth author's surname was incorrectly given as Weiche. The form above is the correct one.