els which place most of the $s_{1/2}$ strength in that state.

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[1] E. G. Adelberger and A. B. McDonald, Nucl. Phys. A145, 497 (1970).

[2] T. Engeland, Nucl. Phys. 72, 68 (1965).

[3] A. P. Zuker, Phys. Rev. Lett. 23, 983 (1969).

[4] S. Kahana, Phys. Rev. C 5, 63 (1972).

[5] G. E. Brown, in *International Congress on Nuclear Physics, Paris, 1964, Proceedings*, edited by P. Gugenberger (Centre National de la Recherche Scientifique, Paris, France, 1964), Vol. 1, p. 129.

[6] H. G. Benson and B. H. Flowers, Nucl. Phys. A126, 332 (1969).

[7] P. J. Ellis and T. Engeland, Nucl. Phys. A144, 161 (1970).

[8] R. Middleton and D. J. Pullen, Nucl. Phys. 51, 63 (1964).

[9] J. L'Ecuyer, R. D. Gill, K. Ramavataram, N. S. Chant, and D. G. Montague, Phys. Rev. C 2, 116 (1970).

[10] W. R. Falk, R. J. Kidney, P. Kulisic, and G. K. Tandon, Nucl. Phys. A157, 241 (1970).

[11] R. A. Paddock, Phys. Rev. C 5, 485 (1972).

[12] For relevant DWBA calculations, see Ref. 10.

[13] J. A. Nolen, Jr., and J. P. Schiffer, Ann. Rev. Nucl. Sci. 19, 471 (1969).

Can the Charge Symmetry of Nuclear Forces be Confirmed by Nucleon-Nucleon Scattering Experiments?*

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The Coulomb interaction is subtracted from the proton-proton scattering data in the $^1S_0$ partial wave. Contrary to traditional belief, the resulting nuclear effective-range parameters depend strongly on the unknown nuclear potential at small distances. The comparison between proton-proton and neutron-neutron data is therefore ambiguous.

Nucleon-nucleon scattering is believed to provide direct and reliable evidence on the isospin properties of the nuclear interaction. Whereas proton-proton (pp) and neutron-proton (np) experiments have been performed with great precision at a variety of energies, no corresponding neutron-neutron (nn) experiment has been carried out by now. All available nn information is extracted from the final-state interaction in few-body reactions. Only the $^1S_0$ effective-range parameters, scattering length a_{nn}, and effective range r_{nn} have been determined this way, and even their best values,[1]

$$a_{nn} = -16.4 \pm 0.9 \text{ fm}, \quad r_{nn} = 2.8 \pm 0.5 \text{ fm}, \qquad (1)$$

are still plagued with large uncertainties. The scarcity of experimental nn data and their uncertainty are usually blamed for the missing detailed comparison between pp and nn scattering and for the lack of a rigorous experimental confirmation of charge symmetry.

However, charge symmetry solely applies to the hadronic part of the nucleon-nucleon interaction. Even in the advent of accurate and abundant nn data, nucleon-nucleon scattering will experimentally prove charge symmetry only to the extent that electromagnetic effects can theoretically be removed from the data in an unambiguous manner. The traditional belief is that this can be done. However, this claim is questioned in the present paper.

The subtraction of the Coulomb force is our concern. It is the most important correction of pp data for electromagnetic effects, though a sophisticated comparison[2] between pp and nn scattering should also take vacuum polarization, the magnetic-moment interaction, and the neutron-proton mass difference into account. The technique for subtracting the Coulomb force is standard: A nuclear potential is fitted to the data such that together with the Coulomb potential V_C it reproduces the experimental phase $\eta_{pp}{}^C(k)$ well.

Then V_C is dropped and the purely nuclear phase $\eta_{pp}(k)$ is calculated. The pp momentum k refers to the relative motion. The effective-range parameters a_{pp} and r_{pp} in the low-energy expansion

$$k\cot\eta_{pp}(k) = -1/a_{pp} + \tfrac{1}{2}r_{pp}k^2 + O(k^4) \qquad (2)$$

of the ${}^1S_0$ phase $\eta_{pp}(k)$, which have been obtained from this procedure, appear rather independent of the nuclear potential chosen. They group around the currently accepted[1] values

$$a_{pp} = -17.1 \pm 0.2 \text{ fm}, \quad r_{pp} = 2.84 \pm 0.03 \text{ fm}, \qquad (3)$$

which include all electromagnetic corrections. The error of 1% assigned to them is supposed to reflect their small residual model dependence; it is not experimental. If the Coulomb-corrected pp effective-range parameter values were as well determined as stated by (3), their comparison with the corresponding nn quantities (1) would be meaningful. Indeed, they turn out to be in agreement with them and therefore seem to support charge symmetry.

However, the claim that the Coulomb subtraction is as independent of nuclear potential models as (3) suggests is really ill founded. By counter examples, we shall demonstrate that the nuclear pp effective-range parameters (2) are not at all uniquely determined by pp scattering and the theoretical knowledge of the long-range part of the nuclear interaction. They strongly depend on the form of the nuclear potential at small distances, where it is theoretically unknown.

In order to study the model dependence of the Coulomb-subtracted ${}^1S_0$ phase $\eta_{pp}(k)$ and its effective-range parameters, a variety of equivalent nuclear potential models is needed: They have to have the theoretically required one-pion-exchange tail and all have to fit the experimental data equally well. Assuming the soft-core Reid potential[3] V_R to account for the experimental pp data with sufficient accuracy, the nuclear potentials

$$\tilde{V}_R = U(K + V_C + V_R)U^+ - K - V_C \qquad (4)$$

generated from V_R have the desired properties. In Eq. (4), K denotes the kinetic-energy operator of relative motion and U is an arbitrary unitary operator of short range. All potentials $\tilde{V}_R$ are equivalent to V_R by construction:

(1) Together with the Coulomb potential they yield the same phase $\eta_{pp}{}^C(k)$ as the Reid potential at all energies. The scattering states $|\tilde{\varphi}(k)\rangle$ of $\tilde{V}_R$ are related to those of V_R, $|\varphi(k)\rangle$, by $|\tilde{\varphi}(k)\rangle = U|\varphi(k)\rangle$, and both $|\tilde{\varphi}(k)\rangle$ and $|\varphi(k)\rangle$ take on the same asymptotic form

$$\langle r|\varphi_a(k)\rangle = C_0(\gamma)\{\cot[\eta_{pp}{}^C(k)]F_0(\gamma, kr) + G_0(\gamma, kr)\},$$
$$C_0(\gamma) = 2\pi\gamma/(e^{2\pi\gamma} - 1), \quad \gamma = e^2M/2\hbar^2k. \qquad (5)$$

In Eq. (5) F_0 (G_0) is the regular (irregular) Coulomb wave function of orbital momentum 0, r is the pp distance, M (e) is the proton mass (charge).

(2) The long-range Coulomb potential appears in Eq. (4) only to guarantee the same experimental phase $\eta_{pp}{}^C(k)$ for all $\tilde{V}_R$. The potentials $\tilde{V}_R$ are purely nuclear *without* Coulomb. They are nonlocal at small distances, but reduce to the local tail of V_R where U reduces to unity. The simple form

$$U = 1 - 2|g\rangle\langle g|, \qquad \langle r|g\rangle = Cr(1 - \beta r)e^{-\alpha r}, \qquad (6)$$

is employed. With the extra factor r included in (6), the radial part of the volume element for integration is dr. The constant C ensures the normalization of $|g\rangle$. The range of U is controlled by α. Choosing α sufficiently large, the potentials $\tilde{V}_R$ have the one-pion-exchange tail of V_R as required by theory.

The Schrödinger equation for the Coulomb-subtracted Hamiltonian $K + \tilde{V}_R$ is solved in momentum space.[4] Its scattering states $|\tilde{\psi}(k)\rangle$ have the asymptotic form

$$\langle r|\tilde{\psi}_a(k)\rangle = \cot[\eta_{pp}(k)]\sin(kr) + \cos(kr), \qquad (7)$$

which yields the nuclear phase $\eta_{pp}(k)$. Its effective-range parameters are obtained according to Eq. (2). They are calculated for the family of potentials $\tilde{V}_R$ derived from the Reid potential with the unitary operator (6). In (6) the range parameter α is taken to be 3 fm^{-1} and the parameter β ranges from 0 to 5 fm^{-1}. In Eq. (4) the point-Coulomb interaction $V_C{}^P = e^2/r$ is used for V_C as is common practice. The resulting nuclear pp effective-range parameters are displayed by the solid lines in Figs. 1 and 2.

The results are not at all stable and in general not close to the values of the Reid potential, $a_{ppR} = -17.1$ fm and $r_{ppR} = 2.80$ fm. They vary drama-

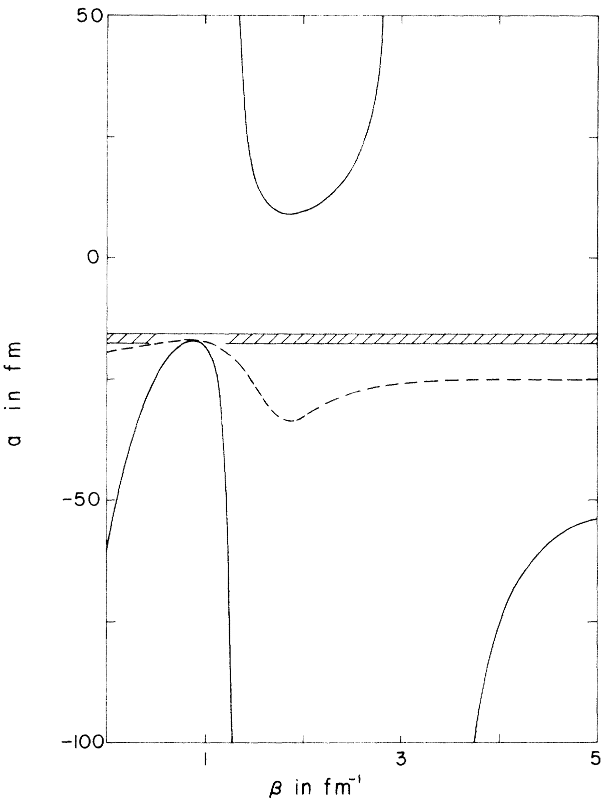

FIG. 1. Nuclear scattering length. The pp value a_{pp} of (2) is shown for the nuclear potentials $\tilde{V}_R$ as function of the parameter β in the unitary transformation (6). $\alpha = 3\ \mathrm{fm}^{-1}$. Solid line, obtained with the point-Coulomb potential in (4); dashed one, with the finite-size Coulomb potential. Cross-hatched strip, range of values consistent with a_{nn} and charge symmetry.

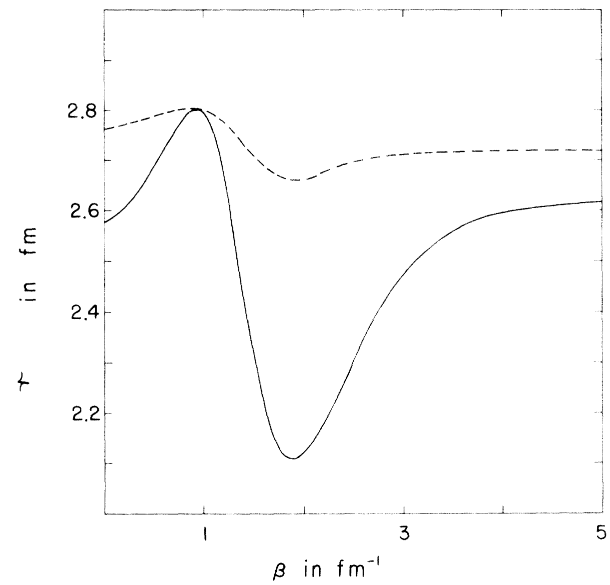

FIG. 2. Nuclear effective range. The pp value r_{pp} of (2) is shown. See Fig. 1 for details. The spread of values consistent with r_{nn} is not indicated.

tically with the potential $\tilde{V}_R$ chosen. The scattering length is especially sensitive to changes in the potential, since it is large compared to the pion Compton wavelength. It can even turn positive. Thus, there are potentials which fit the data as well as the Reid potential, but which support a bound state, if the Coulomb potential is turned off. The spread in the results is smaller for the effective range, but is still sizable. How is this unexpected model dependence of the effective-range parameters possible?

The known exact relation[5] between the phase shifts $\eta_{pp}{}^C(k)$ and $\eta_{pp}(k)$ is applied to $\tilde{V}_R$ and V_R which are equivalent with respect to $\eta_{pp}{}^C(k)$. One obtains

$$k\cot\eta_{pp}(k) - k\cot\eta_{ppR}(k) = -(M/\hbar^2)[\langle\tilde{\psi}(k)|V_C{}^P|\tilde{\varphi}(k)\rangle - \langle\psi(k)|V_C{}^P|\varphi(k)\rangle - \langle\tilde{\psi}_a(k) - \psi_a(k)|V_C{}^P|\varphi_a(k)\rangle], \tag{8}$$

where the quantities $|\psi(k)\rangle$, $|\psi_a(k)\rangle$, and $\eta_{ppR}(k)$ refer to V_R. Because of cancelations, the integrals in the matrix elements can be terminated at the range of the nuclear interaction. This is understood for the following argument. The repulsion in V_R at small distances suppresses the wave functions $|\varphi(k)\rangle$ and $|\psi(k)\rangle$ where $V_C{}^P$ is strong. However, this is not the case for $|\varphi(k)\rangle$, whose small-distance behavior is dominated by $-2\langle r|g\rangle \times \langle g|\varphi(k)\rangle$. Despite the Coulomb repulsion, $\tilde{V}_R$ allows the two protons to approach each other closely. In contrast to $|\varphi(k)\rangle$, $|\tilde{\varphi}(k)\rangle$ is enhanced where $V_C{}^P$ is strong. Since $|\tilde{\varphi}(k)\rangle$ approximates $|\tilde{\psi}(k)\rangle$ well at small distances, the term $\langle\tilde{\psi}(k)|V_C{}^P \times |\tilde{\varphi}(k)\rangle$ might be equated to $\langle\tilde{\varphi}(k)|V_C{}^P|\tilde{\varphi}(k)\rangle$, which is positive. It dominates the right-hand side of Eq. (8). Thus, $\cot\eta_{pp}(k) < \cot\eta_{ppR}(k)$ and $a_{pp}{}^{-1} > a_{ppR}{}^{-1}$. This explains the trend in the results of Fig. 1.

The unitary transformations (6) peak at small r. Depending on the zero β^{-1} in $\langle r|g\rangle$, they exploit the $1/r$ singularity of the point-Coulomb interaction with varying degree. This is the reason for the large spread in the results.

However, the $1/r$ Coulomb singularity is physically removed by the finite size of the proton. The effective-range parameters are therefore recalculated substituting the finite-size[6] Coulomb potential

$$V_C^F(r) = (e^2/r)[1 - e^{-x}(1 + \tfrac{11}{16}x + \tfrac{3}{16}x^2 + \tfrac{1}{48}x^3)] \quad (9)$$

for V_C in Eq. (4). In (9) $x = 2\sqrt{3}\,r/R_p$, and the proton rms radius R_p is taken to be 0.80 fm. The Reid potential with V_C^F fits the experimental data almost as well as with V_C^P. This is because the Reid wave functions are suppressed where V_C^F differs from V_C^P. Together with V_C^F, the potentials $\tilde{V}_R$ remain phase-equivalent with the Reid potential. The results using the finite-size Coulomb in Eq. (4) are shown as dashed curves in Figs. 1 and 2. Indeed, the model dependence of the effective-range parameters is greatly reduced. It remains, however, disturbingly large and is certainly beyond the limits of uncertainty suggested by (3). Thus, the following conclusions appear inescapable:

(1) Given the experimental pp data and our theoretical knowledge on the tail of the nuclear potential, we are unable to arrive at approximately unique answers for the nuclear pp scattering amplitude. Since an approximately unique answer is required for any meaningful comparison with nn scattering, charge symmetry cannot be confirmed by nucleon-nucleon experiments in a model-independent way.

(2) The traditional claim that the Coulomb subtraction in the pp data is only weakly model dependent rests heavily on the assumption of suppressed wave functions at small distances. This wave-function model appears physically sound. It is surely not beyond any doubt, since the nucleon-nucleon potential is theoretically unknown at small distances.

(3) The magnitude of the model dependence in the nuclear pp scattering amplitude sensitively depends on the form and strength of the electromagnetic interaction at small distances. The observed strong model dependence might therefore even be amplified by magnetic-moment corrections, which are omitted in this study.

The analysis presented here is not intended to question the charge symmetry of nuclear forces. The assumption of charge symmetry has worked so well for a wealth of phenomena in nuclear and particle physics that its validity appears to be established with high accuracy. Instead of trying to confirm charge symmetry experimentally, it can rather be taken as a theoretical principle which provides a constraint on nuclear potential models. As Fig. 1 demonstrates, most potentials $\tilde{V}_R$ violate the charge-symmetry constraint with respect to the scattering length. On this ground they have to be discarded, though they fit the pp data as well as the Reid potential. The experimental error in the nn effective range is still too large to provide a serious additional constraint.

The model dependence of the Coulomb subtraction also makes the comparison between pp and np scattering ambiguous. Thus, nucleon-nucleon scattering experiments are unable to establish the degree of charge dependence in the nuclear force in a model-free way. Instead, charge independence might be assumed as a theoretical principle which holds approximately. Then, most potentials $\tilde{V}_R$ fail dramatically to account for the experimental np data, e.g., the np scattering length[1] of -23.715 fm. They therefore violate charge independence by so much that they have to be rejected also on this ground.

The nuclear potential models[7] $U(K+V_R)U^+ - K$ are used in nuclear-structure calculations to study off-shell effects. By construction, their nuclear effective-range parameters are those of the Reid potential and are therefore in sufficient agreement with charge symmetry and approximate charge independence. However, these potentials miss the part $UV_CU^+ - V_C$ which preserves the fit of the Reid potential to experiment. One therefore has to suspect that many of the nuclear potentials used to create off-shell changes in $^1S_0$ fail to account for the experimental phase $\eta_{pp}{}^C(k)$. The relation between the potentials $U(K+V_R)U^+ - K$ and those of Eq. (4) is studied in a forthcoming publication.

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[1]E. M. Henley and D. H. Wilkinson, in *Few Particle Problems in the Nuclear Interaction,* edited by I. Slaus, S. A. Moszkowski, R. P. Haddock, and W. T. H. van Oers (North-Holland, Amsterdam, 1972), pp. 229 and 242, and references there.

[2]H. P. Noyes, Annu. Rev. Nucl. Sci. 22, 465 (1972), and references there.

[3]R. V. Reid, Ann. Phys. (New York) 50, 411 (1968).

[4]G. E. Brown, A. D. Jackson, and T. T. S. Kuo, Nucl.

Phys. A133, 481 (1969).

[5]M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1965), p. 297, Eq. (348).

[6]N. Auerbach, J. Hüfner, A. K. Kerman, and C. M. Shakin, Rev. Mod. Phys. 44, 48 (1972).

[7]F. Tabakin, in *The Two-Body Force in Nuclei*, edited by S. M. Austin and G. M. Crawley (Plenum, New York, 1972), p. 101, and references there.

Asymmetry in Peripheral Production Processes

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As a consequence of local excitation, an asymmetry with respect to the leading particle in peripheral production processes is predicted. It is interpreted in terms of heat propagation in hadronic matter.

Consider a peripheral collision of two hadrons accompanied by particle production. The peripheral nature of this collision manifests itself in the fact that there exists a leading particle which retains a high proportion of the incoming momentum and which is scattered through a small angle. Classically such a process can be viewed as a grazing collision in which the projectile (leading particle) touches only the surface of the target, which for reasons of simplicity is supposed to be spherical (Fig. 1).

Quantum mechanically the localization occurs if $q \gg R^{-1} \sim m_\pi$, where q is the exchanged momentum and R the radius of the target; peripheralism is assured by the condition $q \ll p_i$, where p_i is the incoming momentum. In the following we shall assume that these two conditions are fulfilled. Radiation (particle emission), however, will take place not from the hot spot but from a region of dimensions comparable with the range of strong interaction forces $\sim R$, because only in this case does the number of particles become defined.[1] Hence it follows that there will be a time delay, $\tau_0 \sim R/c$, before particle emission starts (this follows also from the indeterminacy principle since at a given energy transfer q_0 it takes a certain time $\tau_0 > q_0^{-1}$ after which this energy can be measured), since the excitation concentrated initially in a small region has to propagate with *finite* (sound) velocity c until it extends over a region $\sim R$. In the half-space below the tangent plane at N in Fig. 1, there is the body of the target, characterized by a thermal heat conductivity K and a radiation constant λ, and along the line NS (Fig. 1) a temperature gradient develops so that at the moment τ_0 when radiation starts, the temperature at the south pole S is lower than that at the north pole N.

In the half-space above NM there is "vacuum" and no such gradient of temperature is expected to occur, i.e., no diffusion should take place. This leads us to expect a yet unobserved asymmetry in peripheral collisions for particles which are produced in the target, i.e., for target "fragments." (The same is obviously true for projectile fragments.) The asymmetry has two experimental consequences: (i) Particles produced in the hemisphere $\vec{q}_\perp$ will have, on the average, smaller momenta than those in the hemisphere $-\vec{q}_\perp$. (ii) Heavier particles (e.g., kaons or antinucleons) will preferably be produced in the hemisphere $-\vec{q}_\perp$. This should be especially true if, because of quantum-number conservation, these particles can be produced only in pairs.

In general the smaller the angle φ at which particles are emitted, the smaller the corresponding momentum and mass.[2] The size of this

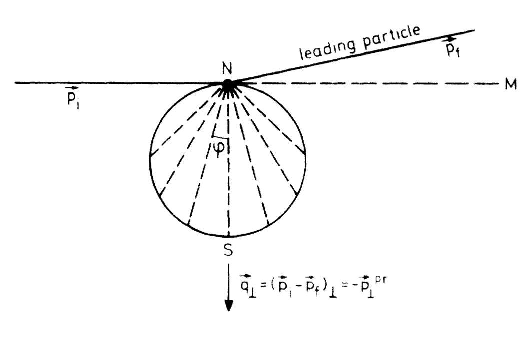

FIG. 1. Local excitation of a spherical target in a peripheral collision.