Regge Poles and Forward-Angle Anomalies in Transfer Reactions

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We use Regge-pole states for the distorted waves in a transfer reaction and compare the results with data showing the forward-angle anomaly.

Several recent Letters[1-4] have reported non-grazing-angle–peaked angular distributions for one- and two-particle transfer reactions induced by heavy ions on intermediate-mass nuclei. Chasman, Kahana, and Schneider[4] show a DWBA (distorted-wave Born approximation) calculation which successfully reproduces the forward-angle oscillation they observe in the 50-MeV transfer reaction ^{48}Ca(^{14}N, ^{13}C)^{49}Sc. They argue that semiclassical physics[5,6] explains the DWBA result. Underlying this explanation is the assumption that DWBA partial-wave amplitudes are sufficiently smooth in the angular momentum, l, of the exit channel to allow their parametrization by an entire function of l.

The distorted waves contain Regge poles which correspond to peripheral states so broad that their lifetimes are commensurate with the direct reaction time scale[7]; i.e., they generally decay before the ions can orbit one another and we shall refer to them as "peripheral fly-off resonances" (PFOR). These states, obviously nonstationary in angle, can be described using complex angular momenta.[7] If the states are sufficiently broad, the corresponding Regge pole occurs at an angular momentum with a large imaginary component and the associated DWBA amplitude varies slowly with changing l allowing the application of the usual semiclassical arguments. On the other hand, forward-angle structure in transfer reactions arises from the interference between projectile paths on opposite sides of the target nucleus.[4] If this interference is appreciable, then the ions must have been brought around one another by the attractive nuclear interaction and the Regge pole which describes this partial orbiting can play a dominant role.

Underlying this conjecture is first the extreme peripherality of heavy-ion direct reactions, and second the assumption that PFOR's dominate the peripheral distorted waves. Austern[8] has shown that peripherality follows from (a) "blackness," which depends on an impedence match for the distorting potential which allows the smaller-l partial waves to enter the absorptive region without reflection, and (b) the large radial momentum of the ions as they enter the reaction region. These conditions are generally well satisfied at energies above the Coulomb barrier for the distorting potentials used in heavy-ion direct reactions except for peripheral partial waves where the reflection coefficients are rising toward unity and the racial momentum is cmall. It is important to note that peripherality depends on surface properties of the distorting potentials and only requires absorptive potentials sufficiently large to damp the distorted wave before it emerges from the reaction region.

Since the PFOR's describe the longest lived peripheral components of the distorted wave, they can be expected to dominate peripheral reactions. Further they provide a *mechanism* to bring the ions around one another and therefore their presence should be most prominant in reactions displaying forward-angle oscillations. Potentially PFOR domination of peripheral reactions provides a large saving of DWBA computation time since the PFOR wave function factors into radial and angular components.[7] This means that radial integrals need not be evaluated for each partial wave. We will return to this point when discussing our calculation.

This Letter examines the simplest realization of the PFOR conjecture where the reaction process is assumed to be dominated by a single PFOR in both the entrance and exit channels. The reader is referred to previous work[7,9,10] for some of the background of the following necessarily abbreviated argument. We assume physi-

cal energies and complex angular momenta. The PFOR component of the distorted-wave function is

$$\psi_\alpha(r,\theta)=\xi_\alpha f_\alpha(r)P_\alpha(\cos\theta) \quad (1)$$

near the reaction surface and in its shadow. In the region "lit" by the beam the same PFOR gives

$$\psi_\alpha(r,\theta)=\xi_\alpha f_\alpha(r)e^{i\pi\alpha}P_\alpha(-\cos\theta), \quad (2)$$

where $P_\alpha(\cos\theta)$ is the Legendre function, and $\alpha=\alpha_1+i\alpha_2$ is the complex Regge-pole angular momentum. In order to evaluate the DWBA matrix element in the usual way, it is necessary to expand the distorted wave in partial waves, i.e., states of the physical angular momentum l. The extremely broad resonances under consideration correspond to angular momenta with an appreciable imaginary part.[7] Therefore, the pole is generally "felt" over a range of physical angular momenta (two or more) making it necessary to consider the background which, although a regular function of l, is not slowly varying.

The factor ξ_α in Eqs. (1) and (2) includes the background evaluated at $l=\alpha$. To determine the background we offer the following admittedly nonrigorous argument. The distorted-wave resonance is contained in the outgoing component of its partial-wave expansion,

$$\psi_{\text{o.g.}}(r,\theta)=(2ik)^{-1}\sum(2l+1)S_l(k)f_l^{(+)}(k,r)P_l(\cos\theta). \quad (3)$$

$f_l^{(+)}(k,r)$ satisfies the angular-momentum–independent boundary condition $f_l^{(+)}(k,r)\sim\exp[i(kr-\eta\ln 2kr)]$ and is therefore an entire function of l for fixed k and r.[11] At the Regge pole $f_l^{(+)}(k,r)$ is a regular, outgoing solution of the Schrödinger equation.[11] The usual WKB argument indicates that the regular wave function picks up most of its phase external to the classical turning point which for peripheral partial waves is near the interaction region. If this phase is to vanish asymptotically, it must include the negative of the centripetal-plus-Coulomb phase, $\frac{1}{2}\pi\alpha-\sigma(\alpha)$. To this we add a component $\tilde{\delta}(\alpha)$ to account for the nuclear interaction in the surface and analytically continue the phase $\frac{1}{2}\pi\alpha-\sigma(\alpha)+\delta(\alpha)$ to $l=\alpha$. Factoring the long-range Coulomb contribution out of the S matrix as $S_l(k)=\exp(2i\sigma_l)\bar{S}_l$ and taking for the "nuclear" S matrix $\bar{S}_l$, the resonance contribution gives

$$S_l^{\,p}=\beta_\alpha[(l-\alpha)(l+\alpha+1)]^{-1}\exp(2i\sigma_l), \quad (4)$$

where β_α is the Regge-pole residue. We now assume that $\tilde{\delta}(l)$ can be approximated in the neighborhood of the Regge pole by expanding to first order around $l=\alpha_1$. Upon incorporating all l-independent contributions into a factor $\gamma_\alpha f_\alpha(r)$, we obtain the peripheral Regge-pole contribution to Eq. (3):

$$\psi_\alpha(r,\theta)=\gamma_\alpha f_\alpha(r)\sum(2l+1)[(l-\alpha)(l+\alpha+1)]^{-1}\exp\{i[\sigma_l+(\tfrac{1}{2}\pi+\tilde{\delta}')l]\}P_l(\cos\theta). \quad (5)$$

Using the Sommerfeld-Watson transformation to convert the sum into an integral, being careful with the required analytic continuation of the integrand,[9] gives Eqs. (1) and (2) for the pole contribution to Eq. (5) at $l=\alpha$.

The significance of the phase factor of Eq. (5) can be inferred from Fig. 1 where we have plotted the angular part of Eq. (5). The phase determines the "starting angle" of the PFOR. This is most apparent in the upper plot where the Coulomb interaction is negligible. This plot is peaked at about $\frac{1}{2}\pi+\tilde{\delta}'$ and then starts the exponential decrease at the rate, toward oscillation with the period, expected for the Regge pole.[7] The centripetal contribution, $\frac{1}{2}\pi l$, would alone give a starting angle of $\frac{1}{2}\pi$ corresponding to the "most peripheral" resonance. The Coulomb phase pushes the starting angle back and broadens the starting peak. Both effects can be understood by picturing the diverging peripheral trajectory in

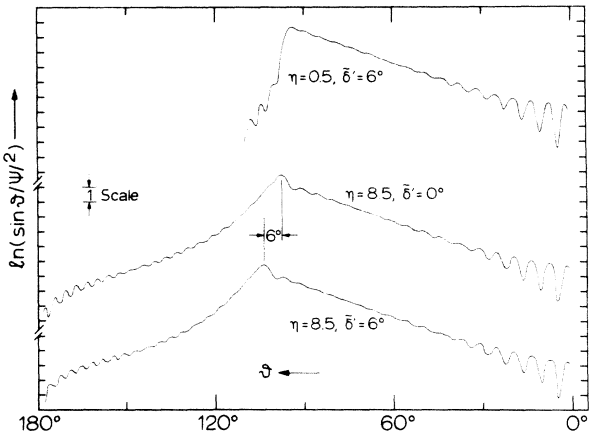

FIG. 1. The angular part of the PFOR calculated from Eq. (5) for the parameters indicated. The beam is incident from the left with the shadow to the right. The Regge pole is $\alpha=19.5+2i$. The Sommerfeld Coulomb parameter is denoted by η; the nuclear background phase is $\tilde{\xi}'$.

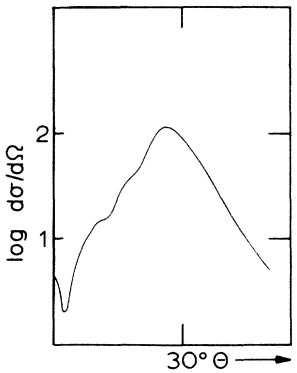

FIG. 2. PFOR-DWBA angular distribution showing grazing-angle–peak. All parameters are the same as those for the 60-MeV ^{40}Ca(^{13}C, ^{12}C)^{41}Ca transfer calculation shown in Fig. 3, except $\alpha_2 = 8$.

a repulsive Coulomb field. A positive $\tilde{\delta}'$ arises from the attractive nuclear force bending this trajectory toward the nucleus, pushing the starting angle further back. These concepts obviously apply only to extremely broad resonances which decay before the ions orbit the reaction region.

The exit-channel wave function follows from Eq. (5) in the usual way.[12] For each angular momentum transferred, in the recoilless approximation, the portion of the matrix element arising from integration over channel radial variables factors out of the summation over entrance and exit channel partial waves. If a single angular momentum is transferred, the angular distribution does not depend on the radial part of the form factor; only the magnitude of the cross section depends on the matrix element of the form factor with the entrance and exit channel radial Regge-pole wave functions.

For sufficiently broad distorted-wave PFOR's this model gives a grazing-angle peak. From Eq. (5) it is apparent that the Coulomb phase can contribute the major phase variation within a resonance so broad that $\alpha_2 \gtrsim \eta$. For the grazing-angle–peaked angular distribution shown in Fig. 2, only α_2 (=8) in both the entrance and exit channels differs from the parameters used in the 60-MeV calculation shown in Fig. 3 for the reaction ^{40}Ca(^{13}C, ^{12}C)^{41}Ca.

Surface transparency now has an immediate interpretation. Decreasing the absorption in the surface[1,2,4] decreases α_2, thereby bringing the PFOR further around the interaction surface. Roughly, for $\alpha_2 \lesssim \frac{1}{2}\eta$, the PFOR can overcome the repulsion of the Coulomb force. Figure 3 shows PFOR-DWBA calculations for the Brookhaven National Laboratory data[13] for the reaction ^{40}Ca(^{13}C, ^{12}C)^{41}Ca which shows a prominent forward-angle anomaly. Only the shape of the calculated angular distribution is significant since we do not evaluate the radial matrix element; i.e., this model calculates the shape of the angular distribution from amplitudes which only require the analytic evaluation of angular integrals which are well known in the recoilless DWBA.[12] The 40-MeV calculation used Regge poles obtained from the distorting potentials[7] and varied the background angle to obtain the fit. The peak in this calculation at 44° results from the background angle, $\tilde{\delta}'$ and is not a grazing-angle peak. In all calculations the same background angle was used in the entrance and exit channels. In the 60- and 68-MeV calculations the Regge poles, set equal in the entrance and exit channels, were fitted to the data. Roughly, α_1 determines the period of angular oscillation; α_2, the average rate of falloff once the angular distribution breaks downward; $\tilde{\delta}'$, the angle where the distribution breaks.

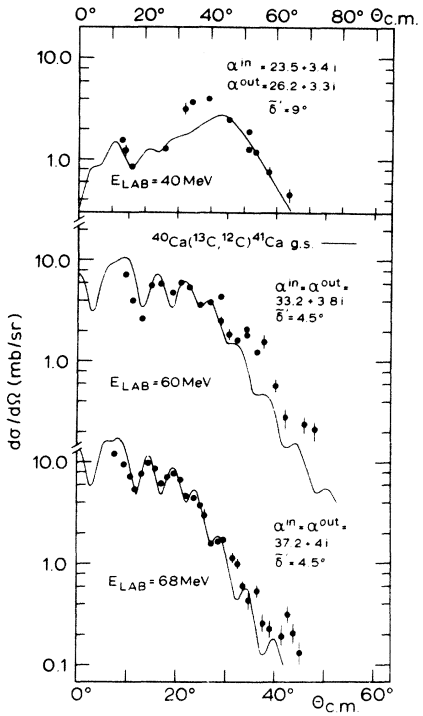

FIG. 3. PFOR-DWBA calculations with parameters indicated and normalized to the data taken from Ref. 13.

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Practical Equations for Three-Particle Scattering Calculations

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A new method is presented for solving the singular integral equations that arise in the Faddeev theory of three-particle scattering. The method is tested by means of an example and found to be practical.

In general, it is more difficult to perform three-particle calculations above the breakup threshold than below. In the Faddeev[1] formalism for nonrelativistic three-particle systems, this difficulty can be attributed to the presence of certain logarithmic singularities in the kernels of the momentum-space integral equations. Three successful techniques for handling these singularities are contour rotation,[2] a method based on the use of Padé approximants to sum a multiple-scattering series,[3] and a modification of the method of moments.[4] The purpose of the present note is to present an alternative approach, which appears to have some advantages over these methods.

The work of Alt, Grassberger, and Sandhas[5] shows that, in general, it is possible to reduce three-particle collision problems to the solution of equations that have the same structure as those which arise when separable two-particle interactions are assumed. Accordingly, here I shall deal with only the equation that arises when each of the pair interactions consists of a single separable term. Furthermore, for the sake of simplicity I shall assume that all of the particles are identical and spinless, and that the two-particle bound state is an s state. This example suffices to illustrate the method; the generalizations to more complicated interactions are not difficult to carry out.

With the assumptions just stated, the two-particle transition operator becomes

$$t(s) = |g\rangle T(s)\langle g|, \tag{1}$$

where s is a complex energy parameter, and $|g\rangle$ is related to the two-particle bound-state wave function $|B\rangle$ with binding energy B by the relation

$$|g\rangle = (-B - \boldsymbol{H}_0)|B\rangle. \tag{2}$$

Here H_0 is the kinetic-energy operator. The propagator T is given by

$$[T(s)]^{-1} = (s+B)\langle B|(s-H_0)^{-1}|g\rangle. \tag{3}$$

Clearly, it has a simple pole at $s=-B$. With this this interaction, it is well known[2,5] that the half-off-shell partial-wave amplitudes for the scattering of one particle from a bound state of the other two can be obtained by solving the equations

$$X_L(q,k;s) = Z_L(q,k;s) + \int_0^\infty Z_L(q,q';s)q'^2\,dq'\,T(s-\tfrac{3}{4}q'^2)X_L(q',k;s), \quad L=0,1,2,\ldots, \tag{4}$$

where

$$Z_L(q,q';s) = \int_{-1}^{1} dx\,P_L(x)g(|\tfrac{1}{2}\vec{q}+\vec{q}'|)g(|\tfrac{1}{2}\vec{q}'+\vec{q}|)/(s-q^2-\vec{q}\cdot\vec{q}'-q'^2), \quad x=\hat{q}\cdot\hat{q}', \tag{5}$$

$$s = -B + \tfrac{3}{4}k^2 + i\epsilon = E + i\epsilon.$$