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Longitudinal Resonance and Domain Effects in the *A* and *B* Phases of Liquid Helium Three

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We report measurements of the longitudinal NMR resonance in the *A* phase of liquid He^3 on the melting curve below 2.7 mK, and show them to be consistent with that phase being the state proposed by Anderson and Brinkman. Strong field dependence of the *B*-liquid transverse NMR line shape, including the existence of a broad high-frequency tail, is presented and interpreted as domain effects in a Balian-Werthamer state. Absorption in the longitudinal mode in the *B* phase is also reported.

Following the observations made by Osheroff *et al.*[1] that below 2.7 mK liquid He^3 exhibits a shifted NMR frequency, considerable theoretical effort has gone into understanding magnetic resonance in anisotropic superfluid states with triplet pairing. The most comprehensive work is that of Leggett.[2] His theory not only explains the shift in the ordinary (transverse) NMR resonance that is observed, but predicts that a longitudinal resonance whose frequency is independent of the applied field should be observable when the rf field $\vec{H}_1$ and the static field $\vec{H}_0$ are parallel. We present experimental confirmation of Leggett's work, and show that our measurements of the longitudinal resonance indicate that the *A* phase of liquid He^3 should be identified with the particular Anderson-Morel (AM) state[3] suggested by Anderson and Brinkman (ABM).[4] Various phenomena involving the lower-temperature *B* liquid are also discussed including frequency shifts of the transverse resonance and clear evidence of absorption in the longitudinal mode. These are interpreted in terms of possible domain structures.

The compression cell used for refrigeration in our experiments is conceptually similar to that described by Osheroff, Richardson, and Lee,[5] although the cell is larger, and the capacitative strain gauge has been situated up inside the bellows of the He^3 chamber, allowing an NMR tail section to be added below. This feature, coupled with careful choices of nonmagnetic cell materials, has limited magnetic field inhomogeneities over our NMR region to less than 5×10^{-5} from 15 Oe to 1.1 kOe.

The NMR tail section consisted of a cylinder of epoxy 6.3 cm long, 1.6 cm in diameter, with a 6.4-mm bore down the center. Two NMR coils, each consisting of a Helmholtz pair the diameter of the tail piece, were wound on grooves machined into the center of the exterior body of the tail section. One coil, the longitudinal coil, had its axis parallel to the axis of the tail section, and the other, the transverse coil, had its axis perpendicular (horizontal) to the axis of the tail section.

Using a superconducting solenoid, a static magnetic field uniform to 5×10^{-5} over the NMR region could be applied parallel to the axis of the tail section, and maintained constant to better than 1×10^{-6}/h. Additional field coils were employed to sweep the static field and produce a field gradient in the vertical direction when desired.

To prevent He^3 solid formation in the region of the rf field, heaters were located in the bore of the tail section 2 cm above and below the NMR coils and were shielded from the rf field by copper foils. These were used to nucleate solid formation in the early stages of compression. By compressing slowly enough, we were able to cool below 2 mK with as little as 4% of the NMR signal contributed by the solid. This small fraction was easily identifiable in "position profiles."[1] The NMR spectrometer used in our work consisted of a simple Rollins circuit driven by a frequency synthesizer with 0.1-Hz resolution, and all data were taken in the cw unsaturated mode.

In the ABM state,[3,4] which we associate with the *A* phase of liquid He^3, each pair of quasiparticles has $l=1$ with $m_l=1$ in some arbitrary direction $\vec{l}$. Each pair also has $s=1$ with $m_s=0$ along another axis $\vec{k}_\chi$. To minimize the dipolar energy, $\vec{l}$ and $\vec{k}_\chi$ orient parallel to one another. In the presence of a magnetic field, Zeeman energy will tend to orient $\vec{k}_\chi$ perpendicular to the field, and at a surface depairing effects will strongly orient $\vec{l}$ perpendicular to the surface. In our geometry, $\vec{l}$ should always lie in a plane perpendicular to the tail axis when $H_0\neq0$.

In transverse resonance in the *A* phase, the precession of the magnetization about $\vec{H}_0$ forces

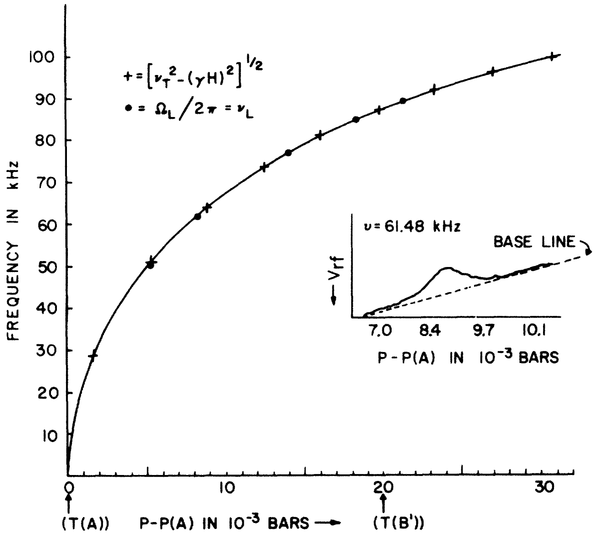

FIG. 1. Comparison of our transverse NMR frequency-shift data $[\nu_T^2-(\gamma H_0)^2]^{1/2}$ with the observed longitudinal resonance frequency in the A phase as a function of pressure difference $P-P(A)$. The solid line has been drawn through the transverse data points.

$\vec{k}_\chi$ to oscillate about $\vec{l}$. It is the restoring force between $\vec{k}_\chi$ and $\vec{l}$ that increases the resonant frequency of the A liquid. By applying an oscillating field along $\vec{H}_0$, one can cause the angle between $\vec{k}_\chi$ and $\vec{l}$ to oscillate directly, thereby exciting a "longitudinal resonance." Of the unitary p-wave states only in the ABM state does one expect the frequency of the longitudinal resonance ν_L to be related to the frequency of the transverse resonance ν_T exactly by $\nu_L^2=\nu_T^2-(\gamma H_0)^2$.

In Fig. 1 we compare the results of our longitudinal-resonance experiments with our measured transverse-resonance shifts. The crosses represent values of $[\nu_T^2-(\gamma H_0)^2]^{1/2}$ measured at 740 and 1150 kHz plotted against strain-gauge pressure differences. The scatter of the data between these frequencies was equivalent to pressure errors of less than 3×10^{-5} bars at all pressures. Plotted along with the transverse data as solid circles are frequencies and pressures at which we observed the longitudinal resonance. The slight scatter is thought to be largely due to the rather broad longitudinal-resonance widths. A representative longitudinal signal is shown in the inset in Fig. 1. The very fine agreement of our ν_L data with the ν_T data is convincing evidence that the A liquid is indeed the ABM state.

In obtaining the longitudinal data we swept the temperature of our cell rather than the rf frequency. This procedure did not broaden transverse-resonance linewidths appreciably, and it was observed that the longitudinal-resonance linewidths were independent of the rate of change of temperature with time. Several features of the longitudinal resonance can be briefly summarized as follows:

(1) The magnitude of the longitudinal absorption equals, to within a factor of 2, the transverse-resonant signal observed at the same frequency in the Fermi liquid with the same NMR coil (by applying a small field perpendicular to the coil axis). This agrees with the Leggett theory.

(2) The longitudinal-resonance linewidth varied from about 2.5 at 50 kHz to about 4 at 90 kHz, and appears to be nearly independent of the magnitude of the applied field (10 Oe to 1.1 kOe), and the rf level (2×10^{-4} to 2×10^{-3} G), although a slight broadening at high rf levels may have occured.

(3) By contrast, the transverse-resonance linewidth was broadened by only about 10 Hz at 350 Oe, but almost 700 Hz at 74 Oe. Thermal or other inhomogeneous broadening of ν_L is inadequate to explain the extremely broad ν_T at low frequencies, and is inconsistent with the sharp ν_T at high frequencies.

(4) In fields of less than about 5 Oe, the longitudinal signal as observed using the longitudinal coil decreased in magnitude and was unobservable at zero field. However, at zero applied field the longitudinal signal could be observed in the transverse coil, although it could not be seen there at fields above as little as 15 Oe. This behavior is consistent with a model in which the orientation of $\vec{l}$ and $\vec{k}_\chi$ changes at low fields to be parallel to the axis of the tail section. We think this behavior may be caused by thermal gradients along the axis of the tail section which align $\vec{l}$ in such a fashion so as to maximize the thermal conductivity by maximizing the quasiparticle current density in that direction.

The Balian-Werthamer[6] (BW) class of paired states are characterized by forming states with all pairs having $\vec{l}=1$, $s=1$, and $j=0$, 'and then performing an arbitrary rotation of the spin variables relative to the orbital variables. The dipolar forces fix the angle of rotation to be $\cos^{-1}(-\frac{1}{4})$, but the axis of rotation $\vec{n}$ is arbitrary. Leggett shows that depairing effects weakly align $\vec{n}$ parallel to an applied field $\vec{H}_0$. We feel that similar effects will weakly align $\vec{n}$ perpendicular to surfaces. One also expects a longitudinal resonance in a BW state with

$$\Omega_{L\,\mathrm{BW}}^2=\tfrac{5}{2}(\chi/\chi_0)(\Delta_{\mathrm{BW}}^2/\Delta_{\mathrm{ABM}})\Omega_{L\,\mathrm{ABM}}^2,$$

where χ/χ_0 is the ratio of the Fermi liquid to the B-phase susceptibility, and the Δ's are the average gaps in the two phases. If $\vec{n}$ is not along $\vec{H}_0$, one expects a shift in the B-phase transverse resonance, which for large fields yields $\Omega_T^2 = (\gamma H_0)^2 + \Omega_{L\,\mathrm{BW}}^2 \sin^2\theta$, where θ is the angle between $\vec{n}$ and $\vec{H}_0$.

In longitudinal-resonance experiments at frequencies of 228 and 240 kHz with an applied field of over 500 Oe we have observed an abrupt decrease in the rf voltage across our tank circuit as the supercooled A-phase liquid snaps over to the B phase indicating absorption in the B phase. This absorption was very broad, however, and no obvious peak was observed. The absorption was clearly absent, however, when the same experiment was performed at fields below 100 Oe.

In Fig. 2 we show transverse NMR signals of the Fermi liquid (dashed line) and B-phase liquid (solid line) obtained in a field of 280 Oe just below $T(B')$. We have accurately subtracted a small solid signal from each trace for clarity. (The linewidth was artificially broadened slightly to facilitate the subtraction.) Notice that the B-liquid peak is quite small, and shifted upward in frequency by about 14 Hz at the peak and about 30 Hz at the low-frequency edge. Notice also the long high-frequency tail on the B-liquid curve. In some runs a "trough" was observed in the B-liquid profile (cf. Osheroff *et al.*[7]) corresponding to missing susceptibility. When this was seen, we observed additional absorption in the high-frequency tail. Such "domain effects" could be annealed out of the liquid by warming until the A-B phase boundary passed below the NMR region, and then cooling back before the B phase was entirely destroyed.

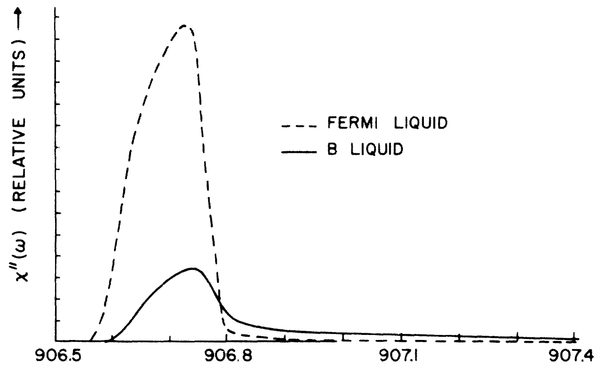

FIG. 2. Comparison of the Fermi- and B-liquid transverse NMR absorption signals showing the B-liquid high-frequency tail.

By increasing the magnetic field at constant pressure, the peak height of the B-liquid absorption signal could be caused to increase dramatically in size. Shown in Fig. 3 are pressure contours of the B-liquid absorption signals normalized to the Fermi-liquid signal, $\chi_B''(\omega_0)/\chi_{\mathrm{F\,liq}}''(\omega_0)$, where ω_0 is the frequency of the maximum Fermi-liquid absorption level. These contours clearly approach zero at nonzero frequencies, and saturate at sufficiently high frequencies. Care must be taken not to extrapolate the curves shown to infinite frequency, however, as a variable fraction of the shifted absorption signal is included in each B-liquid measurement. Since the B-liquid linewidths were always broader than the corresponding Fermi-liquid linewidths, we take these contours as lower limits of the actual ratio $\chi_B/\chi_{\mathrm{F\,liq}}$. Even so, at the highest frequency we measure $\chi_B''(\omega_0)/\chi_{\mathrm{F\,liq}}''(\omega_0) \approx 0.26$ at our highest pressure $[P - P(A) \approx 52.5$ mbar$]$ which probably corresponds to a temperature near 1.0 to 1.3 mK. This is inconsistent with $l = 0$ or $l = 2$ pairing for which much smaller values are expected. Also, the extrapolation of the highest-frequency data to the pressure of the B' transition in zero field yields a value of $\chi_B''(\omega_0)/\chi_{\mathrm{F\,liq}}''(\omega_0)$ of ≈ 0.47, considerably higher than the comparable static magnetization measurements reported by Paulson, Johnson, and Wheatley[8] at the equivalent temperature $(T/T_c \approx 0.8)$. In measurements at 3.7 MHz in which we include more of the shifted B-liquid sig-

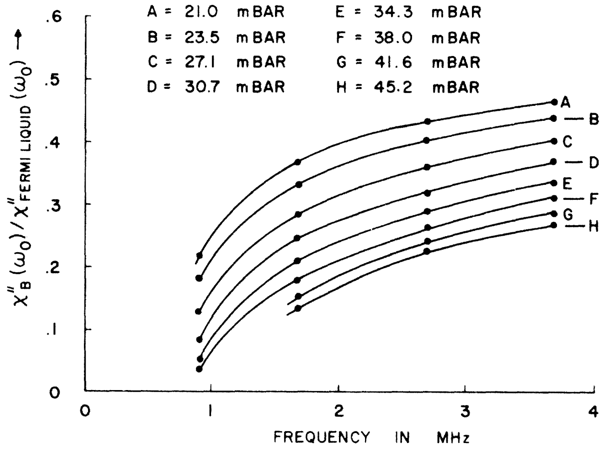

FIG. 3. Pressure contours of the B-liquid absorption signal normalized to the Fermi-liquid absorption signal, $\chi_B''(\omega_0)/\chi_{\mathrm{F\,liq}}''(\omega_0)$, where ω_0 is the frequency at which $\chi_{\mathrm{F\,liq}}''$ is maximum, as a function of resonant frequency. Pressure differences are $P - P(A)$.

nal we obtain values of $\chi_B''(\omega_0)/\chi_{\mathrm{F\,liq}}''(\omega_0)$ as high as 0.51, and consider it likely that at higher fields our measurements would be consistent with a BW curve properly corrected for all Fermi-liquid effects.

Our estimates of the surface pinning, bending, and field orientational energies in the BW state show that the surface and bending energies dominate the field energy below about 100 Oe and we therefore expect that θ, the angle between $\vec{n}$ and $\vec{H}_0$ discussed above, should increase in a gentle arc from zero at the center of the bore to some fraction of $\frac{1}{2}\pi$ at the walls. Such a model is entirely consistent with all our observations, including the broad absorption in the B-longitudinal-resonance experiments and the smeared shape of the transverse-B-resonance signal.

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Thickness of the Moving Superfluid Film at Temperatures below 1°K*

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The thickness of a flowing He II film has been measured at temperatures below 1°K. The film's thickness varies quadratically with the flow velocity, in agreement with theory. The implications of this observation are discussed with regard to similar measurements performed at higher temperatures.

In recent years attempts have been made to explain an apparent violation of basic fluid mechanics applied to a moving superfluid film. This paper presents experimental results which resolve this problem. In the first part of this Letter we briefly describe the essence of the problem. Then we present our experiment and finally make some comments on the consequences of the observed phenomena.

In 1956 Kontorovich[1] showed that in thermodynamic equilibrium, the energy terms describing a flowing film of He II are linked by the equation

$$\rho_s v_s^2/2\rho + gz - \beta d^{-n} = 0, \tag{1}$$

where ρ_s/ρ is the superfluid fraction, v_s is the superfluid velocity, g is the gravitational acceleration, z is the height of the film above the bulk liquid's surface, n is a constant approximately equal to 3, β is a constant describing the strength of the Van der Waals interaction between the substrate and the film, and d is the film thickness. This equation implies that a flowing film should be thinner than a static film. In particular for v_s not too large the film gets thinner by the amount

$$\Delta d = \rho_s d_0 v_s^2/2n\rho g z, \tag{2}$$

where d_0 is the thickness of the static film. Although several experimenters[2,3] looked for this thinning effect it was not observed, and finally Keller[4] in an elegant experiment conclusively showed that the effect was not present (at temperatures $T > 1.12$°K). Several ingenious suggestions have been made to explain Keller's experiment.[5-8] The most convincing explanation points out that the prediction of Eq. (2) leads to an instability between the vapor and the liquid at the free surface.[8] If the film gets thinner the vapor at the interface must be at a pressure greater than the saturated vapor pressure. As