Methods in the Theory of Nonlinear Oscillations (Gordon and Breach, New York, 1962).
[6]M. Nambu and T. Tamao, to be published.
[7]T. Tamao, Phys. Fluids 12, 1458 (1969).
[8]T. H. Dupree, Phys. Fluids 11, 1049 (1968).
[9]V. M. Vasyliunas, J. Geophys. Res. 73, 2839 (1968).
[10]C. S. Wu, Phys. Fluids 11, 683 (1968); G. Bateman, Phys. Fluids 15, 2295 (1972).

Observations of Ion-Acoustic Cylindrical Solitons

Noah Hershkowitz and Thomas Romesser*
Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa 52242
(Received 24 January 1974)

Cylindrical solitons are seen to evolve from compressive cylindrical pulses in a collisionless plasma. The properties of these solitons are found to be consistent with the known properties of one- and three-dimensional solitons.

Soliton solutions are now well known for at least seven distinct *one*-dimensional wave systems.[1] In particular, the soliton solutions of the Korteweg–de Vries (KdV) equation have been extensively studied both theoretically and experimentally during the last decade and KdV is now known to approximately describe many systems which include nonlinear and dispersive effects.[1] Washimi and Taniuti[2] have shown that slightly nonlinear one-dimensional ion acoustic waves in collisionless plasmas with cold ions are described by KdV. In a recent Letter, Maxon and Viecelli,[3] following the procedure of Ref. 2, have derived a modified KdV equation for spherically symmetric ingoing waves. In this Letter we present experimental observations of cylindrical solitons in a collisionless plasma.

One-dimensional solitons have several distinguishing characteristics.[4-11] Among them are the following: (1) Arbitrary positive (compressive) density perturbations evolve after sufficient time into a superposition of spatially separated solitons (solitary pulses). (2) The number and amplitude of the solitons is determined by the solution of an appropriate time-independent Schrödinger equation with a potential well that is proportional to the initial spatial density perturbation. One soliton is formed for each bound state with soliton amplitude proportional to the energy eigenvalues. (3) The soliton velocity is given by $[1+\frac{1}{3}(\delta n/n)]c_s$, where $\delta n/n$ is the maximum density perturbation of each soliton and c_s is the ion acoustic velocity. (4) The spatial widths are proportional to $(\delta n/n)^{-1/2}$, which implies that the product of the square root of the maximum soliton amplitude multiplied by the width is a constant. (5) Solitons retain their identity upon collision with other solitons.

All of these properties have recently been verified experimentally with collisionless plasmas. Linear double-plasma (DP) devices were used by Ikezi, Taylor, and Baker[12] to verify all but the second property, and by Hershkowitz, Romesser, and Montgomery[13] to verify the connection with the underlying Schrödinger equation [property (2)]. Cohn and MacKenzie[14] investigated solitons resulting from very large density perturbations produced by photoionization. A summary of much of the experimental evidence has been given by Ikezi.[15]

In the first work, which considers solitons of dimensionality greater than one, Maxon and Viecelli[3] have numerically determined that spherical solitons have the following four properties. First, an ingoing soliton increases in amplitude while decreasing in width, thus retaining its identity as a *single* soliton. Second, the product of the square root of the maximum soliton amplitude multiplied by the width is a constant. Third, a small residue develops and moves inward behind the soliton, taking up a measurable percentage of the total momentum; and fourth, the soliton velocity is somewhat greater than the velocity of a corresponding one-dimensional soliton.

In this Letter we present data showing that cylindrical solitonlike objects exist and that their properties are consistent with those of one- and three-dimensional solitons. These results are to our knowledge the first experimental evidence for solitons of dimensionality greater than 1.

Experiments were carried out using a cylindrical DP device which had previously been used to study the ion-ion beam instability of cylindrical beams and background plasma.[16] Two concentric cylindrical plasmas (length 30 cm) are separated by two closely spaced, fine-mesh, concentric cylindrical screens with inner screen diameter equal to 20 cm. The outer screen is negatively biased to prevent the flow of electrons between the plasmas, and the inner screen is grounded. The ion density was approximately 10^9 cm^{-3} and the ion and electron temperatures were approximately 0.2 and 3 eV, respectively. Positive half-sine-wave pulses are applied to the outer plasma to launch cylindrical density perturbations in the inner plasma.

Signals are detected by a positively biased Langmuir probe which has variable radial position. No azimuthal dependence was observed. Figure 1(a) shows the perturbed electron number density as a function of time at several radial positions for both large and small initial density perturbations. For the small-amplitude pulse at $r = 9$ cm we can identify an ingoing pulse, which is quite similar to the applied pulse, followed at a later time by a similar outgoing pulse that has propagated from the opposite side and through the center. As the probe is moved closer to the center the ingoing and outgoing pulses approach each other, merging at the center. For the large-amplitude compressive pulse at $r = 9$ cm the ingoing pulse is seen to be similar to the applied pulse, but three solitons can be identified in the outgoing pulse. The traces at other radial positions indicate how the initial density perturbation evolves into the solitons. The increased velocity of the first two solitons compared to the ion acoustic velocity is evident. Once formed, the largest soliton is seen to be much narrower than the applied pulse. We find that the average velocity of the largest ingoing soliton is approximately $1.17c_s$. The application of a negative (rarefactive) density perturbation is not found to evolve into solitons.

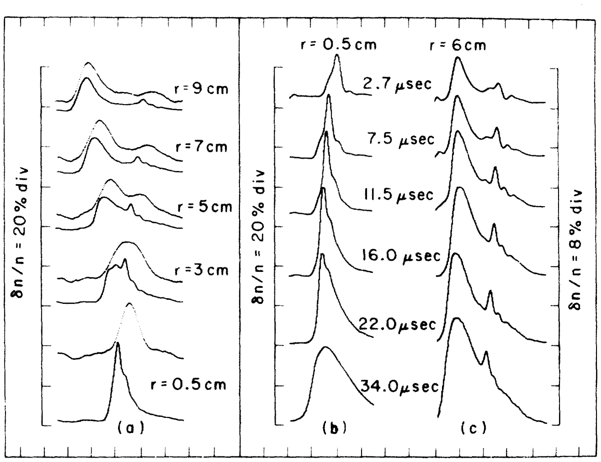

FIG. 1. (a) Perturbed electron number density as a function of time at several radial positions. Upper traces, linear ($\delta n/n < 1\%$) ion acoustic pulses (with amplitude adjusted for comparison). Lower traces, nonlinear pulses propagating, steepening, and breaking into solitons. (The received signals are digitized and stored on magnetic tape for later analysis. This is the cause of the observed steplike structure.) (b) Perturbed electron number density detected at $r = 0.5$ cm labeled by the applied pulse widths. (c) Received signals at $r = 6$ cm labeled by applied pulse width.

As in the one-dimensional DP device, the maximum applied voltage is limited by the electron temperature.[13] For applied voltages larger than the electron temperature (here ≈3 eV), particle bursts (pseudowaves) are detected. Data were taken with the largest initial density perturba- that could be obtained without launching pseudowaves. The width of the applied pulse was then varied to determine how the soliton number depended on the initial density perturbation (see Fig. 1). This procedure was identical to that followed in our earlier measurements (Ref. 13). Figure 1(b) shows how the signal received, $r = 0.5$ cm, depends on applied pulse width. *Widening* the applied pulse results in increased amplitude, decreased width, and increased velocity in the received signal. In the top trace we see one well-defined soliton. In the second trace the first soliton has grown and narrowed and a second soliton is apparent. In the third trace the first two solitons have grown, narrowed, and speeded up. A third soliton is barely apparent. We find that the square root of the maximum amplitude multiplied by the width is constant to within 10% for the first four traces. In the fourth trace the third soliton is seen to grow as well. For further increases in width the solitons no longer have sufficient time to separate from the initial perturbation. Figure 1(c) shows the signals corresponding to the same six applied pulse widths as detected at $r = 6$ cm.

For small applied pulse width it is possible to launch single solitons whose amplitude depends on the applied pulse width. The amplitude of an incoming soliton was found to be approximately constant over much of its trajectory as a result of a competition between damping and geometric growth (see Fig. 1). This simplified the determination of the amplitude dependence of the velocity. Figure 2 shows the soliton velocities, determined from individual soliton trajectories,

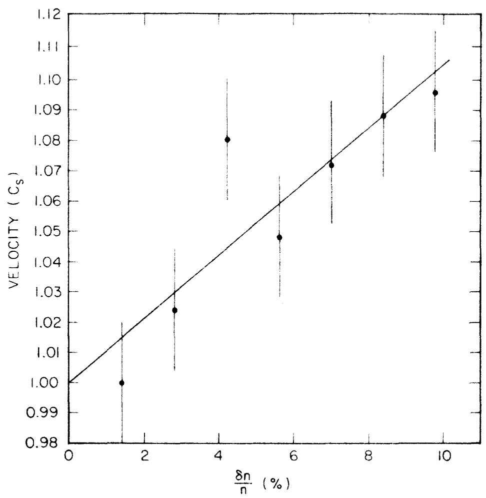

FIG. 2. Velocity of single solitons as a function of the maximum soliton amplitude.

versus soliton amplitude. The best least-squares fit to these data gives $v = [1 + a(\delta n/n)]c_s$, where $a = 1.05 \pm 0.20$. This is faster than a corresponding one-dimensional soliton.

This experiment differs from an idealized one in at least two respects. First, as in one-dimensional experiments, damping is present. After accounting for a geometric increase which goes like $(r_0/r)^{1/2}$, the ion acoustic pulse is seen to damp by about a factor of 3 in propagating 9 cm, and the soliton damps by a factor of 1.5. In the absence of damping, the geometrical growth factor (r_0/r) should only be expected to hold for linear ion acoustic pulses. Second, the density perturbation is not found to diverge at the center. We attribute this result in part to broadening of the received pulse as a result of variations in the radius of the cylindrical screens of the order of 0.3 cm and to finite size of the probe and of an insulating glass cylinder (0.5 cm) which covers all but the last centimeter of the probe.

We summarize the measured properties of *two*-dimensional solitons. Compressive density perturbations evolve into solitons. The number of the solitons is determined by the width and amplitude of the applied pulse. Rarefactive perturbations do not evolve into solitons. The solitons retain their identity after converging (colliding) at the center. All of these are well-known properties of *one*-dimensional solitons. In addition we find that the soliton width multiplied by the square root of the maximum soliton amplitude is approximately constant even though the amplitude, width, and velocity are functions of time. The velocity is somewhat greater than the velocity of a corresponding one-dimensional soliton. Both of these properties of three-dimensional solitons and the first holds for one-dimensional solitons as well.

Maxon has recently derived a modified KdV equation for cylindrical solitons.[17] Detailed comparison with numerical solutions of this equation will be presented in a later publication. Attempts will be made to compare the amplitude, width, and propagation speed as a function of time with numerical solutions of a modified KdV equation for cylindrical solitons which includes damping when such results become available.

We thank S. Maxon for helpful discussions and Alfred Scheller for construction of much of the apparatus.

*Work supported in part by the National Aeronautics and Space Administration under Grant No. NGL-16-001-043.

[1]A. C. Scott, F. Y. F. Chu, and D. W. McLaughlin, Proc. IEEE 61, 1443 (1973).

[2]H. Washimi and T. Taniuti, Phys. Rev. Lett. 17, 996 (1966).

[3]S. Maxon and J. Viecelli, Phys. Rev. Lett. 32, 4 (1974).

[4]N. J. Zabusky and M. D. Kruskal, Phys. Rev. Lett. 15, 240 (1965).

[5]N. J. Zabusky, Phys. Rev. 168, 124 (1968).

[6]C. S. Gardner, J. M. Greene, M. D. Kruskal, and R. M. Miura, Phys. Rev. Lett. 19, 1095 (1967).

[7]V. I. Karpman, Phys. Lett. 25A, 708 (1967).

[8]Yu. A. Beresin and V. I. Karpman, Zh. Eksp. Teor. Fiz. 46, 1880 (1964), and 51, 1557 (1966) [Sov. Phys. JETP 19, 1265 (1964), and 24, 1049 (1967)].

[9]R. Miura, J. Math. Phys. (N.Y.) 9, 1202 (1968); R. Miura, C. S. Gardner, and M. D. Kruskal, *ibid.* 1204 (1968); C. H. Su and C. S. Gardner, *ibid.* 10, 536 (1969); C. S. Gardner, *ibid.* 12, 1548 (1971); M. D. Kruskal, R. M. Miura, C. S. Gardner, and N. J. Zabusky, *ibid.* 11, 952 (1970).

[10]F. D. Tappert and N. J. Zabusky, Phys. Rev. Lett. 27, 1774 (1971).

[11]M. Wadati and M. Toda, J. Phys. Soc. Jap. 32, 1403 (1972).

[12]H. Ikezi, R. J. Taylor, and D. R. Baker, Phys. Rev. Lett. 25, 11 (1970).

[13]N. Hershkowitz, T. Romesser, and D. Montgomery, Phys. Rev. Lett. 29, 1586 (1972).

[14]D. B. Cohn and K. R. MacKenzie, Phys. Rev. Lett. 30, 258 (1973).

[15]H. Ikezi, Phys. Fluids 16, 1668 (1973).

[16]N. Hershkowitz and T. Romesser, Bull. Amer. Phys. Soc. 18, 1260 (1973).

[17]S. Maxon, private communication.