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New Class of Renormalizable Vector-Meson Theories?*

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(Received 2 November 1973)

We propose a new class of apparently renormalizable heavy–vector-boson theories. These models are spontaneously broken gauge theories, modifed by the addition of arbitrary mass terms for vectors associated with invariant Abelian subgroups. The vacuum is *not* invariant under these subgroups. Such theories are probably renormalizable since their multiparticle S-matrix elements are unitarily bounded in the tree approximation. As illustrations, the Higgs [U(1)] and Weinberg gauge theories are modified in this way.

The only known renormalizable systems of heavy vector bosons are either spontaneously broken gauge theories (SBGT) or "conserved current" models. In an SBGT[1] the field variables can always be chosen so that the Lagrangian is locally gauge invariant. In the language of these field variables spontaneous symmetry breaking is the origin of the vector-boson masses. Massless vector mesons have conserved source currents. On the other hand, "conserved current" models always contain at least one massive vector boson, whose source current is conserved. Massive quantum electrodynamics is the simplest system of this type. The general prescription for constructing conserved current models can be stated as follows: (1) Begin with a Lagrangian which is invariant under a nonsemisimple group of local gauge transformations (i.e., a group of transformations containing an invariant Abelian subgroup). (2) Arrange for spontaneous symmetry breaking (if any) such that the vacuum expectation value of the scalar field is invariant under at least one invariant (single-parameter) Abelian subgroup (thus, at this stage the corresponding Abelian vector is massless and coupled to a conserved current). (3) Add (in the R gauge) an arbitrary mass term for the same Abelian vector. Notice that the resulting Lagrangian is not locally invariant under Abelian gauge transformations.

This paper suggests that there is a third class of heavy–vector-boson interactions which may be renormalizable.[2] The models of this new class are constructed according to the above three-part prescription,[3] except that in step (2) spontaneous symmetry breaking is arranged so that the vacuum expectation value of the scalar field is *not* invariant under at least one invariant (single-parameter) Abelian subgroup [thus, the corresponding Abelian vector, possibly in linear combination with other vectors, would acquire a mass at step (2)]. Models of this type are different from SBGT systems, since the R-gauge Lagrangian is not locally gauge invariant under the entire group; they differ from conserved-current theories since, in general, there is no massive vector boson with a conserved source current. Instead, these models are "hybrid" systems which interpolate between the SBGT and conserved-current theories: In the limit in which the added Abelian vector mass term vanishes, an SBGT is recovered; on the other hand, as the scalar-field vacuum expectation value is altered so that it becomes invariant under the Abelian subgroup in question, the "hybrid" model becomes a conserved-current theory. The "hybrid" theories are probably renormalizable since we can show that they are "tree unitary" (i.e., unitarily bounded in the tree approximation). Specifically, because couplingconstant relations force the cancelation of bad high-energy behavior, the N-particle S-matrix elements diverge no more rapidly than E^{4-N} in the high-energy limit (i.e., in the limit in which all angles are fixed and the overall energy scale E increases to infinity). In the following these features are illustrated by constructing hybrid versions of the Higgs[4] [U(1)] and Weinberg[5] [SU(2) $\otimes$ U(1)] SBGT's. In both cases the final particle spectrum contains only massive vector bosons.

Consider the following Lagrangian, constructed from a real vector field A_μ and a two-dimensional array of real scalar fields Π:

$$\mathcal{L} = -\tfrac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \tfrac{1}{2}M_0^{\,2}A_\mu^{\,2} + \tfrac{1}{2}(\partial_\mu \Pi + igA_\mu D\Pi)^2 - h(\Pi^2 - f)^2. \tag{1}$$

Here, D is a two-dimensional, imaginary, antisymmetric matrix with $D^2 = 1$. We take $h>0$ in order to guarantee stability and $f \equiv \lambda^2 > 0$ in order to have $\langle \Pi \rangle_0 \neq 0$. If M_0 were zero, Eq. (1) would describe the usual Higgs U(1) SBGT; however, we are interested in studying the "hybrid" case, $M_0 > 0$. Notice that the Lagrangian is not locally gauge invariant; but it is invariant under the global U(1) transformations, $\Pi \to e^{igD\Lambda}\Pi$, where Λ is a real constant. Since $\langle \Pi \rangle_0 \neq 0$, this symmetry is spontaneously broken. It is convenient to define a set of new real field variables, φ_1, φ_2, and W_μ:

$$\Pi \equiv \exp\left(\frac{ig^2\lambda\varphi_1 D}{MM_0}\right)\begin{bmatrix}\lambda + \varphi_2 \\ (M_0/M)\varphi_1\end{bmatrix}, \quad W_\mu \equiv A_\mu + \frac{g\lambda}{MM_0}\partial_\mu \varphi_1,$$

where $M \equiv (M_0^{\,2} + g^2\lambda^2)^{1/2}$. In terms of these field variables, the Lagrangian is

$$\mathcal{L} = -\tfrac{1}{4}(\partial_\mu W_\nu - \partial_\nu W_\mu)^2 + \tfrac{1}{2}M^2 W_\mu^{\,2} + \tfrac{1}{2}(\partial_\mu\varphi_1)^2 + \tfrac{1}{2}(\partial_\mu\varphi_2)^2 + g^2\lambda W_\mu^{\,2}\varphi_2 + (gM_0/M)W_\mu(\varphi_2\partial^\mu\varphi_1 - \varphi_1\partial^\mu\varphi_2)$$
$$+\tfrac{1}{2}g^2 W_\mu^{\,2}[(M_0^{\,2}/M^2)\varphi_1^{\,2} + \varphi_2^{\,2}] - h[2\lambda\varphi_2 + (M_0^{\,2}/M^2)\varphi_1^{\,2} + \varphi_2^{\,2}]^2. \tag{2}$$

Since the change of variables is "canonical," the Lagrangians in Eqs. (1) and (2) have identical S matrices.[6]

Notice that in terms of either set of field variables there are three massive vector modes and two scalar modes; the field A_μ was massive from the beginning and was not able to "soak up" a scalar mode. The most important feature of this model is that all S-matrix elements in the tree approximation are unitarily bounded at high energy.[7] This type of gentle high-energy behavior suggests that the theory is renormalizable. The Lagrangian also has interesting symmetry properties. It is invariant under global transformations which have the infinitesimal form

$$\delta\varphi_1 = \frac{g\Lambda M}{M_0}(\lambda + \varphi_2), \quad \delta\varphi_2 = \frac{-g\Lambda M_0}{M}\varphi_1, \quad \delta W_\mu = \frac{g^2\lambda\Lambda}{M_0^{\,2}}\partial_\mu\varphi_2,$$

where Λ is an infinitesimal constant. The corresponding conserved Noether's current, J_μ, is

$$J^\mu = \frac{-gM_0}{M}\varphi_1\left(\partial^\mu\varphi_2 - \frac{gM_0}{M}\varphi_1 W^\mu\right) + \frac{gM}{M_0}(\varphi_2 + \lambda)\left(\partial^\mu\varphi_1 + \frac{gM_0}{M}\varphi_2 W^\mu\right) - \frac{g^2\lambda}{M_0^{\,2}}\partial_\nu\varphi_2(\partial^\mu W^\nu - \partial^\nu W^\mu). \tag{3}$$

The associated symmetry is realized dynamically, since J_μ couples to the massless (Goldstone) boson φ_1. Note that this conserved current is not the source current of W_μ.

The above hybrid model smoothly connects the Higgs U(1) SBGT and a conserved-current theory (massive-photon scalar electrodynamics). We have already observed that the original Lagrangian in Eq. (1) becomes the R-gauge Higgs model as $M_0 \to 0$ with $\lambda \neq 0$. It is easy to check that in the same limit Eq. (2) describes the U-gauge Higgs Lagrangian. As $M_0 \to 0$, the Goldstone boson field φ_1 becomes free, leaving an interacting (Higgs) theory of one massive vector and one scalar. In the same limit the conserved current J^μ is ill defined; this is expected since the Higgs model has no conserved current. As $\lambda \to 0$ with $M_0 \neq 0$, the original Lagrangian of Eq. (1) becomes the "electrodynamics" of massive "photons" and massless scalars. The Lagrangian of Eq. (2) reaches the same limit. In this limit, the massless scalar φ_1 remains coupled to other fields, but it decouples from J_μ. Therefore, J_μ changes into a conserved current which is associated with an algebraic symmetry of the S matrix. In fact, J_μ becomes the "electromagnetic" current which is the source current of the massive "photon."

As another example, consider the Lagrangian constructed from four real vector fields $A_{a\mu}$ and a two-dimensional array of complex scalar fields Π:

$$\mathcal{L} = -\tfrac{1}{4}(\partial_\mu A_{a\nu} - \partial_\nu A_{a\mu} - d_{abc}A_{b\mu}A_{c\nu})^2 + \tfrac{1}{2}M_0{}^2 A_{4\mu}{}^2 + \tfrac{1}{2}|(\partial_\mu - \tfrac{1}{2}ig\vec{\tau}\cdot\vec{A}_\mu + i\tfrac{1}{2}g'A_{4\mu})\Pi|^2 - h(\Pi^\dagger\Pi - f)^2. \tag{4}$$

Here, the structure constants d_{abc} are zero when any index is 4 and are otherwise equal to $-g\epsilon_{abc}$. As before, we take $h>0$ to assure stability and $f\equiv\lambda^2>0$ in order to have $\langle\Pi\rangle_0\neq 0$. If M_0 were zero, Eq. (4) would describe the Weinberg SU(2)⊗U(1) SBGT with no fermions; instead, we study the hybrid case, $M_0>0$. Notice that $\mathcal{L}$ is invariant under the global gauge transformations,

$$\Pi \to \exp(-\tfrac{1}{2}ig\vec{\tau}\cdot\vec{\Lambda} + \tfrac{1}{2}ig'\Lambda_4)\Pi, \quad A_{a\mu}\to[\exp(-i\Lambda_e t_e)]_{ab}A_{b\mu},$$

where $(t_e)_{bc}\equiv id_{ebc}$ and Λ_e are four real constants. Since $\langle\Pi\rangle_0\neq 0$, this symmetry is spontaneously broken down to the U(1) subgroup which leaves $\langle\Pi\rangle_0$ invariant. In order to simplify calculations we adopt the following notation: First, choose $\langle\Pi\rangle_0 = \binom{\lambda}{0}$. Then, the mass matrix, which characterizes the quadratic vector field terms in L, is $M_{ab}{}^2$:

$$M_{11}{}^2 = M_{22}{}^2 = M_{33}{}^2 = \tfrac{1}{4}g^2\lambda^2, \quad M_{44}{}^2 = M_0{}^2 + \tfrac{1}{4}g'^2\lambda^2, \quad M_{34}{}^2 = M_{43}{}^2 = -\tfrac{1}{4}gg'\lambda^2;$$

all other $M_{ab}{}^2=0$. This matrix can be diagonalized by an orthogonal matrix R which is a rotation in the "3-4 plane": $(R^{-1}M^2R)_{ab} = M_a{}^2\delta_{ab}$. The eigenvalues $M_a{}^2$ must be greater than zero and satisfy the trace and determinant conditions

$$M_1{}^2 = M_2{}^2 = \tfrac{1}{4}g^2\lambda^2, \quad M_3{}^2 + M_4{}^2 = M_0{}^2 + \tfrac{1}{4}\lambda^2(g^2+g'^2), \quad M_3{}^2M_4{}^2 = \tfrac{1}{4}g^2\lambda^2M_0{}^2. \tag{5}$$

We can always solve these equations for R_{ab} and $M_a{}^2$ as functions of g, g', λ, and M_0; however, the main features of this model can be understood without doing this. The next step is to define a new set of real field variables, ρ, θ_i $(i=1,2,3)$, and $W_{a\mu}$:

$$\Pi \equiv \exp(\tfrac{1}{2}ig\vec{\tau}\cdot\vec{\theta})\begin{pmatrix}\lambda+\rho\\ 0\end{pmatrix}, \quad R_{ab}W_{b\mu} \equiv (e^{-i\vec{\theta}\cdot\vec{t}})_{ab}A_{b\mu} - [(1-e^{-i\vec{\theta}\cdot\vec{t}})/i\vec{\theta}\cdot\vec{t}]_{ai}\,\partial_\mu\theta_i.$$

In terms of these variables the Lagrangian is

$$\mathcal{L} = -\tfrac{1}{4}(\partial_\mu W_{a\nu} - \partial_\nu W_{a\mu} - C_{abc}W_{b\mu}W_{c\nu})^2 + \tfrac{1}{2}M_a{}^2W_{a\mu}{}^2 + \tfrac{1}{2}(\partial_\mu\rho)^2 + \tfrac{1}{2}\lambda^{-2}W_{a\mu}W_b{}^\mu(2\lambda\rho+\rho^2)(M_a{}^2\delta_{ab} - M_0{}^2R_{4a}R_{4b}) - h(2\lambda\rho+\rho^2)^2, \tag{6}$$

where $C_{abc}\equiv d_{a'b'c'}R_{a'a}R_{b'b}R_{c'c}$. Once again, since the change of variables is "canonical," the Lagrangians in Eqs. (4) and (6) have identical S matrices.[6] In terms of the old variables, $A_{a\mu}$ and Π, there were three "massless" vector fields, one massive vector field, and four real scalar fields. In terms of the new variables there are four massive vector fields and one massive scalar; the three θ_i fields became longitudinal modes of vectors and do not appear in Eq. (6).

As before, all N-particle scattering amplitudes in this model are unitarily bounded in the tree approximation.[7] This supports our conjecture that the model is renormalizable. Even though all the vector bosons are massive the Lagrangian has a residual symmetry; it is invariant under $W_{(1\pm i2)\mu}\to e^{\pm i\Lambda}W_{(1\pm i2)\mu}$. The corresponding conserved Noether's current J_μ generates an algebraic symmetry of the S matrix. However, it is not the source current of any single massive vector field; instead, $W_{3\mu}$ and $W_{4\mu}$ couple to linear combinations of J^μ and another (nonconserved) current. Note that the model has no massless vector bosons but has a conserved charge which is carried by two of the massive vectors and their currents.

If $M_0=0$ and $\lambda\neq 0$, the original Lagrangian describes the Weinberg SU(2)⊗U(1) SBGT (with no fermions) in the R gauge. Similarly, as $M_0\to 0$, the Lagrangian in Eq. (6) becomes the Weinberg model in the U gauge. Equation (5) shows that one vector boson becomes a massless photon in that limit. It can be demonstrated from the equations of motion that the corresponding vector field becomes coupled only to J^μ, which is the electromagnetic current of Weinberg's model.

These hybrid theories were not discovered by accident. We are now completing a systematic search for tree-unitary systems of heavy vector mesons,[8] which are described by a Lagrangian of mass dimension less than or equal to 4. At the end of that search the hybrid, SBGT, and conserved current models appear as the only tree-unitary theories.[9] A dimensional argument makes it plausible that any renormalizable theory has to be tree unitary. Our result then indicates strongly that the above three types of models represent the only renormalizable heavy-vector-boson theories.

If the hybrid models are truly renormalizable, there are several possibilities to be explored: (1) "Hybridization" might be a useful way of regulating the infrared behavior of massless vectors in ordinary SBGT's. For instance, in the usual Weinberg model the photon can be given a mass according to the hybrid prescription without destroying renormalizability. (2) Are the hybrid theories asymptotically free[10] and thereby suitable for describing the strong interactions? (3) So far, we have discussed these models in the tree approximation; the one-loop solutions might be qualitatively different. For example, consider the system in Eq. (1) at the point $f=0$, $M_0^2>0$. The tree-approximation solution describes the "electrodynamics" of a massive "photon" and two massless scalars. An analogy with the work of Coleman and Weinberg[11] suggests the existence of one-loop solutions at the same point, which resemble hybrid models with one massive vector, one massive scalar, and one massless (Goldstone) scalar coupled to a conserved current. (4) It is possible to construct hybrid theories (e.g., the hybrid Weinberg model) with a conserved charge which is carried by vector bosons and their currents and which is not coupled to a massless vector. Are there realistic models in which that conserved charge is muon number, lepton number, or baryon number?

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*Work supported in part by the National Science Foundation.

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