and the slab ~20 MeV thick, the geometrical efficiency for electrons >10 MeV depositing at least 10 MeV in a detector element was ~0.8. The 10-MeV threshold resulted in ~30% loss of decay electrons and the finite oscilloscope display time further reduced the detection efficiency by ~40%. Combining these factors yields an overall decay-electron detector efficiency

$$\eta = 0.8\times0.7\times0.6 = 0.34.$$

The limit on the nucleon decay rate $\tau_N$ is obtained from

$$\tau_N = \frac{N}{dN/dt} = \frac{1.1\times10^{31}\times0.34}{5/2.6} = 1.9\times10^{30}\text{ yr}.$$

The number of decays seen is not inconsistent with that expected from $\nu$-produced muons, so that it is reasonable to state the limit for decay modes which result in muons as

$$\tau > 2\times10^{30}\text{ yr}.$$

It is useful in planning an improved experiment to contemplate the rate associated with the present equipment,

$$R = \frac{5}{20\times2.6\times0.34} = 0.3/\text{ton yr}.$$

The overall detector efficiency can be raised to ~100%, and any case of $\pi\to\mu$ decay observed as well, by employing a detector of linear dimensions larger than the range of the charged nucleon decay products sought. Such a detector with 100 tons of scintillator would enable an order-of-magnitude increase in sensitivity to nucleon decay. Further it should be possible from the total energy associated with the first pulse (or pulses) of the delayed coincidence to discriminate to some extent against the neutrino-produced muons and in favor of the lower-energy muons expected from nucleon decay.

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# Electron's Anomalous Moment and Its Spin-Precession Frequency Shift

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We derive the anomalous moment of the electron from the quantum-electrodynamic equations of motion for the electron spin precession.

The anomalous moment of the electron[1] occupies a special place in quantum electrodynamics. Unlike the Lamb shift, it is a property of the free electron interacting only with the electromagnetic field and uncomplicated by problems of binding. Despite this, and despite the excellent agreement between the theoretical predictions and experimental determinations of the moment anomaly, it is widely remarked[2] that the usual methods of perturbative calculation do not provide clear physical insights into the origin of the anomaly or allow an intuitive understanding of even the sign of the lowest-order correction to the moment.

Existing quantitative treatments[1,3] of the moment anomaly are almost universally based on a point of view which has elementary electron-photon scattering events at the root of the effect. As an alternative, it is possible to construct a qualitative and intuitive treatment by considering the effect of vacuum magnetic field fluctuations on the electron spin interaction $\mu\vec{\sigma}\cdot\vec{B}$. Unfortunately, it has been long known[2a] that the result will not even have the correct sign.[1]

Recently, a number of new attempts have been made[5] to overcome these difficulties. These have been unsatisfactory for a variety of reasons including the introduction of *ad hoc* correlation functions, and the suggestion that new renormalization techniques may be needed.

Nevertheless, in this recent work a point of view has been emphasized which is attractive enough to study carefully. It has been proposed[5,6a] that an intuitive understanding of the moment correction follows directly from viewing the basic electron-photon interaction as an example of a *resonance precession* effect. Such a viewpoint is even suggested by the techniques that are currently used to measure the $g$-factor anomaly (Ref. 1a, Sect. III). In this view a quantitative estimate of the anomaly would follow from a calculation of the radiative frequency shift of the resonant spin-flip transition.

At first glance the resonance precession picture is extremely simple. Qualitatively, the electron spin precesses in an applied field $B_0$ at the frequency $\omega_0 = 2(eB_0/2mc)$, and generates magnetic dipole radiation as it flips over into the lower spin state. This radiation itself interacts with the spin, the effect of the radiation reaction being to alter slightly the natural precession frequency to $\omega_0' = g(eB_0/2mc)$. In the limit $B_0 \to 0$, the ratio $a_e = (g-2)/2$ is termed the $g$-factor anomaly.

However, experience with other radiative frequency shifts such as the Lamb shift dictates a certain amount of caution. Virtual transitions to nearby nonresonant levels can be as important as the real resonant transition in determining the size of the shift. In the present case it is very simple to include such virtual transitions because there are so few nearby nonresonant levels, only two in fact: the two negative-energy spin states of the electron. Thus we have to deal merely with a four-level quantum system, with energies which we label $E_{1,2} = m \pm \frac{1}{2}\omega_0$, $E_{3,4} = -(m \pm \frac{1}{2}\omega_0)$.

In order to exploit the resonance precession point of view we use the Heisenberg-equation-of-motion method of Ackerhalt and co-workers.[6] At its simplest level our approach is merely a careful analysis of the precession of the electron spin operator $\vec{\sigma}$ in the low-energy limit. In the paragraphs below we outline how one may construct the spin-operator precession equations, and then identify the effective resonance frequency $\omega_0'$ in them. A calculation of $a_e$ is then straightforwardly made.

To account properly for all four energy levels it is best to start with the Dirac Hamiltonian, and later take the low-energy limit. As is well known, equations of motion obtained from the Dirac Hamiltonian suffer from the complexity of *Zitterbewegung* and are quite involved when the electron interacts with the radiation field. It is possible, however to separate the "mean" motion from the *Zitterbewegung* by transforming the Hamiltonian in the Foldy-Wouthuysen (F-W) manner. In our problem, a very convenient transformation exists[7] that diagonalizes the unperturbed Hamiltonian and transforms the interaction Hamiltonian in closed form.[8]

In the presence of the quantized radiation field and of a static classical magnetic field $\vec{B}_0 = \nabla \times \vec{A}_0$ along the $z$ axis, the Dirac Hamiltonian has the form (in units so that $\hbar = c = 1$)

$$H = \beta m + \vec{\alpha}\cdot(\vec{p} - e\vec{A}_0 - e\vec{A}_{\rm rad}) + H_F, \tag{1}$$

where $H_F = \sum_\lambda \omega_\lambda a_\lambda^\dagger a_\lambda$ and $[a_{\lambda'}, a_\lambda^\dagger] = \delta_{\lambda\lambda'}$ as usual, and where $\lambda$ indexes both polarization and wave vector. After the F-W transformation, $H \to e^{iu} \times He^{-iu}$ with $u = \frac{1}{2}i\vec{\alpha}\cdot\vec{\Pi}_0\theta$, where $\xi\theta = \tan^{-1}(\xi/m)$, $\xi^2 = (\vec{\alpha}\cdot\vec{\Pi}_0)^2$, and $\vec{\Pi}_0 = \vec{p} - e\vec{A}_0$, we arrive at the following working Hamiltonian in the low-momentum limit:

$$H' = \beta m + \tfrac{1}{2}\beta\omega_0\sigma_3 + (\beta/2m)\Pi_0^2 + H_{\rm int} + H_F, \tag{2}$$

where $\omega_0 = eB_0/mc$ is the unperturbed Larmor-Zeeman precession frequency.

The interaction term, containing the coupling of the electron with the radiation field, is[9]

$$H_{\rm int} = \sum_\lambda f_\lambda \beta\vec{\alpha}\cdot\hat{\epsilon}_\lambda(e^{i\vec{k}\cdot\vec{r}}a_\lambda + \text{H.c.}) + \sum_\lambda i g_\lambda(\omega_\lambda/2m)\beta\vec{\sigma}\cdot\hat{\epsilon}_\lambda\times\hat{k}(-e^{i\vec{k}\cdot\vec{r}}a_\lambda + \text{H.c.}). \tag{3}$$

We use the abbreviations $\vec{k}$ for $\vec{k}_\lambda$, $E_k$ for $(m^2+k^2)^{1/2}$,

$$f_\lambda = -e(2\pi/V\omega_\lambda)^{1/2}[(m+E_k)/2E_k]^{1/2}, \quad g_\lambda = -e(2\pi/V\omega_\lambda)^{1/2}2m[2E_k(m+E_k)]^{-1/2}.$$

The Heisenberg equation $idO/dt = [O, H']$ can now be applied to the electron and field operators to determine their temporal evolution. There are so many operator variables $[\vec{\alpha}, \beta, a_\lambda, a_\lambda^\dagger, \vec{\sigma}, \text{etc.}]$ that it pays to be selective. We are ultimately only interested in the dynamics of the spin operator, so we concentrate on the equations of motion for its transverse components $\sigma_x = \sigma_{12} + \sigma_{21}$ and $\sigma_y = -i(\sigma_{12} - \sigma_{21})$. Here $\sigma_{12}$ and $\sigma_{21}$ are the spin raising and lowering operators connecting positive-energy levels $E_1$ and $E_2$. As a notational convenience we rewrite all of the electron operators in the Hamiltonian entirely in terms of raising and lowering operators $\sigma_{ij}$ which have all matrix elements zero except for the $ij$ element which is unity. In particular, $H_{\rm int}$ takes the form, after some straightforward algebra,

$$H_{\rm int} = \sum_\lambda f_\lambda[(\sigma_{14} - \sigma_{32})\epsilon_- + (-\sigma_{41} + \sigma_{23})\epsilon_+ + (\sigma_{13} - \sigma_{24} - \sigma_{31} + \sigma_{42})\epsilon_3][\exp(i\vec{k}\cdot\vec{r})a_\lambda + \text{H.c.}] + i\sum_\lambda g_\lambda(\omega_\lambda/2m)[(\sigma_{12} - \sigma_{34})\tilde{\epsilon}_- + (\sigma_{21} - \sigma_{43})\tilde{\epsilon}_+ + (\sigma_{11} - \sigma_{22} - \sigma_{33} + \sigma_{44})\tilde{\epsilon}_3] \times [-\exp(i\vec{k}\cdot\vec{r})a_\lambda + \text{H.c.}]. \quad (4)$$

Here $\epsilon_\pm \equiv (\hat{x} \pm i\hat{y})\cdot\hat{\epsilon}_\lambda$, $\epsilon_3 \equiv \hat{z}\cdot\hat{\epsilon}_\lambda$, and $\tilde{\epsilon}_\pm$ and $\tilde{\epsilon}_3$ are similarly defined with respect to $\hat{\epsilon}_\lambda \times \hat{k}$.

An important consequence of having $H_{\rm int}$ in the form (4) above is that one sees clearly that two types of interaction are important in general. Both types contribute significantly to the $g$-factor anomaly. Those terms in (4) which contain $\sigma_{ij}$ operators connecting $E_1$ with $E_2$, or $E_3$ with $E_4$, or any of the $E_i$'s with itself, arise solely from the interaction of the electronic *spin* with the field. The remaining $\sigma_{ij}$ operators connect either $E_1$ or $E_2$ with either $E_3$ or $E_4$. They arise from the interaction of the electronic *charge* with the field. These charge interaction terms have been neglected altogether in analogous earlier work.[5] On the other hand in the nonrelativistic limit of the Drell-Pagels treatment,[3] the spin interaction terms have been omitted. As the work of Koba and Tani[4] suggests, the charge terms should be responsible for the positive sign of $a_e$.

The equation of motion for the positive-energy spin-flip operator is

$$i\dot{\sigma}_{12} + \omega_0\sigma_{12} = [\sigma_{12}, H_{\rm int} + H_F] = \sum_\lambda f_\lambda[(\sigma_{42} + \sigma_{13})\epsilon_+ + (-\sigma_{14} + \sigma_{32})\epsilon_3][\exp(i\vec{k}\cdot\vec{r})a_\lambda + \text{H.c.}] + i\sum_\lambda g_\lambda(\omega_\lambda/2m)[(\sigma_{11} - \sigma_{22})\tilde{\epsilon}_+ - 2\sigma_{12}\tilde{\epsilon}_3][-\exp(i\vec{k}\cdot\vec{r})a_\lambda + \text{H.c.}]. \quad (5)$$

In order to compute the values for $\exp(i\vec{k}\cdot\vec{r})a_\lambda(t)$ required in (5) it is only necessary to integrate to first order in $e$ the Heisenberg equation for the mode operator.[6] We find the following lengthy expression:

$$\exp(i\vec{k}\cdot\vec{r})a_\lambda(t) = [\exp(i\vec{k}\cdot\vec{r})a_\lambda(t)]_F - if_\lambda[(\sigma_{14}I_{14} - \sigma_{32}I_{32})\epsilon_- + (-\sigma_{41}I_{41} + \sigma_{23}I_{23})\epsilon_+ + (\sigma_{13}I_{13} - \sigma_{24}I_{24} - \sigma_{31}I_{31} - \sigma_{42}I_{42})\epsilon_3] + g_\lambda(\omega_\lambda/2m)[(\sigma_{12}J_{12} - \sigma_{34}J_{34})\tilde{\epsilon}_- + (\sigma_{21}J_{21} - \sigma_{43}J_{13})\tilde{\epsilon}_+ + (\sigma_{11} - \sigma_{22} - \sigma_{33} + \sigma_{44})J_{11}\tilde{\epsilon}_3]. \quad (6)$$

Here $[\,]_F$ denotes the free-field solution, and the bracketed terms are the charge and spin parts, respectively, of the operator reaction field generated by the electron. The coefficients $I_{ij}$ and $J_{ij}$ arise in the integration of the field-operator equations of motion. They express field-electron energy conservation, taking account of electron recoil as well as possible virtual transitions among the four energy states:

$$I_{ij} = G[E_i - E_j + (\omega - \beta\omega^2/2m)], \quad J_{ij} = G[E_i - E_j + (\omega + \beta\omega^2/2m)],$$

where $G$ is the singular function $G(x) = \pi\delta(x) - iP(1/x)$. After inserting (6) and its Hermitian adjoint into (5), the effect of the electric and magnetic reaction fields on the precession frequency is apparent. By taking vacuum expectation values in a positive energy state we find that the spin-flip operator obeys the remarkably simple equation

$$i\langle\dot{\sigma}_{12}\rangle + \omega_0\langle\sigma_{12}\rangle = -(C + S)\langle\sigma_{12}\rangle, \quad (7)$$

where $C$ and $S$ are numerical factors arising from the coupling of the electron's charge and spin, respectively, to its operator reaction fields, and are given by

$$C = \lim_{\beta\to1} -i\sum_\lambda f_\lambda^2\epsilon_3^2(I_{42} + I_{31}^*), \quad (8a)$$

$$S = \lim_{\beta\to1} i\sum_\lambda g_\lambda^2(\omega_\lambda/2m)^2\tilde{\epsilon}_+\tilde{\epsilon}_-(J_{12} + J_{21}^*). \quad (8b)$$

The remaining terms from (6) are either smaller by an extra power of $e^2$ or have vanishing vacuum expectation value.

The net result is that the true precession frequency of the electronic spin, to order $e^2$, may be found by inspection of (7) to be not $\omega_0$ but $\omega_0' = \omega_0 + \mathrm{Re}C + \mathrm{Re}S$. Thus the $g$-factor anomaly is simply $a_e = \lim_{\omega_0 \to 0}[\mathrm{Re}(C+S)/\omega_0]$. Note that we do *not* need to renormalize our expressions for $C$ and $S$: The self-mass terms cancel in our frequency-shift expressions.

By substituting for $I$ and $J$, converting the mode sums to integrals, and performing the polarization sum and angular integration, one easily finds that Re$C$ and Re$S$ are strictly positive and negative, respectively:

$$C = \frac{4e^2\omega_0}{3\pi}\int_0^{\omega_c} \frac{(m^2+\omega^2)^{1/2}+m}{2(m^2+\omega^2)^{1/2}} \frac{\omega\, d\omega}{(\omega - \omega^2/2m - 2m)^2}, \tag{9a}$$

$$S = i\frac{e^2\omega_0^3}{3m^2} - \frac{8e^2\omega_0}{3\pi}\int_0^{\omega_c} \frac{1}{2(m^2+\omega^2)^{1/2}[(m^2+\omega^2)^{1/2}+m]} \frac{\omega^3\, d\omega}{(\omega+\omega^2/2m)^2}. \tag{9b}$$

The sum $C+S$ is convergent because electron recoil has been accounted for. However, a cutoff $\omega_c$ has been supplied as a reminder of the low-energy character of our calculation. The imaginary part of $C+S$ gives the correct magnetic dipole Einstein $A$ coefficient and arises wholly from the spin-flip or magnetic-dipole source-field term $S$ as expected. There is no decay rate associated with the charge source field. One expects $\omega_c/2m \approx 1$ to be a natural cutoff in a low-energy theory. We find, in fact, that if $\omega_c/2m \approx 0.97$ the entire order $e^2$ correction, $a_e = \alpha/2\pi$, is provided by our analysis.

It seems fair to conclude that our simple quantum-electrodynamic resonance-precession picture[5,6] provides a sound basis for understanding the electron's anomalous moment both intuitively and quantitatively. Compared with covariant perturbative methods, a direct study of the electron's spin precession is more transparent, closer to being classical in spirit, and conceptually practically identical with the experimental approach. Moreover, these intuitive advantages are not won at the expense of quantitative accuracy. We have found, in one straightforward low-energy Heisenberg-picture calculation, the correct magnetic decay rate for the precessing spin, the expected absence of decay due to the charge interaction, and a value for $a_e$ that has both the correct sign and magnitude, and is relatively independent of the imposed cutoff. Renormalization is unnecessary.

In addition to having a calculation which is conceptually direct, and operationally straightforward, it would be nice if it were also trivial. We concede that this is not the case. However, the physical necessity for the complexity is now clear. Just as with the Lamb shift, the possibility of virtual transitions to nearby nonresonant levels is very important to the size of the precession frequency shift. Only because of these virtual transitions does the electron's *electric reaction field* enter into the correction to its *magnetic moment*. Because the magnetic reaction field is weaker than the electric by a factor $\omega_\lambda/2m$, the electric interaction dominates. The neglect of the electric interaction in earlier work[5] explains the traditional sign difficulty[2] in low-energy calculations of the anomaly.

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## New Class of Renormalizable Vector-Meson Theories?*

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We propose a new class of apparently renormalizable heavy–vector-boson theories. These models are spontaneously broken gauge theories, modifed by the addition of arbitrary mass terms for vectors associated with invariant Abelian subgroups. The vacuum is *not* invariant under these subgroups. Such theories are probably renormalizable since their multiparticle $S$-matrix elements are unitarily bounded in the tree approximation. As illustrations, the Higgs [U(1)] and Weinberg gauge theories are modified in this way.

The only known renormalizable systems of heavy vector bosons are either spontaneously broken gauge theories (SBGT) or "conserved current" models. In an SBGT[1] the field variables can always be chosen so that the Lagrangian is locally gauge invariant. In the language of these field variables spontaneous symmetry breaking is the origin of the vector-boson masses. Massless vector mesons have conserved source currents. On the other hand, "conserved current" models always contain at least one massive vector boson, whose source current is conserved. Massive quantum electrodynamics is the simplest system of this type. The general prescription for constructing conserved current models can be stated as follows: (1) Begin with a Lagrangian which is invariant under a nonsemisimple group of local gauge transformations (i.e., a group of transformations containing an invariant Abelian subgroup). (2) Arrange for spontaneous symmetry breaking (if any) such that the vacuum expectation value of the scalar field is invariant under at least one invariant (single-parameter) Abelian subgroup (thus, at this stage the corresponding Abelian vector is massless and coupled to a conserved current). (3) Add (in the $R$ gauge) an arbitrary mass term for the same Abelian vector. Notice that the resulting Lagrangian is not locally invariant under Abelian gauge transformations.

This paper suggests that there is a third class of heavy–vector-boson interactions which may be renormalizable.[2] The models of this new class are constructed according to the above three-part prescription,[3] except that in step (2) spontaneous symmetry breaking is arranged so that the vacuum expectation value of the scalar field is *not* invariant under at least one invariant (single-parameter) Abelian subgroup [thus, the corresponding Abelian vector, possibly in linear combination with other vectors, would acquire a mass at step (2)]. Models of this type are different from SBGT systems, since the $R$-gauge Lagrangian is not locally gauge invariant under the entire group; they differ from conserved-current theories since, in general, there is no massive vector boson with a conserved source current. Instead, these models are "hybrid" systems which interpolate between the SBGT and conserved-current theories: In the limit in which the added Abelian vector mass term vanishes, an SBGT is recovered; on the other hand, as the scalar-field vacuum expectation value is altered so that it becomes invariant under the Abelian subgroup in question, the "hybrid" model becomes a conserved-current theory. The "hybrid" theories are probably renormalizable since we can show that they are "tree unitary" (i.e., unitarily bounded in the tree approximation). Specifically, because coupling-