Unity of All Elementary-Particle Forces

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Strong, electromagnetic, and weak forces are conjectured to arise from a single fundamental interaction based on the gauge group SU(5).

We present a series of hypotheses and speculations leading inescapably to the conclusion that SU(5) is the gauge group of the world—that all elementary particle forces (strong, weak, and electromagnetic) are different manifestations of the same fundamental interaction involving a single coupling strength, the fine-structure constant. Our hypotheses may be wrong and our speculations idle, but the uniqueness and simplicity of our scheme are reasons enough that it be taken seriously.

Our starting point is the assumption that *weak and electromagnetic forces are mediated by the vector bosons of a gauge-invariant theory with spontaneous symmetry breaking.* A model describing the interactions of leptons using the gauge group $SU(2) \otimes U(1)$ was first proposed by Glashow, and was improved by Weinberg and Salam who incorporated spontaneous symmetry breaking.[1] This scheme can also describe hadrons, and is just one example of an infinite class of models compatible with observed weak-interaction phenomenology. If we assume that *there are as few fermion fields as possible* and, in particular, that there are no unobserved leptons, the Weinberg model becomes unique up to extensions of the gauge group: The observed leptons may be described by six left-handed Weyl fields $(e_L{}^-, \mu_L{}^-, \nu_L, \nu_L', e_L{}^+, \mu_L{}^+)$ and their charge conjugates. If the gauge couplings do not mix leptons with quarks, these six fields must transform as a representation of the gauge group: one of the 23 subgroups of U(6) containing an SU(2) $\otimes$ U(1) subgroup in which the leptons behave as they do in the Weinberg model.

To include hadrons in the theory, we must use the Glashow-Iliopoulos-Maiani (GIM) mechanism and introduce a fourth quark p' carrying charm.[2] Still, decisions must be made: Should the quarks have fractional or integer charges? Should there be one quartet of quarks or several? Bouchiat, Iliopoulos, and Meyer suggested what seems the most attractive alternative: *three quartets of fractionally charged quarks.*[3] This combination

of the GIM mechanism with the notion of colored quarks[4] keeps the successes of the quark model and gives an important bonus: Lepton and hadron anomalies cancel so that the theory of weak and electromagnetic interactions is renormalizable.[5]

The next step is to include strong interactions. We assume that *strong interactions are mediated by an octet of neutral vector gauge gluons* associated with local color SU(3) symmetry, and that there are no fundamental strongly interacting scalar-meson fields.[6] This insures that parity and hypercharge are conserved to order α,[7] and does not lead to any new anomalies, so that the theory remains renormalizable. The strongest binding forces are in color singlet states which may explain why observed hadrons lie in qqq and $q\bar{q}$ configurations.[8] And, it gives another important bonus: Since the strong interactions are associated with a non-Abelian theory, they may be asymptotically free.[9]

Thus, we see how attractive it is for strong, weak, and electromagnetic interactions to spring from a gauge theory based on the group $\mathcal{F} = SU(3) \otimes SU(2) \otimes U(1)$. Alas, this theory is defective in one important respect: It does not truly unify weak and electromagnetic interactions. The $SU(2) \otimes U(1)$ gauge couplings describe two interactions with two independent coupling constants; a true unification would involve only one.

Electric charge is observed to be quantized. This has no natural explanation in the framework of conventional quantum electrodynamics, but it is necessarily true in any unified theory[10]—yet another reason to search for a true unification.

We must assume that the gauge group is larger than $\mathcal{F}$. Suppose it is of the form $SU(3) \otimes \mathcal{W}$ where $\mathcal{W}$ contains $SU(2) \otimes U(1)$ but has a unique gauge coupling constant. $\mathcal{W}$ must be simple, or the direct product of isomorphic simple factors with discrete symmetries which interchange them. This embedding of the Weinberg model implies a relationship between the coupling constants of the SU(2) and U(1) subgroups. Because leptons are singlets under color SU(3), leptons and quarks

must lie in separate representations of $\mathcal{W}$. If only the six observed lepton states are involved, $\mathcal{W}$ must be one of the 23 relevant subgroups of U(6). The only candidates involving a single gauge coupling constant are SU(3), SU(3) ⊗ SU(3), and SU(6).[11] For each of these cases, the mixing angle is fixed so that $\sin^2\theta_w = \frac{1}{4}$. None of these schemes can describe hadrons: The generator corresponding to electric charge does not admit fractional charges, nor, being traceless, can it explain why the sum of the quark charges is not zero. No gauge group of the form SU(3) ⊗ $\mathcal{W}$ works.

We see that we cannot unify weak and electromagnetic interactions independently of strong interactions. The remaining possibility is that the gauge group $\mathcal{G}$ contains $\mathcal{F}$ as a subgroup but is itself simple or the direct product of isomorphic simple factors. Leptons and quarks must lie together in the same irreducible representations of such a group: Some gauge fields carry lepton number and quark number. The same coupling strength—the fine-structure constant—characterizes all three kinds of interaction. This outrageous possibility may seem palatable after the following discussion about asymptotic freedom and its complement, infrared slavery.

Asymptotic freedom is a property of non-Abelian Yang-Mills field theories which promises to explain the pointlike structure of hadrons at high energy.[9] Unfortunately, these theories do not appear to describe strong interactions correctly since they involve massless strongly interacting vector bosons. The obvious solution is to introduce strongly interacting scalar mesons which develop vacuum expectation values, spontaneously break the gauge symmetry, and generate vector meson masses by the Higgs mechanism. But the scalar-meson Lagrangian involves additional renormalizable couplings which may spoil asymptotic freedom. Sadly, no one has found an asymptotically free model in which the gauge symmetry is completely broken and all the vector mesons develop mass.[12]

Weinberg, and Gross and Wilczek,[13] propose an astonishingly radical solution: to leave the gauge symmetry unbroken. *While the Yang-Mills Lagrangian appears to describe massless vector bosons, the hideous infrared divergences of the theory conspire to prevent their appearance in physical states.* This could explain the absence of physical states that are not color singlets and answer the old saw: Why don't the quarks get out? We have nothing to say about the merits of

this picture; we assume that it works and use it.

The essential thing about a theory of strong interactions based on an unbroken non-Abelian gauge symmetry is that the strength of strong interactions no longer depends on the existence of a large coupling constant. Even if the gauge coupling constant is small, say of order e, the infrared divergences of the theory can lead to phenomenological interactions strong enough to keep the quarks bound.[14] *What we want is not asymptotic freedom but infrared slavery.*

The theory we have in mind involves a unifying gauge group $\mathcal{G}$ whose only coupling constant is the unit of electric charge and which contains—in an appropriate way—the subgroup $\mathcal{F}$. The symmetry is spontaneously broken leaving only the direct product of color SU(3) and electromagnetic gauge invariance as exact local symmetries. Color SU(3) is an unbroken non-Abelian gauge symmetry causing infrared slavery and leading to strong interactions. Electromagnetic gauge invariance is Abelian and commutes with color SU(3). Since the photon has no direct couplings to the gauge fields of color SU(3), electromagnetism is free of insolvable infrared-divergence problems, and photons may be freely emitted and absorbed. All other gauge fields develop masses through the Higgs mechanism. Those associated with the subgroup SU(3) ⊗ U(1), aside from the photon, mediate ordinary weak interactions and the neutral-current effects of the Weinberg model. The rest, which are colored and massive, mediate new and presumably even weaker interactions.

Our unifying group $\mathcal{G}$ must be of rank at least 4. There are exactly nine rank-4 local Lie groups which can involve only one coupling strength: $[SU(2)]^4$, $[O(5)]^2$, $[SU(3)]^2$, $[G_2]^2$, O(8), O(9), Sp(8), F_4, and SU(5).' The first two are unacceptable since they do not contain SU(3). To proceed, we review the behavior of quarks and leptons under $\mathcal{F}$.

We use the Weyl notation in which all fermion fields are left-handed two-component spinors. There are thirty such fields in our picture of nature: four leptons $(\mu^-, \nu', e^-, \nu)_L$, two antileptons $(\mu^+, e^+)_L$, twelve quarks $(p_i', p_i, n_i, \lambda_i)_L$, and twelve antiquarks $(\bar{p}_i', \bar{p}_i, \bar{n}_i, \bar{\lambda}_i)_L$, where the color index i assumes three values. Under the subgroup SU(3) ⊗ SU(2), the leptons are SU(3) singlets and SU(2) doublets; the antileptons are singlets under both groups; the quarks are SU(3) triplets as well as SU(2) doublets; and, finally, the anti-quarks are SU(3) $\underline{3}^*$'s but SU(2) singlets.

The SU(3) ⊗ SU(2) content of the thirty fields is
$2(\underline{1}, 2) \oplus 2(\underline{1}, 1) \oplus 2(\underline{3}, 2) \oplus 4(\underline{3}^*, 1)$ in an evident nota-
tion.

This representation is complex, not equivalent
to its complex conjugate. So also is the corre-
sponding representation of $\mathfrak{G}$. Of our nine candi-
dates only $[SU(3)]^2$ and SU(5) admit complex rep-
resentations. We have already considered and
rejected $[SU(3)]^2$ in our discussion of the synthe-
sis of just weak and electromagnetic interactions.
We are left with SU(5).

Under the subgroup SU(3) ⊗ SU(2), the funda-
mental five-dimensional representation of SU(5)
transforms like $(\underline{1}, 2) \oplus (\underline{3}, 1)$. The complex con-
jugate $\underline{5}^*$ transforms like $(\underline{1}, 2) \oplus (\underline{3}^*, 1)$. The ir-
reducible ten-dimensional representation given
by the antisymmetrized tensor product of two
$\underline{5}$'s transforms like $(\underline{1}, 1) \oplus (\underline{3}^*, 1) \oplus (\underline{3}, 2)$. If the
thirty left-handed fermions transform like two
$\underline{10}$'s and two $\underline{5}^*$'s, the $\mathfrak{F}$ content is just right to
describe physics. In order to display these rep-
resentations, we replace the two $\underline{5}^*$'s of left-
handed fields by two $\underline{5}^*$'s of their right-handed
charge conjugates. The representations contain-
ing electrons are then a $\underline{5}$ and a $\underline{10}$:

$$\begin{bmatrix} n_1 \\ n_2 \\ n_3 \\ e^+ \\ \nu \end{bmatrix}_R, \quad \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & \bar{p}_3 & -\bar{p}_2 & -p_1(\theta) & -n_1 \\ -\bar{p}_3 & 0 & \bar{p}_1 & -p_2(\theta) & -n_2 \\ \bar{p}_2 & -\bar{p}_1 & 0 & -p_3(\theta) & -n_3 \\ p_1(\theta) & p_2(\theta) & p_3(\theta) & 0 & -e^+ \\ n_1 & n_2 & n_3 & e^+ & 0 \end{bmatrix}_L,$$

where $p(\theta) = p \cos\theta - p' \sin\theta$. The $\underline{5}$ and $\underline{10}$ con-
taining muons are obtained from these by the re-
placements $e^+ \to \mu^+$, $\nu \to \nu'$, $n \to \lambda$, $\bar{p} \to \bar{p}'$, and
$p(\theta) \to p'(\theta) = p' \cos\theta + p \sin\theta$.

Having the representations before us, we an-
swer the obvious questions: Are there anomalies?
What Higgs mesons are necessary? What mixing
angle is predicted? What new interactions are
predicted?

While we already know that the $\mathfrak{F}$ subgroup is
free of anomalies for the representation we have
chosen, the full unifying group might not be. But
it is! Remarkably, the $\underline{5}^*$ and $\underline{10}$ have equal and
opposite anomalies: Our theory is entirely anom-
aly free. Indeed, SU(5) is the only group of any
rank with a thirty-dimensional, anomaly-free
representation with the correct $\mathfrak{F}$ content.

Two irreducible representations of Higgs mes-
ons are needed. We need a multiplet with a very
large vacuum expectation value to break the
SU(5) symmetry down to $\mathfrak{F}$. This is done most
simply with 24 real scalar-meson fields trans-
forming like the adjoint representation. It is the
analog to the superstrong breaking discussed by
Weinberg in his treatment of SU(3) ⊗ SU(3).[11] All
the vector bosons except the twelve associated
with generators of $\mathfrak{F}$ develop superheavy masses
and can hopefully be neglected. We also need
Higgs mesons to give mass to the fermions and
the weak-interaction intermediaries. For the
most general zeroth-order mass matrix consis-
tent with exact color SU(3) symmetry, we need
five complex scalar-meson fields transforming
like the fundamental representation and 45 com-
plex scalar-meson fields transforming like the
$\underline{45}$ contained in $\underline{5}^* \times \underline{10}$. If only the $\underline{5}$ is present,
the p and p' masses and the Cabibbo angle are
arbitrary, but the other masses satisfy the re-
lations $m_n = m_e$ and $m_\lambda = m_\mu$. Does this mean that
the muon-electron mass splitting has the same
origin as SU(3) breaking?

For the mixing angle, the theory predicts
$\sin^2\theta_w = \frac{3}{8}$.

Finally we come to a discussion of superweak
interactions and SU(3)-colored superheavy vec-
tor bosons. In addition to mediating such bizarre
interactions as $K^0 \to \mu^+ e^-$, they make the proton
unstable. For instance, there is a superheavy
colored vector boson which causes the virtual
transitions $p_1 + p_2 \to W \to \bar{n}_3 + e^+$. Exchange of this
vector boson contributes directly to the decay
$p \to \pi^0 + e^+$. Since the proton is rather stable,[15]
this vector boson must be very massive.[16] The
Higgs mesons can also mediate proton decay,
and must also be very massive.

From simple beginnings we have constructed
the unique simple theory. It makes just one
easily testable prediction, $\sin^2\theta_w = \frac{3}{8}$. It also pre-
dicts that the proton decays—but with an unknown
and adjustable rate. More theoretical work is
needed to determine whether the idea of infrared
slavery, necessary for our unification, actually
makes sense.

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[10]In a general gauge theory, the Lie algebra of the gauge group is a direct sum of a semisimple and an Abelian Lie algebra. Only if the Abelian term is absent—as in a unified theory—is the gauge group necessarily compact, and charge necessarily quantized. Assume that electric charge Q were not quantized in such a theory, i.e., that its eigenvalues were not commensurate. The topological closure of the one-parameter subgroup $\{\exp(i\alpha Q)\}$ would be a compact Abelian Lie group with at least two parameters. Let its Lie algebra A be spanned by $Q, Q_1, \ldots, Q_n$, where $n > 0$. Because the gauge group is compact, its Lie algebra contains A and the Q_i are associated with gauge fields other than the photon. Because $\exp(i\alpha Q)$ is an unbroken local symmetry, the Q_i generate unbroken local symmetries and their associated gauge fields are massless. Since there is only one massless gauge field, we conclude that Q is quantized.

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[16]A naive calculation indicates that the vector boson mass must be greater than 10^{15} GeV $\simeq 10^{-9}$ g! Let the reader who finds this hard to swallow double the number of fermion states and put quarks and leptons in different (but equivalent) thirty-dimensional representations. He must introduce both weakly interacting quarks and strongly interacting leptons. Now quark number is conserved modulo two and the proton is stable. The deuteron decays via the exchange of four superweak vector bosons, but this is not a serious problem.

Measurement of the p-p Total Cross Section at 200 and 300 GeV/c*

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We have measured total cross sections for p-p scattering with the results $\sigma_T = 40.42 \pm 0.27$ mb at 200 GeV/c and 40.40 ± 0.28 mb at 300 GeV/c. Our 300-GeV/c result is significantly higher than published data from the CERN intersecting storage rings. Our data, taken together with the Serpukhov data, indicate that the cross section rises ≈ 2 mb between 60 and 250 GeV. The variation of the cross section with energy may be more complicated than the $a + b \ln^\alpha s$ behavior commonly assumed for $E_{\text{lab}} \gtrsim 50$ GeV.

We have carried out a measurement of proton-proton total cross sections at 200 and 300 GeV/c with a proton beam at the National Accelerator Laboratory (NAL). The standard good-geometry transmission technique was used with a liquid-hydrogen target and scintillation counters.

The experimental arrangement is shown schematically in Fig. 1. The proton beam was made by slightly modifying an existing neutral beam by the addition of two small bends, each about 0.7 mrad. The beam was taken off at an angle of 1 mrad from a beryllium target in an external pro-