What Can We Learn from Three-Body Reactions?*

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A new method of analysis is applied to n-d elastic scattering and deuteron breakup in order to test the sensitivity of such reactions to details of the NN force. The results suggest that no off-shell information can be obtained from these processes which is not already implicit in the value of the n-d doublet scattering length.

It has generally been accepted that a study of three-particle systems can yield new information regarding details of the two-particle interactions and, in particular, the short-distance behavior of the nuclear force. This possibility has stimulated a great deal of work on the trinucleon system during the past decade. However, if one restricts oneself to phenomena below the threshold for deuteron breakup, only the triton binding energy E_T, the position of the diffraction minimum in the ^{3}He charge form factor, and the n-d doublet scattering length a_2 exhibit any sensitivity, and at least two of these are largely correlated.[1]

It therefore appears that the success of this program will depend to a large extent on the isolation of independently sensitive parameters from three-body reactions, in this case n-d elastic scattering and deuteron breakup. Although one would expect such processes to be less sensitive than the bound-state parameters, it is conceivable that a proper selection of scattering observables, particularly in the breakup domain, might nevertheless yield new information. A recent comparison of various local and nonlocal two-nucleon models by Kloet and Tjon[2] has generated some optimism in this respect. However, inasmuch as these models produce far from identical results in the *two*-body problem, such comparisons do not of themselves establish the requisite sensitivity. The purpose of this article is to examine whether such sensitivity in fact exists.

In order to obtain an unambiguous answer to this question, it is necessary to isolate the pertinent degrees of freedom by holding the two-particle phase shifts invariant. A direct approach might involve application of the standard Faddeev formalism to a sufficiently exhaustive set of phase-equivalent potentials. Unfortunately, granting the ability to construct such a set, the effort required by even a single three-body calculation is such that this procedure is simply not practical. Instead, we shall employ an alternative treatment of the three-body problem recently proposed by this author.[3] In this formulation the two-particle phase shifts are imposed by means of a set of boundary conditions on the three-particle wave function in appropriate asymptotic domains. The dynamical input is essentially a statement as to how the wave function approaches these asymptotic limits. One may therefore regard this input as supplying the off-shell information, although it actually represents the sum total of effects which cannot be deduced from two-particle observables, including possible three-body forces. Insofar as three-body reactions are concerned, this distinction turns out to be purely academic.

In practice, application of the boundary conditions to an explicit representation for the three-particle wave function leads to a one-variable integral equation of the form $X = \Omega + KX$, where X completely determines the scattering observables. Furthermore, the kernel can be expressed as $K = K_1 + AK_2$, where the operators Ω, K_1, and K_2 are uniquely specified by the two-particle phase shifts, and A is an arbitrary real operator corresponding to the dynamical input. Two important advantages follow immediately from the form of K: (a) The off-shell content may be varied independently while holding the two-body observables invariant, and (b) by expanding A in some complete set, it is only necessary to solve an integral equation *once* for a given total energy and a specified set of phase shifts; the off-shell analysis then reduces to linear algebra.[3] This makes it possible to compare a wide variety of possibilities with a minimal investment in computer time.

As shown in BC1, there is no particular difficulty in working directly with the experimental phase shifts. However, the precise choice is unimportant for the purposes of this article, and hence a particularly convenient set has been assumed in what follows. The model consists of purely s-wave interactions in the singlet and triplet channels specified by an energy-independent boundary condition on the two-particle wave function.[4] There are thus two parameters in each channel, the core radius and the value of the logarithmic derivative; these are chosen to produce the correct low-energy s-wave phase shifts and the deuteron binding energy. Specifically, we require that

$$\lim_{\epsilon \to 0} \frac{\psi'(a+\epsilon)}{\psi(a+\epsilon)} = (f-1)/a. \qquad (1)$$

The corresponding values are $a_t = 1.095$ fm, $f_t = -0.253$, $a_s = 1.2826$ fm, $f_s = 0.0513$. As compared to experiment, the core radii come out too large in this simple model, a fact which is reflected in the phase shifts falling off too rapidly beyond the effective-range region.

If one takes $A \equiv 0$, this model predicts a value of 7.05 MeV for E_T, while the doublet and quartet scattering lengths come out to be 2.1 and 6.31 fm, respectively. In order to examine the degree of *independent* sensitivity in the reaction data above breakup, an appropriate set of A operators was constructed as follows: Given an *Ansatz* for the functional dependence of A, the overall *scale* was adjusted to produce agreement with the experimental values of $k \cot\delta_4$ and $k \cot\delta_2$ below breakup.[5] In actuality, only the latter is sensitive, and it is completely determined by requiring that $a_2 = 0.41$ fm. The fact that the low-energy scattering is specified by only a single parameter was observed some time ago by Barton and Phillips,[6] and is illustrated in this analysis by a variation of less than 0.1% in $k \cot\delta_2$ over the set of A operators considered.

As shown in BC1, A corresponds to the specification of boundary values in two distinct regions of the three-particle configuration space. There is an inner region, where the two-body forces overlap and three-particle forces may be present, and an outer region in which A is completely determined by the off-shell characteristics of the two-particle interactions. In order to obtain $a_2 = 0.41$ fm a net attraction must be supplied to the system; this could arise in a variety of ways through combinations of attraction and repulsion in these two regions. Consequently, the set of A operators was constructed to be as representative as possible of these qualitatively distinct possibilities, incorporating a broad variety of separable, local, and semilocal forms. Inasmuch as a score of such possibilities were essayed, the author is confident that the results, if anything, *overestimate* the sensitivity to be expected from plausible physical mechanisms.

Given this input, the above formalism was applied to generate a class of predictions for n-d elastic scattering and breakup corresponding to the same low-energy trinucleon parameters and two-nucleon observables. Previous investigations[7] had indicated a sensitivity on the order of 15% at small and large angles in the elastic scattering, depending on the choice of potential model. In this analysis the variations reduced to less than 1%, indicating that no new information is contained in these data. However, although the elastic cross section is expected to be insensitive because of the large quartet contribution, a judicious choice of experimental configurations in the breakup channel can be used to isolate the far more sensitive doublet amplitude. This is illustrated by the lower half of Fig. 1, which corresponds to the most sensitive region observed by Kloet and Tjon in their comparison of potential models.[2] Nevertheless, the strong model dependence at the minimum does not survive under the constraints we have imposed, as evidenced by the upper half of the figure (note that the two upper curves correspond to the *maximum* dispersion displayed over the entire A set). In fact, a complete search of the phase space revealed that such variations as exist are restricted to at most a few percent. It was recently proposed[8] that breakup data be analyzed in such a way as to emphasize certain kinematical situations; the suggested procedure leads to the curves plotted in Fig. 2. As in the preceding example, the variations exhibited are essentially unmeasurable.[9]

Taken in conjunction with preceding investigations, the results of this analysis strongly suggest the following conclusions:

(1) The low-energy trinucleon reactions can provide no new information, being essentially determined by the two-nucleon observables and the value of a_2. It thus appears highly unlikely that this system will enhance our understanding of the short-range nuclear force.[10] In compensation, the striking absence of off-shell effects to the level demonstrated is itself a valuable piece of information, and would seem to demand theoretical attention.

(2) If a three-nucleon force is introduced to explain the charge-form-factor data[11] and/or to correct the value of E_T, the trinucleon properties will be essentially determined. In particular, no scattering measurement will be able to distinguish between this possibility and that of an

exotic off-shell mechanism which produces the same effect.[12]

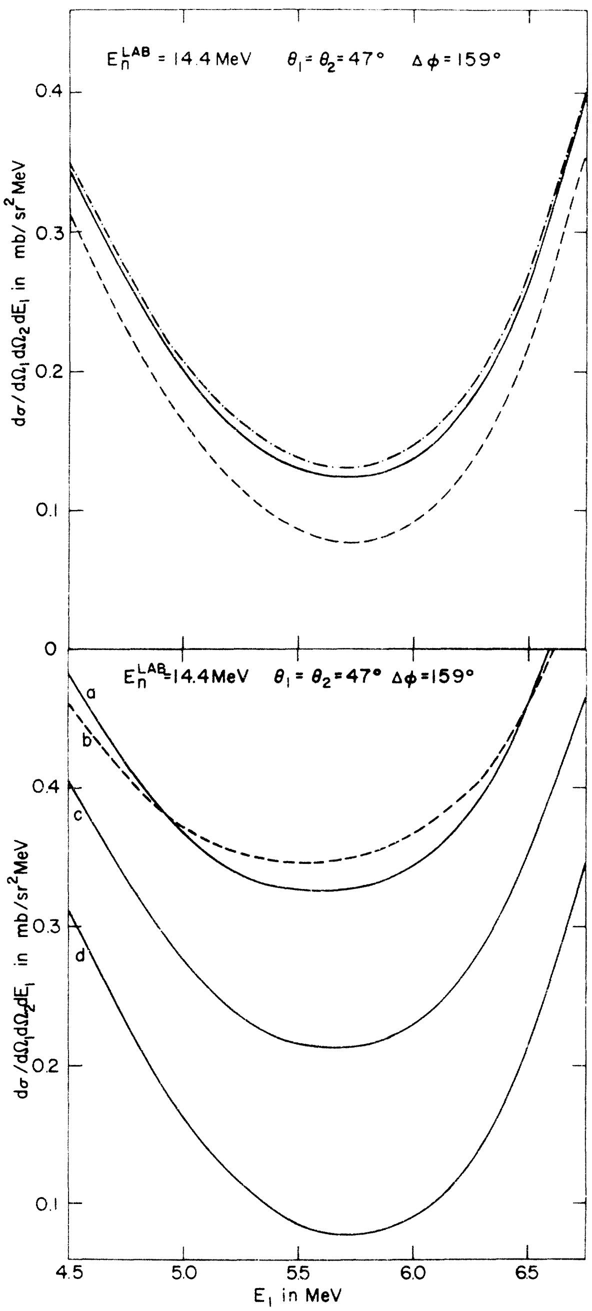

FIG 1. Differential cross section for n-d breakup versus the energy of an outcoming neutron. The lower set of curves arise from models which produce identical behavior in the low-energy two-nucleon system: a corresponds to the Malfliet-Tjon I-IV potential, b to a separable potential, c to the I-III potential, and d to the $A=0$ version of this analysis. The dashed curve in the upper half of the figure is the same as d; the solid and dash-dotted curves represent the maximum differences exhibited over the entire A set.

(3) There is no point in the scheme proposed by Jain, Rogers, and Saylor[8] unless it is demonstrably more accurate than methods for directly measuring a_2.

(4) In testing for "off-shell" sensitivities it is essential to employ phase-equivalent models in order to avoid misinterpretation.

(5) By use of the technique demonstrated in this paper, one can effectively eliminate ambiguities due to off-shell effects. This makes it possible to use three-body reactions as highly accurate probes of hard-to-measure two-body parameters, which may turn out to be their most useful feature.

(6) There is no point in employing complicated "realistic" potentials directly in scattering calculations. A vast amount of labor can be saved by employing the boundary-condition approach, normalizing the input function A to the values of E_T or a_2 produced by the model.

(7) It is highly unlikely that such disagreements as exist between present model calculations and experiment can be resolved via an appeal to off-shell effects. The explanation almost certainly lies in the neglect of small components in the two-particle interaction, e.g., higher partial waves.

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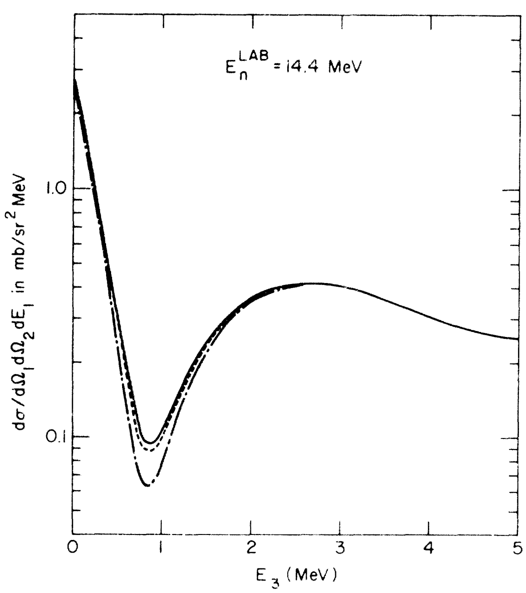

FIG. 2. Differential cross section along the locus prescribed in Ref. 8. The lower curve corresponds to $A=0$; the two upper curves summarize variation over the A set.

of this work, and to acknowledge the cooperation of the Computer Science Center of the University of Maryland where the calculations were performed.

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[1]A strong correlation of E_T and a_2 was first noted by A. C. Phillips, Nucl. Phys. A107, 209 (1968), and has been extensively corroborated by a multitude of subsequent authors. The form-factor minimum is somewhat ambiguous because of unknown exchange corrections, but a correlation with E_T is suggested by the phase-equivalent calculations of M. I. Haftel, in *Few Particle Problems in the Nuclear Interaction*, edited by I. Šlaus, S. A. Moszkowski, R. P. Haddock, and W. T. H. Van Oers (Elsevier, New York, 1973). Unfortunately, the trend here is opposite to what one would wish.

[2]W. M. Kloet and J. A. Tjon, Nucl. Phys. A210, 380 (1973).

[3]D. D. Brayshaw, Phys. Rev. D 8, 952 (1973); hereafter we shall refer to this as BC1. Although mathematically distinct, this work owes much in spirit to the earlier development of H. P. Noyes, Phys. Rev. D 5, 1547 (1972).

[4]A preliminary investigation of this model for a system of three spinless particles, together with a discussion of appropriate numerical techniques, may be found in D. D. Brayshaw, Phys. Rev. D 8, 2572 (1973).

[5]An alternative choice would have been to fix E_T. However, in the absence of tensor components in this model, taking a_2 as the standard seemed more appropriate. The actual curves for $k \cot\delta$ produced are identical with those obtained in a simple model calculation; see D. D. Brayshaw and B. Buck, Phys. Rev. Lett. 24, 733 (1970).

[6]G. Barton and A. C. Phillips, Phys. Lett. 28B, 378 (1969).

[7]W. M. Kloet and J. A. Tjon, in *Few Particle Problems in the Nuclear Interaction*, edited by I. Šlaus, S. A. Moszkowski, R. P. Haddock, and W. T. H. Van Oers (Elsevier, New York, 1973).

[8]M. Jain, J. G. Rogers, and D. P. Saylor, Phys. Rev. Lett. 31, 838 (1973).

[9]One might also question whether variations in the vicinity of such an interference minimum will not be washed out by small components which one can usually ignore, such as higher partial waves in the two-body force.

[10]In this discussion we have not dealt with the spin observables; however, model calculations do not indicate a sensitivity to off-shell effects. See, for example, I. H. Sloan and J. C. Aarons, Nucl. Phys. A198, 321 (1972). Also, although this analysis has been restricted to energies less than 30 MeV, the continued success of simple separable models at much higher energies would seem to preclude much off-shell sensitivity; see J. M. Wallace, Phys. Rev. C 7, 10 (1973).

[11]D. D. Brayshaw, Phys. Rev. C 7, 1731 (1973).

[12]The Phillips correlation was maintained in this analysis to within a shift of 0.4 MeV in E_T, which is comparable to estimates of relativistic corrections. Underlying this conclusion is the assumption of a detailed fit to the two-nucleon data; one must impose all constraints *simultaneously*. A possible exception is the charge form factor, which *may* be capable of discriminating between such alternatives if exchange corrections are unimportant.

Muon Pair Production by Photon-Photon Interactions in e^+e^- Storage Rings

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The photon-photon interaction has been investigated by e^+ and e^- collisions at about 2.7-GeV total energy. Evidence based on 34 well-identified events has been obtained for the process $e^+e^- \to e^+e^-\mu^+\mu^-$, hitherto unobserved. Such a process is found to occur in agreement with theoretical predictions based on the equivalent-photon approximation. Results on 74 events from the process $e^+e^- \to e^+e^-e^+e^-$ are also reported.

Electron colliding beams provide a means, at present unique, for investigating the photon-photon interaction at high energy, as pointed out by many authors.[1] In the present experiment the outgoing $e^{+,-}$ are detected at very small angles with respect to their incident directions, in coinci-