the laser perturbation only.

The results presented here do not agree with the statements made in Ref. 1 where an *ad hoc* argument shows that the implosion is stabilized by ablation. Two-dimensional calculations, using cylindrical symmetric but nonuniform laser heating, have also shown that Taylor instability will occur unless the laser pulse is tailored in frequency and unless a sufficient low-density atmosphere is maintained around the pellet.[12] The ablative stabilization argument must then be interpreted as indicative of the fact that stability can be achieved only if a large enough region of sufficiently hot plasma is maintained between the critical and ablating surfaces.

In conclusion, we have observed that the laser-driven spherical implosion is linearly unstable to perturbations at the ablating surface and that these instabilities are present even with uniform laser heating. Nonuniform heating leads to an earlier and more rapid growth of the instabilities. The perturbations can amplify several orders of magnitude in a few nanoseconds, but as soon as the perturbations begin to grow the linear approximation breaks down. Further computations must be performed to determine the parameter range (laser frequency history, initial low-density atmosphere created by a prepulse, initial heating rate, etc.) in which a stable implosion can be obtained. A realistic multidimensional, nonlinear treatment is required to follow the complete evolution of the disturbances which occur in the unstable cases.

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[1]J. Nuckolls, L. Wood, A. Thiessen, and G. Zimmerman, Nature (London) 239, 139 (1972).

[2]J. S. Clarke, H. N. Fisher, and R. J. Mason, Phys. Rev. Lett 30, 89 (1973).

[3]K. A. Brueckner, IEEE Trans. Plasma Sci. 1, 13 (1973).

[4]J. Taylor, Proc. Roy. Soc., Ser. A 201, 192 (1950).

[5]E. B. Goldman, Plasma Phys. 15, 289 (1973).

[6]L. Spitzer, *Physics of Fully Ionized Gases* (Interscience, New York, 1956).

[7]E. B. Goldman, Laboratory for Laser Energetics, University of Rochester Report No. 16, 1973 (unpublished). The zero-order code reproduces the spherically symmetric hydrodynamic results given in Refs. 1 and 3.

[8]J. N. Shiau and A. Simon, Phys. Rev. Lett. 29, 1664 (1972); J. N. Shiau and E. B. Goldman, Bull. Amer. Phys. Soc. 18, 1358 (1973); D. B. Henderson and R. L. Morse, Bull. Amer. Phys. Soc. 18, 684 (1973). The application of spherical harmonic decomposition of the perturbation is discussed in detail by G. E. Backus, Geophys. J. Roy. Astron. Soc. 13, 71 (1967).

[9]M. S. Plesset, J. Appl. Phys. 25, 96 (1954).

[10]The pulse given in Ref. 1 is of the form $\dot{E}_0(1-t/t')^{-s}$ with $s=\frac{15}{8}$. Here we use $t'=9.595$ nsec and $\dot{E}_0=1.03\times10^{11}$ W.

[11]J. P. Somon, in *Physics of High Energy Densities, Proceedings of the International School of Physics "Enrico Fermi," Course XLVIII*, edited by P. Caldirola (Academic, New York, 1971).

[12]J. Nuckolls, Lawrence Radiation Laboratory, Livermore, Report No. UCRL-74345, 1973 (unpublished).

Symmetry of Laser-Driven Implosions

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A perturbation method is developed for nearly spherical heat and fluid flow. It shows that electron thermal conduction may be sufficiently symmetrizing to allow laser-driven fusion with a single beam.

One-dimensional, spherically symmetric calculations have predicted that small spheres[1,2] or spherical shells[2] of thermonuclear fuel can be made to give economical nuclear-fusion energy yields when imploded and heated by as little as 1 kJ of absorbed laser-pulse energy. These predictions depend in an essential way on the spherical character of the flow. The parameter of merit of the imploded core of the pellet of fuel is ρR (density × radius), and only through a spherically converging flow can the necessary values of about 0.1 g/cm^2 or more[3] be achieved at thermonuclear temperatures with such pulse energies. At that point in an implosion when asphericity of the pellet core becomes of the order of pellet-core dimensions, then ρR, which is proportional to $1/R^2$, stops increasing and the implosion is effectively terminated. Because such premature termination of implosions, especially implosions of large-aspect-ratio shells,[2] appears to be one of the most important problems faced by laser fusion, we have developed a perturbation method for analyzing asymmetries of such spherical flows.

The unperturbed, or zero-order, flow is an angle-independent radial flow. The method is fundamentally Eulerian in that first-order quantities are defined on unperturbed trajectories. It is, however, useful to introduce the perturbed displacement $\vec{\xi}$, defined through

$$d\vec{\xi}/dt-(\vec{\xi}\cdot\nabla)\vec{v}_0=\vec{v}_1, \tag{1}$$

where $\vec{v}_0(\vec{r},t)$ and $\vec{v}_1(\vec{r},t)$ are zero and first-order velocity. Linearizing Euler's equation with respect to pressure, density, and velocity gives

$$\frac{d\vec{v}_1}{dt}+(\vec{v}_1\cdot\nabla)\vec{v}_0=-\frac{\rho_1}{\rho_0}\frac{d\vec{v}_0}{dt}\frac{\nabla p_1}{\rho_0}\equiv\vec{a}_1. \tag{2}$$

The perturbed density is then

$$\rho_1=-\vec{\xi}\cdot\nabla\rho_0-\rho_0\nabla\cdot\vec{\xi}+\rho_0\rho_{1I}/\rho_{0I}\equiv\rho_{1C}+\rho_{1L}. \tag{3}$$

The perturbed pressure, temperature, and other auxiliary state variables are given by expressions of the form

$$\begin{aligned}p_1&=-\vec{\xi}\cdot\nabla p_0+\rho_{1L}(\partial p/\partial\rho)_S+S_{1L}(\partial p/\partial S)_\rho\\&\equiv p_{1C}+p_{1L},\end{aligned} \tag{4}$$

where I, C, and L denote initial, convective, and local parts, respectively. The specific entropy S and the density ρ are taken to be the independent state variables. The local entropy S_{1L} is then advanced according to

$$\frac{dS_{1L}}{dt}=\frac{1}{\rho_0T_0}\left[\frac{dQ_1}{dt}-\left(\frac{\rho_{1L}}{\rho_0}+\frac{T_{1L}}{T_0}\right)\frac{dQ_0}{dt}\right], \tag{5}$$

where Q_1 is obtained from the perturbed viscous dissipation, external heat sources, and heat flow, the last being described by

$$dQ_{1K}/dt=\nabla\cdot(\kappa_1\nabla T_0+\kappa_0\nabla T_1). \tag{6}$$

The Landshoff-Spitzer[4] κ and the ideal-gas equation of state are used in the calculations below.

When all first-order quantities are assumed to have the form $\rho_1(\vec{r},t)=\rho_1(\vec{r},t)Y_l^m(\Omega)$, Eq. (2) can, after some integration, be put in the form

$$\begin{aligned}&dA/dt=a_1/\chi,\quad v_{r1}=A\chi;\\&dB/dt=A\chi^2,\quad \xi_r=B/\chi;\\&dC/dt=l(l+1)p_1/\rho_0,\quad C=r^2(\nabla\cdot\vec{v}_1)_\Omega;\\&dD/dt=C/r^2,\quad D=(\nabla\cdot\vec{\xi})_\Omega;\end{aligned} \tag{7}$$

where $\chi\equiv dr_I/dr$. Equation (6) becomes

$$\begin{aligned}\frac{dQ_{1K}}{dt}&=\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\left(\kappa_1\frac{\partial T_0}{\partial r}+\kappa_0\frac{\partial T_1}{\partial r}\right)\right]\\&\quad-\frac{\kappa_0}{r^2}l(l+1)T_1,\end{aligned} \tag{8}$$

and Eqs. (3)–(5) are unchanged except that now all first-order quantities are coefficients of a particular $Y_l^m(\Omega)$. We have found, therefore, that if the viscosity and thermal conductivity are taken to be scalar, the equations of motion for different l's decouple and are degenerate with respect to m, with obvious important advantages for computation and interpretation.[5] A three-dimensional analysis, not two-dimensional as in (r,z) hydrodynamics codes, is obtained with about the complexity and computational cost of a one-dimensional calculation. Taking only the angular thermal relaxation term from Eq. (8) and the assumed form $T_1\sim e^{-t/\tau}$ gives an angular symmetrizing time

$$\tau=\frac{3}{2}\frac{k(n_e+n_i)r^2}{l(l+1)\kappa_0}=\frac{n_er^2\ln\Lambda_{ei}}{[T(\text{eV})]^{5/2}l(l+1)}\times10^{-21}, \tag{9}$$

the second form being for hydrogen only. For representative values of blowoff-cloud parameters, we take $n_e\gtrsim10^{21}$, $r\gtrsim10^{-2}$, $T\gtrsim5\times10^3$, and $\ln\Lambda_{ei}\simeq5$, and find that τ can be as small as $(10^{-12}\ \text{sec})/l(l+1)$.

In this Letter the equations derived above are used to show an important result: Electron thermal conduction in angular directions has a very significant effect in making the ablation surface pressure, and thus the implosion, much more symmetrical than does the deposition of laser energy at the critical surface.

Whether a pellet starts as a sphere or shell, ablation pressure forms a shell of higher-than-initial density just inside the ablation surface. If the pressure is asymmetric, then parts of this shell will reach the center ahead of others and may prevent the achievement of the necessary high densities. It is, therefore, necessary

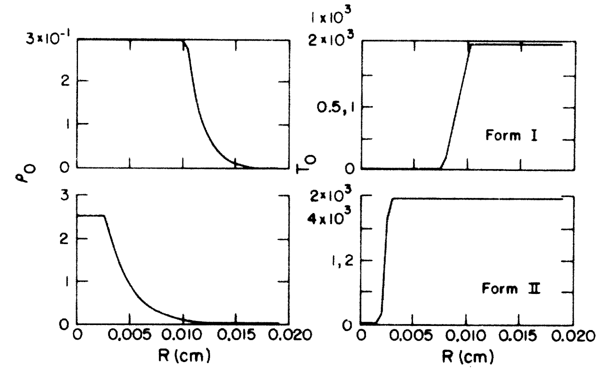

FIG. 1. Zero-order density and temperature profiles. Note two temperature scales for each form. The units throughout are cgs; temperatures are in electron volts.

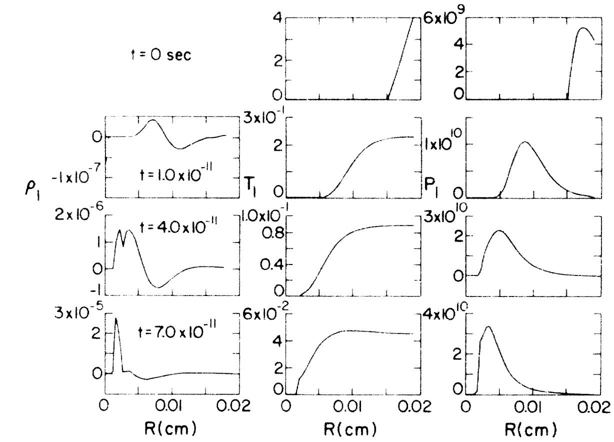

FIG. 2. Time sequences of first-order quantities from form-II, 2-keV, $l = 0$ calculations.

that the distortion

$$\delta_{R,l} \equiv \xi_r / r_s = (1/r_s) \int v_{r1,l}\, dt = (1/r_s) \int_{r_{sI}}^{r_{sF}} (v_{r1,l}/v_{r0})\, dr \qquad (10)$$

be sufficiently small for all l's. The distortion represents the ratio of the perturbed motion of the shell to the instantaneous shell radius r_s. The allowed distortions may be up to 1.0 representing 100% modulation.

An impulse approximation is now applied in which idealized sets of $\rho_0(r), T_0(r)$ profiles of deuterium-tritium mixtures are assumed and held fixed in time, on the ground that they do not change significantly during the time of interest for the required first-order calculations. Calculations with self-consistent zero-order flows have shown the important additional result that the ablation process is itself positively stable. These results will be published separately.

The four zero-order cases used here are the two forms, I and II, in Fig. 1, with two temperature scales each. In both, $r_{\text{crit}} = 2\times10^{-2}$ cm (where $\rho_{\text{crit}} = 5\times10^{-3}$ g cm^{-3}). In I the ratio of critical-to-ablation surface radii is seen to be 2, typical of earlier times; in II the ratio is 8. These density profiles have filled centers but are thought of for present purposes, which do not directly involve the center, as representing a variety of more or less hollow profiles. The temperature is then given the same initial perturbation $T_1(r, t=0)$, localized near r_{crit} for all l. For $l=0$ this perturbation corresponds to an increment of the zero-order absorbed energy, and for $l>0$ to an asymmetry of the absorbed energy which is equal in amplitude to the $l=0$ increment. The contribution of the absorbed-energy input asymmetry at the time represented by the chosen ρ_0, T_0 profiles to the asymmetry of the system at a given l can be estimated by taking the ratio of the $l>0$ and $l=0$ linear responses of the system for the given $T_1(r, t=0)$ to be the asymmetry resulting from 100% input asymmetry at that l. In particular, the ratio $v_{r1,l}/v_{r0}$ in Eq. (10) is taken to be the value of $v_{r1,l}/v_{r1,0}$ at the shell, obtained after the impulse contributions of $T_1(r, t=0)$ to the ablation-driven implosion has occurred, because strong pulse shaping[1,2] makes late-time values of v_{r1}/v_{r0} almost independent of early-time contributions. The asymmetry caused by smaller input modulations is then scaled down from this value. Note that $Y_l^m(\Omega)$ can have l or more maxima, depending on m, which may indicate the correspondence to the number of incident beams.

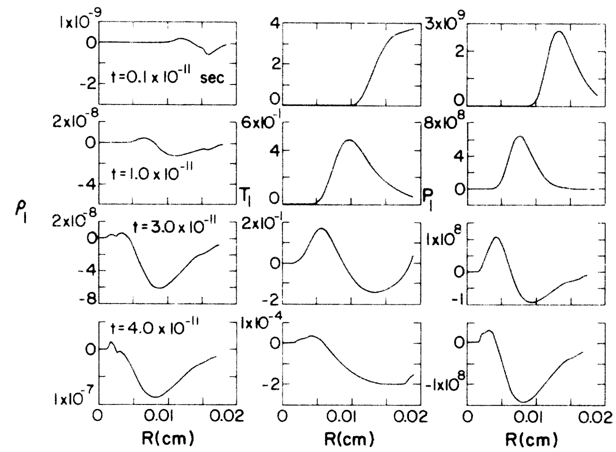

FIG. 3. Time sequences of first-order quantities from form-II, 2-keV, $l = 4$ calculations.

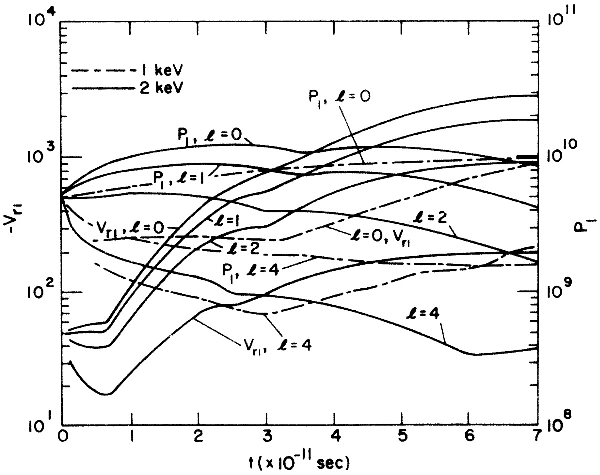

FIG. 4. Time histories of p_1 and v_{r1} maxima for form-I calculations.

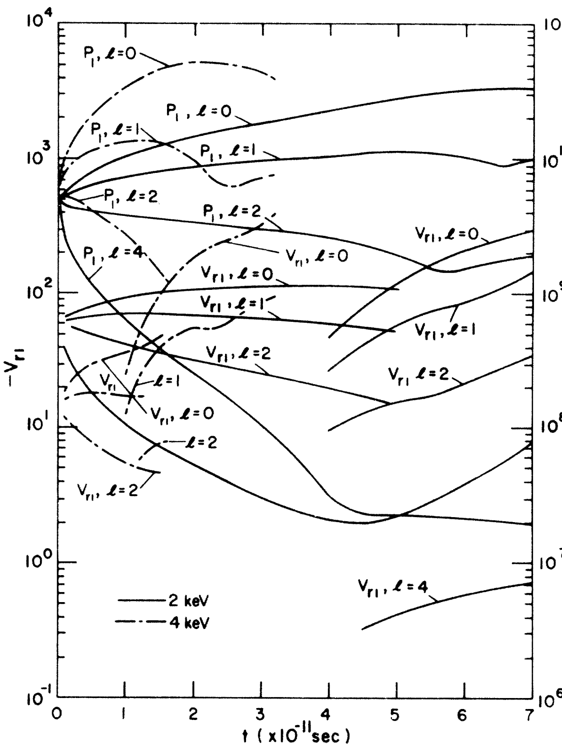

FIG. 5. Time histories of p_1 and v_{r1} maxima for form-II calculations.

Figures 2 and 3 show the computed $l=0$ and 4 responses of the form-II, $T_{0,\max}=2$-keV profiles to the $T_1(r, t=0)$ shown in Fig. 2. The results are insensitive to the particular form of $T_1(r, t=0)$ as long as the form is well localized near the critical surface where the energy is absorbed. The very fast diffusive relaxation of short-wavelength structure makes other details truly unimportant. The chosen magnitude of $T_1(r, t=0)$ is arbitrary because this is a linear theory. Figure 2 shows a thermal wave propagating rapidly inward, giving increasing ρ_1 as it reaches regions of larger ρ_0, causing only a small acoustic response until it reaches the ablation front. There it makes its contribution to ablation by launching strong acoustic waves inward (implosion) and outward (blowoff recoil), at about $t=4.0\times10^{-11}$ sec. The $l=4$ response, Fig. 3, shows the thermal and pressure waves being attenuated very strongly by angular thermal conduction, and to some extent by the angular acoustic response. Figures 4 and 5 show the maxima over r of the $p_1(r)$ and $v_{r1}(r)$ values as a function of time for all four zero-order profiles and assorted l values.

From Figs. 4 and 5 one can see a general improvement in implosion symmetry in going to higher temperatures and larger ratios of critical-to-ablation surface radii, both associated with latter times in a given implosion. *The improvement with increasing critical radius is a strong reason for us to favor larger laser-light wavelengths.*

It is seen from Fig. 4 that energy absorption with 100% angular modulation is sufficiently smoothed by thermal conduction at early implosion times to give a contribution to the dr integration in Eq. (10) of 1.0 or less, only if $l=4$ or larger. At later times and higher temperatures, Fig. 5, $l=2$ symmetry is sufficient if not $l=1$. Moreover, $l=1$ perturbations are not in all cases disruptive since they may only shift the implosion center. Within the limitations of the perturbation and impulse approximations, we conclude that while the equivalent of four or more evenly spaced beams may be required to initiate an adequately symmetric implosion, the subsequent, and more intense, irradiation may be carried by as few as one beam.

[1]J. Nuckolls, L. Wood, A. Thiessen, and G. Zimmerman, Nature (London) 239, 139 (1972).

[2]J. S. Clarke, H. N. Fisher, and R. J. Mason, Phys. Rev. Lett. 30, 89, 249 (1973).

[3]G. S. Fraley, E. J. Linnebur, R. J. Mason, and R. L. Morse, "Thermonuclear Burn Characteristics of Compressed DT Microspheres" (to be published).

[4]R. Landshoff, Phys. Rev. 76, 904 (1949), and 82, 442 (1951); L Spitzer, *Physics of Fully Ionized Gases* (Interscience, New York, 1956), Chap. V.

[5]D. B. Henderson and R. L. Morse, Bull. Amer. Phys. Soc. 18, 684, 1360 (1973). Also R. L. Morse, Bull. Amer. Phys. Soc. 18, 1359 (1973).