and have found that OH fluorescence is independent of water concentration and of the type of carrier gas. Since the amount of OH generated is proportional to water concentration, the observed results can be obtained only if quenching is predominantly due to collisions with water molecules.

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Linear Stability Analysis of Laser-Driven Spherical Implosions*

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A linear approximation method has been developed to study the stability of spherically symmetric hydrodynamic motion. Application of this technique to laser-driven spherical implosions indicates that the implosions are linearly unstable to perturbations at the ablating surface.

The proposed schemes[1-3] for laser-driven implosion of deuterium-tritium pellets to the superdense state necessary for thermonuclear burning assume perfect spherical symmetry. Unavoidable departures from spherical symmetry certainly limit the compressions achievable in practice and raise the laser energy required for breakeven. Departures from symmetry during implosion arise from inherent hydrodynamic instabilities and these can be accelerated by nonuniform laser illumination. Nonuniform laser energy deposition leads to early-time nonuniform accelerations which in turn cause the earlier and more rapid growth of the instabilities.

The implosion consists of a dense layer of plasma accelerated by material being ablated into the hot, low-density blowoff (see Figs. 1 and 2). Similar conditions occur in the Rayleigh-Taylor instability which arises when two superposed fluids of different densities are accelerated in a direction perpendicular to their interface.[4] The interface is unstable if the acceleration is directed from the less dense to the more dense fluid. For two inviscid, incompressible fluids with large density differences, separated by a plane interface, the growth rate of the perturbation on the interface is given[4] by $\gamma = (2\pi a/\lambda)^{1/2}$, where a is the acceleration and λ the wavelength of the perturbation. For laser-driven implosions, the accelerations are $\simeq 10^{16}$ cm/sec^2 giving growth rates of 10^9 sec^{-1} for perturbations with wavelengths of 100 μm. The inviscid growth rates increase for shorter wavelengths but, below a certain wavelength, dissipative effects can no longer be ignored. In addition to compressive effects, the laser-driven implosion differs from the Rayleigh-Taylor problem through the presence of a sharp temperature gradient and the ablating flow. These change the basic unperturbed solutions and warrant a detailed stability analysis.

The evolution of dense laser plasmas is described by a one-velocity, two-temperature set of hydrodynamic equations[5] which include separate species temperatures and thermal conductivities as well as Coulomb energy exchange between the species. To study the stability of the flow, all of the Lagrangian variables are resolved into a zero-order, spherically symmetric component

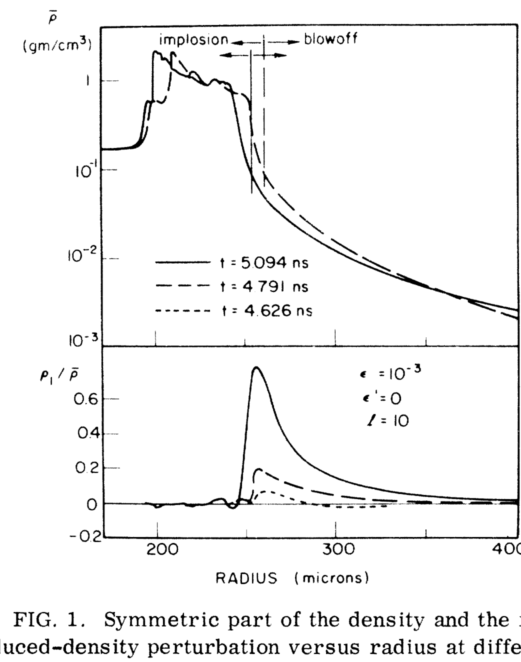

FIG. 1. Symmetric part of the density and the reduced-density perturbation versus radius at different times. Vertical lines indicate positions of ablating surface.

and a small perturbation

$$\psi=\bar{\psi}(j,t)+\sum_{l,m}\tilde{\psi}^{lm}(j,t)Y_{lm}(\theta,\varphi), \qquad (1)$$

where j is the spherical Lagrangian coordinate and the perturbation has been decomposed into spherical harmonics. Here ψ represents any of the set of fluid variables including the radial coordinate, radial velocity, radial vorticity, divergence of the velocity, density, pressure, and temperature. The energy deposition term can also be expanded in the form (1).

Through linearization of the basic equations, we obtain a new self-consistent set of equations for the spectral components of the perturbations. In particular, the equations governing the radial component of the perturbed velocity $\tilde{u}_r$ and the perturbed electron temperature $\tilde{T}_e$ take the form

$$\bar{\rho}\frac{d\tilde{u}_r}{dt}=-\frac{\partial\tilde{p}}{\partial\bar{r}}+\frac{\partial\bar{p}}{\partial\bar{r}}\left(\frac{\tilde{\rho}}{\bar{\rho}}+\frac{\partial\tilde{r}}{\partial\bar{r}}\right), \qquad (2)$$

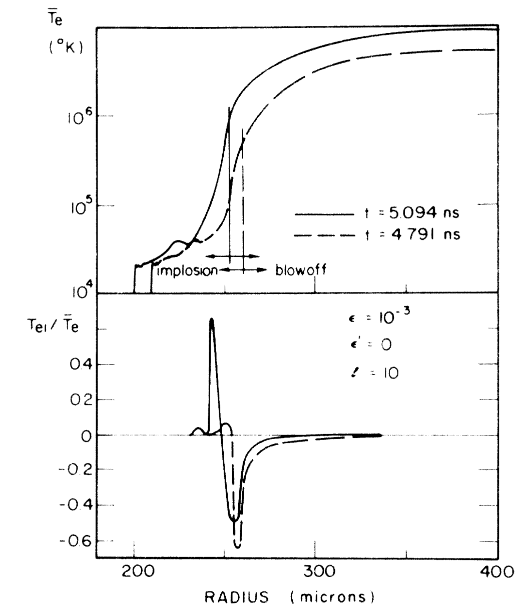

FIG. 2. Symmetric part of the electron temperature and the reduced-electron-temperature perturbation versus radius at different times. Vertical lines indicate positions of ablating surface.

and

$$\bar{\rho}_e c_v\frac{d\tilde{T}_e}{dt}=-\bar{P}_e\tilde{s}-\tilde{P}_e\bar{s}+\bar{P}_e\bar{s}\frac{\tilde{\rho}}{\bar{\rho}}-\frac{1}{\bar{r}^2}\left(2\frac{\tilde{r}}{\bar{r}}+\frac{\partial\tilde{r}}{\partial\bar{r}}+\frac{\tilde{\rho}}{\bar{\rho}}\right)\frac{\partial}{\partial\bar{r}}\left(\bar{r}^2\bar{\kappa}_e\frac{\partial\bar{T}_e}{\partial\bar{r}}\right)+\frac{1}{\bar{r}^2}\frac{\partial}{\partial\bar{r}}\left[\bar{r}^2\bar{\kappa}_e\frac{\partial\bar{T}_e}{\partial\bar{r}}\left(2\frac{\tilde{r}}{\bar{r}}+2.5\frac{\tilde{T}_e}{\bar{T}_e}-\frac{\partial\tilde{r}}{\partial\bar{r}}+\frac{\partial\tilde{T}_e}{\partial\bar{T}_e}\right)\right]$$
$$-\frac{\bar{\kappa}_e}{\bar{r}^2}l(l+1)\left(\tilde{T}_e-\frac{\partial\bar{T}_e}{\partial\bar{r}}\tilde{r}\right)-\frac{3\bar{n}_e k(\tilde{T}_e-\tilde{T}_i)}{2\bar{\tau}_{ei}}+\frac{3\bar{n}_e k(\bar{T}_e-\bar{T}_i)}{2\bar{\tau}_{ei}^{\,2}}\tilde{\tau}_{ei}+\bar{\rho}_e\tilde{W}, \qquad (3)$$

where r is the radial coordinate, p the pressure, ρ the density, s the divergence of the velocity, c_v and κ_e the specific heat and thermal conductivity, respectively, of the electrons (given by Spitzer[6]), τ_{ei} the electron-ion relaxation time, n the number density, k the Boltzmann constant, and $\tilde{W}$ the nonuniform laser-energy deposition (the superscripts have been surpressed). In addition to Eqs. (2) and (3) there are equations for the perturbed density, divergence of velocity, radial component of vorticity, ion energy, and state of the plasma. These equations are decoupled for each l mode and degenerate with respect to the m wave number. The zero-order, spherically symmetric quantities appear as coefficients in these perturbed equations.

The zero-order solutions are computed from a spherically symmetric Lagrangian code[7] which is decoupled from the perturbed solutions. An addition to this code has been developed to compute the evolution of the spectral components simultaneously with the advance of the zero-order results. This initial-value-problem approach to stability analysis has been used previously in the analysis of ionospheric instabilities.[8]

As a check on our methods, we have applied the extended code to some test problems with known analytic solutions. First, we considered the case of a spherical shell of almost incompressible fluid accelerated radially outward by internal pressure. The inner surface is unstable and the computed growth of perturbations on this surface agrees with that obtained analytically.[9] Second, normal modes of sound waves inside a spherical cavity were calculated including the effects of thermal dissipation. The solutions were observed to oscillate with the correct eigenfrequency and to exhibit the correct damping decrement.

To examine the stability of the laser-driven implosions, we consider a deuterium-tritium pellet with a dense 400-μm-radius core surrounded by a low-density corona. The pellet is irradiated by an optimally tailored[10] 60-kJ pulse, and all of the laser energy is assumed to be absorbed at the critical surface into an electron Maxwellian distribution (as has been assumed in previous spherical calculations).[1-3] Typical zero-order

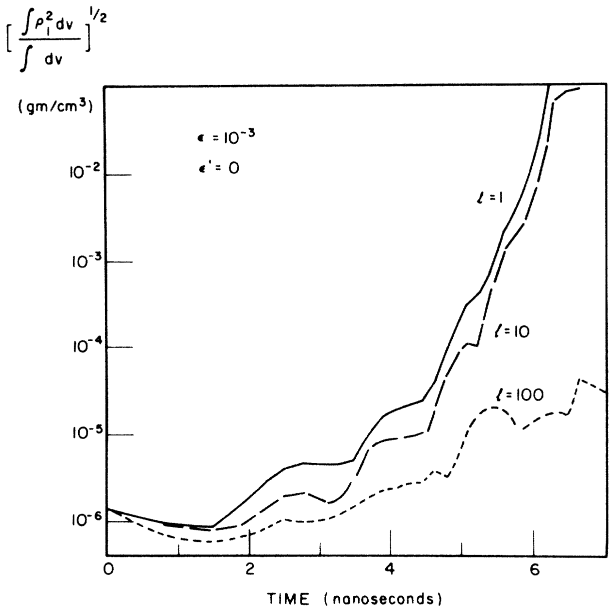

FIG. 3. Root-mean-square density perturbations for different modes as functions of time for the case with initial random density perturbations of level 10^{-3} (uniform heating).

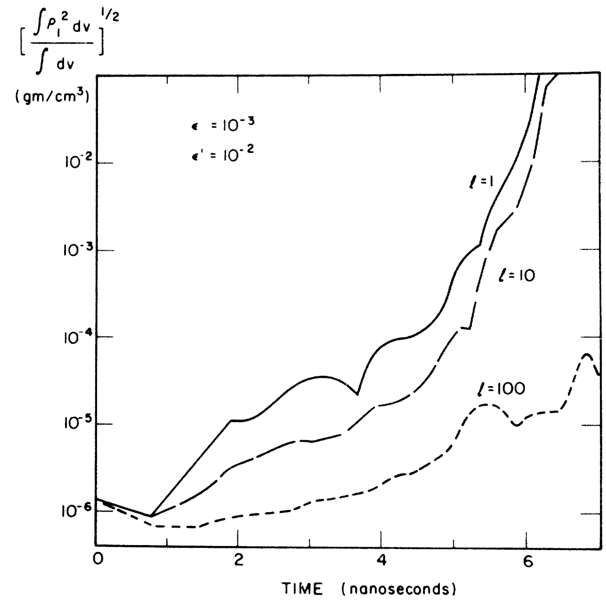

FIG. 4. Root-mean-square density perturbations for different modes as functions of time for initial random density perturbations of level 10^{-3} and nonuniform heating of 1%.

density and temperature profiles are shown in Figs. 1 and 2. To start the perturbation calculation, we consider an initial random density perturbation of amplitude ϵ and observe the spatial distributions of growing modes shown in Figs. 1 and 2. In Fig. 1 we plot the reduced-density perturbations $\rho_1/\bar{\rho}$ as a function of r for the $l = 10$ mode at different times, where $\rho_1 = \tilde{\rho} - (\partial\bar{\rho}/\partial\bar{r})\tilde{r}$ is the Eulerian density perturbation. Figure 2 is a similar plot for the electron temperature perturbations. These results are representative of the linear instabilities found in all of our calculations.

The initial random density perturbation was observed to grow several orders of magnitude in a few nanoseconds. The perturbations grow at the ablating surface and are propagating inward faster than the ablating flow can convect them away. To characterize the growth rates, we plot the rms density perturbations versus time for different modes in Fig. 3. Dissipation such as arises in the $l(l+1)$ lateral-heat-conduction term in Eq. (3) tends to damp the modes with shorter wavelengths. It can be seen that modes with large l are damped, while the growth rates for lightly damped modes amplify with time as the radial acceleration grows. We also observe some slow growth of the perturbations at shock fronts, and these may be related to instability of converging shocks.[11] (As another check, we ran the code without the laser pulse and observed no anomalous growth of the initial perturbations.)

In a reference frame moving locally with the ablating surface, one sees an approximately stationary density gradient. This reference frame is accelerated radially inward and fluid elements in this frame experience a radially outward inertia force. This force, coupled with the zeroth-order density gradient, drives the instability, and initial density perturbations at or near the ablating surface are thus amplified.

We have also performed a series of computations with a perturbation of level ϵ' in the laser illumination [$\tilde{W}$ in Eq. (3)] in addition to the density perturbation of level ϵ. The rms density perturbations are shown in Fig. 4. Although lateral heat conduction will eventually smooth the laser-generated temperature perturbations in the region between the critical and ablating surfaces,[1] the early-time temperature perturbations lead to enhanced density perturbations. This is easily seen in a comparison of Figs. 3 and 4 where the lightly damped modes go unstable earlier with the added heating perturbation and grow more rapidly. A similar type of behavior is also observed when we perform the computation with the laser perturbation only.

The results presented here do not agree with the statements made in Ref. 1 where an *ad hoc* argument shows that the implosion is stabilized by ablation. Two-dimensional calculations, using cylindrical symmetric but nonuniform laser heating, have also shown that Taylor instability will occur unless the laser pulse is tailored in frequency and unless a sufficient low-density atmosphere is maintained around the pellet.[12] The ablative stabilization argument must then be interpreted as indicative of the fact that stability can be achieved only if a large enough region of sufficiently hot plasma is maintained between the critical and ablating surfaces.

In conclusion, we have observed that the laser-driven spherical implosion is linearly unstable to perturbations at the ablating surface and that these instabilities are present even with uniform laser heating. Nonuniform heating leads to an earlier and more rapid growth of the instabilities. The perturbations can amplify several orders of magnitude in a few nanoseconds, but as soon as the perturbations begin to grow the linear approximation breaks down. Further computations must be performed to determine the parameter range (laser frequency history, initial low-density atmosphere created by a prepulse, initial heating rate, etc.) in which a stable implosion can be obtained. A realistic multidimensional, nonlinear treatment is required to follow the complete evolution of the disturbances which occur in the unstable cases.

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Symmetry of Laser-Driven Implosions

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A perturbation method is developed for nearly spherical heat and fluid flow. It shows that electron thermal conduction may be sufficiently symmetrizing to allow laser-driven fusion with a single beam.

One-dimensional, spherically symmetric calculations have predicted that small spheres[1,2] or spherical shells[2] of thermonuclear fuel can be made to give economical nuclear-fusion energy yields when imploded and heated by as little as 1 kJ of absorbed laser-pulse energy. These predictions depend in an essential way on the spherical character of the flow. The parameter of merit of the imploded core of the pellet of fuel is ρR (density $\times$ radius), and only through a spherically converging flow can the necessary values of about 0.1 g/cm^2 or more[3] be achieved at thermo-