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Evidence for Higher Shape Deformations in Muonic X Rays*

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We have fitted the K x rays and hyperfine splittings from muonic ^{232}Th and ^{238}U by a model with a Fermi charge distribution with distortion terms of the type $\beta_{2n} Y_{2n,0}$, $n \leq 3$. We have also reanalyzed for $n \leq 2$ the data for rare-earth deformed muonic atoms. Our results are compared with those of inelastic proton and α scattering as well as Coulomb excitation. Such muonic analyses may provide the preferred determinations of quadrupole moments.

We have used the measured values of the K x rays, hyperfine splittings, and electric quadrupole moments from muonic ^{232}Th and ^{238}U to fit the parameters of a distorted Fermi-type nuclear charge distribution of the form

$$\rho(r, \theta) = \rho_0\{1 + \exp[r/a - (1 + \beta_2 Y_{20} + \beta_4 Y_{40} + \beta_6 Y_{60})c/a]\}^{-1}. \tag{1}$$

Normalization to the total nuclear charge Z determines ρ_0; a is related to the skin thickness t by

$$t = 4a \ln 3; \tag{2}$$

c is the spherical half-density radius and the β_n are the usual deformation parameters. Equation (1) is an extension to higher deformations of the charge distribution used by Hitlin *et al.*,[1] thus facilitating the comparison of our analysis with theirs.

The data for ^{232}Th and ^{238}U were collected at the 600-MeV synchrocyclotron at the Space Radiation Effects Laboratory of the National Aeronautics and Space Administration. The pion beam was produced with a thin carbon filament as an internal target. The backward-decaying muons were subsequently focused onto the target. A standard beam-counter telescope was used, and the muonic x-ray detector was a large-volume, high-resolution Ge(Li) detector. The Space Radiation Effects Laboratory IBM 360/44 computer was used to facilitate the collection of the data.

Peaks in the muonic x-ray spectra were found by fitting a skewed Gaussian line shape to the data. The energies were determined by using a calibration based on known γ-ray energies from sources and on the known energy difference between the single and double escape peaks. Table I lists the experimental muonic K x-ray energies for ^{232}Th and ^{238}U.

The analysis procedure made use of an automatic routine[2] which varied each of the parameters a, c, and β_n in the potential derived from Eq. (1), calculated the physical quantities of in-

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TABLE I. Experimental muonic K x-ray energies (in keV) for ^{232}Th and ^{238}U.

^{232}Th	^{238}U
6455.1	6564.3
6405.9	6558.4
6382.8	6518.2
6353.5	6480.3
6316.2	6461.8
6304.5	6453.8
6270.0	6416.0
6103.0	6409.1
6077.9	6379.6
6070.5	6167.3
6053.1	6148.8
6021.6	6140.4
5966.0	6122.0
	6096.6
	6046.4

TABLE II. Nuclear charge parameters resulting from fits to muonic x-ray data. Also included are the resulting quadrupole moment, number of degrees of freedom, and χ^2.

Nucleus	a (F)	c (F)	β_2	β_4	β_6	Q (b)	n	χ^2
^{232}Th	0.484	6.981	0.248	0.035	-0.022	9.69	9	7.1
^{238}U	0.471	7.050	0.277	0.013	-0.025	11.13	11	42.4
^{152}Sm[a]	0.542	5.903	0.287	0.063	-	5.94	2	11.7
^{162}Dy[a]	0.550	6.007	0.333	0.042	-	7.55	2	15.9
^{170}Er[a]	0.441	6.269	0.337	-0.001	-	8.02	2	21.2
^{182}W[a]	0.485	6.408	0.254	0.000	-	6.76	2	17.7

[a] Reanalyzed data from Ref. 1.

terest, and compared these quantities with the experimental values until χ^2 was minimized. The physical quantities (K x-ray energies, hyperfine splittings, and electric quadrupole moment) were obtained by numerically integrating the Dirac equation with a monopole potential derived from Eq. (1). To the resulting eigenvalues the following significant corrections were added: (a) vacuum polarization,[3] (b) Lamb shift,[4] and (c) nuclear polarization.[5] The fitting routine calculated (a) and (b) at each pass while (c) was added to the input data. The quadrupole moment was determined by numerical integration using Eq. (1). The quadrupole interaction Hamiltonian was then diagonalized in a basis formed from the product of the Dirac solutions and the wave functions of the symmetric, deformed-rotator nuclear model. Up to four nuclear excited states ($I \leq 8$) were used to form the model space. The results of the fitting procedure are presented in Table II. The uncertainty in the nuclear charge parameters can be estimated by noting the sensitivity of χ^2 to the variation of the values of the parameters about those values which yield the minimum χ^2. Keeping all the parameters fixed except one, this one parameter was varied until χ^2 increased by 1.0. This procedure, when applied to ^{232}Th and ^{238}U, leads to an uncertainty of 0.001 for a, c, and β_2, 0.002 for β_4, and 0.009 for β_6. However, the total uncertainty in any one parameter is also affected by the uncertainties in the theoretical corrections, particularly the nuclear polarization.

The published data from the rare-earth deformed muonic atoms[1] were reanalyzed using the same procedure, and the results are included in Table II. In both cases we have not fitted to the intensities since they are the most poorly fit (in Ref. 1 they account for about 85% of the total χ^2), and at least for the actinide data the number of K x-rays is sufficient to give a meaningful fit. Our values of χ^2 per degree of freedom are somewhat larger than those of Ref. 1. We have extended the analysis to include β_4 and have not fitted to either the L x-ray energies or K and L x-ray relative intensities, which are not as sensitive to the details of the nuclear shape as are the K x-ray energies.

In Table III we compare our results with proton[6] and α[7] inelastic scattering and Coulomb excitation.[8,9] In general the agreement is good although certain exceptions stand out. In particular for ^{232}Th and ^{238}U, the values of β_4 determined by α scattering and Coulomb excitation are about 3 to 5 times larger than those determined in this analysis or by proton scattering. The agreement of the several methods for the lighter deformed nuclei is reasonably good. The only disagreement is for ^{182}W. Again the α-scattering results suggest a β_4 of much larger magnitude than our analysis. Our rare-earth results are consistent with the Ref. 1 analysis.

A final comment is in order. Since the charge-shape parameters are only weakly affected by the experimental quadrupole moment (since it accounts for but one degree of freedom with a

TABLE III. Comparison of nuclear deformation parameters as determined by the present analysis of muonic x-ray data with results of other types of experiments.

Nucleus	Deformation parameter	Present muonic x-ray analysis	Proton inelastic scattering[a]	Alpha inelastic scattering[b]	Coulomb excitation[c]
^{232}Th	β_2	0.248	0.23		0.238
	β_4	0.035	0.050		0.130
	β_6	-0.022	--		--
^{238}U	β_2	0.277	0.27	0.237	0.283
	β_4	0.013	0.017	0.067	0.059
	β_6	-0.025	-0.015	-0.012	--
^{152}Sm	β_2	0.287		0.256	0.286
	β_4	0.063		0.061	0.058
^{162}Dy	β_2	0.333			0.33
	β_4	0.042			0.010
^{170}Er	β_2	0.337			0.33
	β_4	-0.001			-0.003
^{182}W	β_2	0.254		0.254	
	β_4	0.000		-0.070	

[a] Ref. 6.

[b] Ref. 7 with second-order corrections.

[c] Ref. 8 for ^{232}Th and ^{238}U. Ref. 9 for ^{152}Sm, ^{162}Dy, and ^{170}Er.

relatively high error), it would seem that the preferred way to determine ground-state nuclear quadrupole moments would be from precise measurements of the muonic x rays. This has the defect that the quadrupole moments are then dependent upon the chosen form of the charge distribution, a fact which can be seen by comparing our values of the quadrupole moments with those of McKee.[10] However, this seems preferable to the use of less accurately measured values to fix the charge parameters.

A more complete description of the experimental details as well as the theoretical analysis is in preparation.

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