Observation of an A_3- Amplitude Phase Change*

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A spin and parity decomposition is presented of the $(\pi^+\pi^+\pi^-)$ final state formed opposite a proton by incident 13-GeV/c π^+ mesons. The A_3 enhancement is identified as the 2^- amplitude decaying to $f^0\pi^+$ via an S wave. A change in relative phase is noted between the 2^-S amplitude and the other principal contributions; this is not incompatible with analyses of the $(3\pi)^-$ system. The method employs the University of Illinois three-body partial-wave analysis program.

A full partial-wave analysis has been performed[1] of the $\pi^+\pi^+\pi^-$ system produced opposite a proton and a coherent deuteron by incident 13-GeV/c π^+ mesons. The mass region covered goes from 0.9 to 1.9 GeV/c^2, thus including the A_3 enhancement. Past experiments[2,5] have favored the 100% decay mode $A_3 \to f^0\pi^+$ and it has been claimed that the A_3 is not a simple resonance but is a diffractive $f\pi$ effect similar to the A_1 in the $\rho\pi$ system.

In a large experiment[5-7] with the negatively charged 3π system it has been found that, although the 2^-S amplitude undergoes a Breit-Wigner-like change in intensity, the phase relative to the other principal constituents remains constant. The analysis herein uses the same maximum-likelihood program,[8] developed at the University of Illinois, which treats the total amplitude as factorizable into production and decay terms such that the event distribution probability may be written in the form

$$W = \Sigma M_{m1\eta}^{J1P1} \rho_{m1m2\eta}^{J1P1J2P2} M_{m2\eta}^{J2P2*},$$

where the elements of the production density matrix, $\rho_{m1m2\eta}^{J1P1J2P2}$, are fitted. The decay amplitude for a state of definite spin and parity J^P, z component m, and reflection parity η is expanded as a sum over intermediate two-body states, each having definite orbital angular momentum l between the π^+ and $\pi^+\pi^-$ systems,

$$M_{m\eta}^{JP} = \sum_l C_l^{JP} M_{m\eta}^{JPl}.$$

In this assumption of coherence, the complex numbers C_l^{JP} form another set of parameters relating the phase and magnitude of the different quasi-two-body decay amplitudes. The decay is presumed to proceed by intermediate "ϵ"π, $\rho\pi$, or $f\pi$ states with the exception of an uncorrelated phase-space background ("FLAT") having isotropic distributions in all variables. Experimentally, this latter amplitude has been found to have a contribution compatible with zero at all 3π masses. This gives some credence to a further assumption that there are no significant pion rescattering effects.

The data for this publication derive from 750 000 pictures of the Stanford Linear Accelerator Center 82-in. bubble chamber with hydrogen exposed to an incident π^+ beam at 13 GeV/c, yielding 40 303 events of the reaction

$$\pi^+p \to \pi^+\pi^+\pi^-p. \tag{1}$$

The reaction has a strong $\Delta^{++}(1231)$ channel, especially in the region of interest. Whereas only ~8% of events are removed by a Δ^{++} cut in the 3π mass interval 1.0 to 1.4 GeV/c^2, some 27% are removed in the region 1.4 to 1.9 GeV/c^2, leaving 5250 events with $|t'| < 0.5$ (GeV/c)^2. In this analysis the $p\pi^+$ masses have been cut both for experimental data and in the (symmetrized) theoretical expression. Because of the size of the resulting correction which must be made and the possible danger of tail effects, various different cuts were tried. The sensitivity of the results to these cuts was examined in detail[1] and there were no gross changes in any parameters in going from the large cut $M(p\pi^+) < 1.42$ GeV/c^2 to that finally adopted to maximize statistics, $1.18 \le M(p\pi^+) \le 1.38$ GeV/c^2. There is negligible contamination from Δ^0 or N^*'s ($\lesssim 4\%$).

Quasi-two-body parametrization.—The propagator for the $\pi^+\pi^-$ system in the decay matrix element is parametrized by a relativistic Breit-Wigner form with spin = 0, 1, 2 barrier factors corresponding to "ϵ^0", ρ^0, and f^0 decay, respectively. In separate tests in the A_1-A_2 and the A_3 regions it was determined that none of the fitted parameters was sensitive to the inserted masses and widths of these states.[1,9]

Partial-wave analysis.—The parameters of the fit are the factors C_l^{JP} and $\rho_{mm\eta}^{JPJP}$, presumed to be functions of the 3π mass and the square of the momentum transfer ($t' = t - t_{\min}$) from the beam to the 3π system. The maximum-likelihood program was used to fit the data in 50-MeV/c^2 mass bins for $|t'| < 0.5$ (GeV/c)^2, where an $e^{At'}$ dependence was imposed on all production intensities, with the slope parameter determined separately as a function of mass. Individual slopes for the states retained in the A_3 region were ascertained later.

Total spin J was limited to ≤ 3, and $l \leq 2$; within these restrictions, other states have been omitted after determining that they were experimentally undetectable. For example, the fits required $< 1\%$ contribution from $\eta = -1$ (natural parity) states, implying almost complete natural-parity exchange in a t-channel model. Furthermore, the unnatural-parity series appears principally in $M = 0$ states [e.g., the $M = 1$ state of 2^- contributes $(4.4 \pm 1.0)\%$ of the total at the A_3 mass] and the only sizable member of the natural-parity series, 2^+, is produced in the $M = 1$ state only.

Results of this analysis are displayed in Fig. 1 where the number of events in a given mass bin have been multiplied by the diagonal terms of the density matrix. This number is corrected for the Δ^{++} cuts made. The dominance of 1^+S at low masses dies away very quickly, and the sharp rise and fall of $2^+D_{M=1}$ in the A_2 region is evident. These enhancements are discussed in an earlier publication.[10] In the region of the A_3 peak it is the $2^-S\, f\pi$ partial wave which undergoes the greatest change in intensity; however, there could also be some structure in $2^-P\, \rho\pi$. 1^+S and 1^+P intensities appear to be quite dissimilar as a function of mass, so it is unlikely that the high 2^-P level is a "following" effect due to the decay coherence assumption. In the $(3\pi)^-$ system the coherence has been checked, and found to be unity within errors.[5] The contributions of spin-3 waves enter at about 1.3 GeV/c^2 and rise sufficiently slowly to be considered, along with the low partial waves, as being almost constant in the region 1.4 to 1.9 GeV/c^2.

In a comparison with the reaction $\pi^-p \to \pi^-\pi^-\pi^+p$ at similar energies,[5] it has been determined that the spin and parity composition appears to be independent of the charge of the incident meson, although there is a suggestion that the 2^- intensity peaks a little earlier and sharper in this experiment.

The states included in the high-mass region are listed in Table I and require 36 parameters for an average number of 525 events per 50-MeV/c^2 mass bin.

A_3 parameters.—In a fit to an S-wave relativistic Breit-Wigner form, the intensity of the 2^-S amplitude corresponds to a mass of 1.60 ± 0.01 GeV/c^2 and a width of 0.31 ± 0.04 GeV/c^2 with a constant, nonresonant background compatible with zero [$(5 \pm 12)\%$]. The mass is somewhat smaller than that obtained in the A_3^- analysis[5] the results of which were $M, \Gamma = 1.66 \pm 0.01, 0.27$

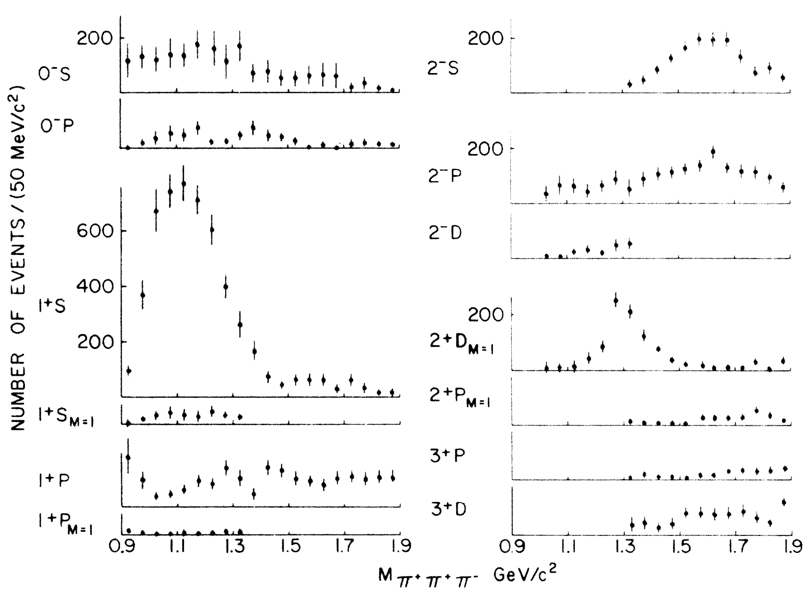

FIG. 1. Events of reaction $\pi^+p \to \pi^+\pi^+\pi^-p$ in each $|J^PlM\rangle$ state as a function of $M(3\pi)^+$ in bins of 50 MeV/c^2 with $0.0 \leq |t'| \leq 0.5$ (GeV/c)^2.

TABLE I. States and slopes ($e^{At'}$) in the A_3^+ ($M_{3\pi}$ = 1.5 – 1.8 GeV/c^2) region.

#	J	P	ℓ	M	η	Decay	A(GeV/c)$^{-2}$
1)				"FLAT"		3π	
2)	0	-	S	0	+	επ	7.7 ± 1.8
3)	0	-	P	0	+	ρπ	3.7 ± 3.2
4)	1	+	P	0	+	επ	8.8 ± 2.0
5)	1	+	S	0	+	ρπ	10.1 ± 1.9
6)	2	-	S	0	+	fπ	8.0 ± 0.5
7)	2	-	P	0	+	ρπ	5.5 ± 0.5
8)	3	+	D	0	+	ρπ	4.8 ± 0.8
9)	3	+	P	0	+	fπ	8.9 ± 2.9
10)	2	+	D	1	+	ρπ	2.8 ± 1.9
11)	2	+	P	1	+	fπ	1.9 ± 1.1

± 0.06 GeV/c^2. The 2^-P "events" will also give a good confidence level to a P-wave Breit-Wigner form but the resulting width moves to 0.43 ± 0.08 GeV/c^2. If this latter parametrization were relevant, it would imply a branching ratio for $A_3 \to \rho\pi$ of $(48 \pm 14)\%$.

Inserting a Breit-Wigner mass dependence for 2^-S and a constant level for the other waves, a 300-MeV/c^2 mass interval may be selected about 1.65 GeV/c^2 and fits obtained in small $|t'|$ bins. In this way the 2^-S "events" were found to fit to an exponential with slope parameter $A = 8.0 \pm 0.5$ (GeV/c)$^{-2}$, approximately the same value obtained with the $(3\pi)^-$ system.[5] The differential cross section slopes of the other principal partial waves in the same mass interval are given in Table I.

If the A_3^+ is defined as the 2^-S wave in the interval 1.5 to 1.8 GeV/c^2 and with $|t'| < 0.5$ (GeV/c)^2, the cross section is 28 ± 5 μb, compatible with results for A_3^-. Corrected for mass and t' limitations, this figure is 55 ± 9 μb, within errors of the result for A_3^+ at 11.7 GeV/c.[4]

Phase-shift analysis.—The current listing of the particle data tables[11] lists the A_3 as not a Breit-Wigner resonance. The principal evidence for this statement is an intensive investigation of the phase of the 2^-S wave in the negatively charged system. This was found to remain constant, similar to the 1^+S phase behavior through the $A_1^\pm$ region, and to be most unlike the 2^+D phase variation in the $A_2^\pm$ region.[10,12] Although there is an undeterminable overall phase, the interference density matrix element between two J^P states and the phase of the appropriate C_l^{JP} gives the relative phase between two amplitudes.

In Fig. 2 the phase differences $\psi(JPl)$ = (phase 2^-S) – (phase J_l^P) are plotted for all other included J^Pl as a function of mass; for a genuine resonance in 2^-S and all other J^Pl phases constant in the mass region, one would expect a $\pi/2$ increase

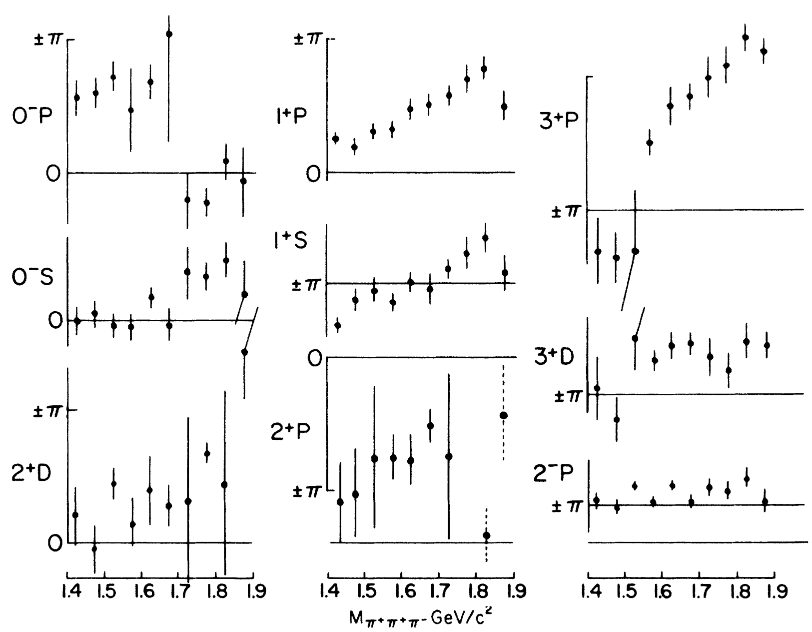

FIG. 2. The phase difference $\psi(JLl)$ = (phase $2^- S$) – (phase J^Pl) for all other contributing states as a function of $M_{\pi^+\pi^+\pi^-}$, for events of reaction $\pi^+p \to \pi^+\pi^+\pi^-p$ in $0.0 \le |t'| \le 0.5$ (GeV/c)^2.

in phase through the ~300-MeV/c^2 full width at half-maximum of the A_3 mass region. By the same argument the 1^+ exhibits no resonant structure in the A_1 region.

If one examines the interferences with partial waves which are of secondary importance after 2^-S viz. 1^+P, 1^+S, 0^-S, and 2^-P, three of the four show this magnitude of change. The approximately constant phase of 2^-S with respect to 2^-P could indicate that the latter is rotating with 2^-S and hence is also resonant. Of the other waves, the phase versus 3^+P shows a very large increase and that versus 3^+D shows a small increase. The 2^-S-2^+P phase has very large errors but it is clear that there is some very violent activity in this neighborhood, perhaps associated with an A_4 which has been claimed at 1.83 GeV/c^2.[12,13]

Of all the phase variations displayed, only that with respect to 0^-P shows a decrease. It should be noted from Fig. 1 that 0^-P has a very small cross section. The contrary behavior of 0^-P exhibited in Fig. 2 can be attributed to some residual Δ^{++} tail contamination coupled with a marked insensitivity of the likelihood function to the 0^-P phase.[1]

The conclusion that the 2^-S partial wave does, in fact, rotate in a positive sense in the Argand diagram is so different from previous analyses and has such far-reaching consequences that it would be advisable to confirm the behavior in another reaction. Therefore, the analysis was performed on $\pi^+d \rightarrow d\pi^+\pi^+\pi^-$ events at the same energy. Here there is negligible d^* production and no problems similar to the required Δ^{++} cut. However, statistics are so low as to prohibit a firm conclusion. Nevertheless there is a general trend of relative phase increase again, and it is possible to say at least there is no contradiction of the proton results.[1]

Although a resonance would be a possible interpretation, it is not clear that confirmation of the phase change would invalidate a description of the A_3 as an $f^0\pi$ diffractive enhancement.[14]

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[9]We employed the same quasi–two-body parametrization as the A_3^- analysis, in which the "ϵ" is a broad structure and contributes substantially in the f^0 region. The results of the analysis were insensitive to reasonable variations of the "ϵ" parameters, which were allowed to vary.

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