Deuteron Wave Function at Small Distances*

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We show that the tensor polarization in elastic electron-deuteron scattering at large but presently attainable momentum transfers is supersensitive to the deuteron wave function within 0.5 fm, but insensitive to the percentage of D state assumed. This may give the first practical method to learn something directly about the two-nucleon interaction in the core.

Recent developments of polarization experiments in nuclear or particle physics greatly added to our understanding by giving detailed information about the spin structure of such reactions and hence formulating severe tests for proposed models. Extending this trend, in this note we use a polarization quantity to yield information about certain spatial features of the wave function of the deuteron, namely its small-distance behavior which so far has been unyielding to experimental probes, as demonstrated by the large number of deuteron wave functions developed during past decades. These agree very well beyond about 1.5 fm, but disagree violently within 0.5 fm. This is so because for the experimental quantities explored so far the wave function usually appears in an integrand weighted heavily toward large distances. Examples are all static properties of the deuteron and most nucleon-nucleon scattering.

One might think that a weight function r^{-n} with a large n might change the situation. Not so, because of formal difficulties at the origin, because of few reactions featuring r^{-n} weight, and because of r^{-n} not decreasing fast enough even with large n.

The solution must be, therefore, to find a weight which is finite at the origin, decreases fast with distance, and then *oscillates* in order to cancel large-distance contributions. This occurs for the spherical Bessel function in electron-deuteron scattering. There the form factor is[1,2]

$$G^2(q) = \frac{d\sigma/d\Omega}{(d\sigma/d\Omega)_{\text{point}}} = G_0{}^2 + G_2{}^2 + [2\tan^2(\tfrac{1}{2}\theta) + 1]\, G_M{}^2, \quad (1)$$

where q is the momentum transfer, θ the scattering angle, and G_0, G_2, and G_M the charge, quadrupole, and magnetic form factors, respectively. Fixing q but varying θ separates G_M. The other two form factors are

$$G_i(q) = 2G_{\text{ES}}(q)F_i(q), \quad i = 0, 2;$$
$$F_0(q) = \int_0^\infty (u^2 + w^2) j_0(\tfrac{1}{2}qr)\, dr, \quad (2)$$
$$F_2(q) = \int_0^\infty 2w(u - 8^{-1/2}w) j_2(\tfrac{1}{2}qr)\, dr,$$

where G_{ES} is the isoscalar nucleon electric form factor, u and w are deuteron S- and D-state wave functions (see Figs. 1 and 2), respectively, and $j_i(qr)$ is the ith spherical Bessel function. The differential cross section alone cannot be used to separate G_0 from G_2; hence comparing wave functions from such a measurement of $G_0{}^2 + G_2{}^2$ failed,[3,4] since this quantity depends almost entirely on the large-distance contributions where all wave functions are virtually identical. For $q^2 \geqslant 10\ \text{fm}^{-2}$ the experiment differs by less than 10% from the closely overlapping bunch of theo-

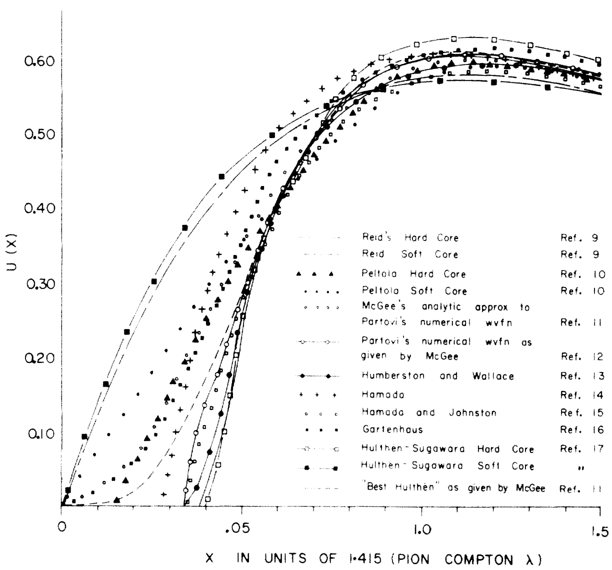

FIG. 1. Various deuteron S-state wave functions published in the literature. The normalization is $\int_0^\infty [u^2(x) + w^2(x)]\, dx = 1$, where x is expressed in pion Compton wavelengths.

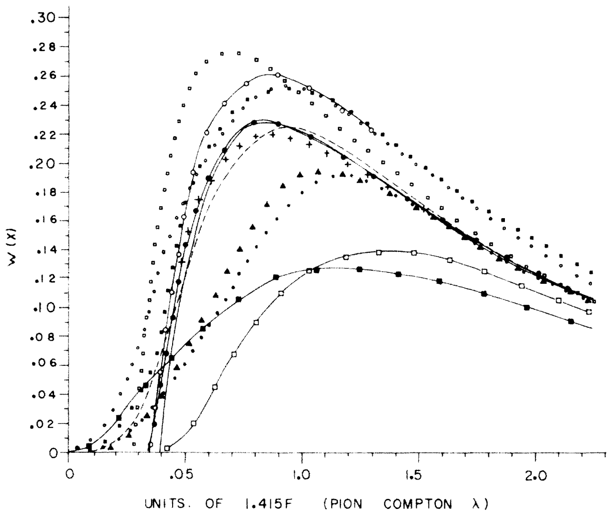

FIG. 2. Same as for Fig. 1, but for D-state wave functions.

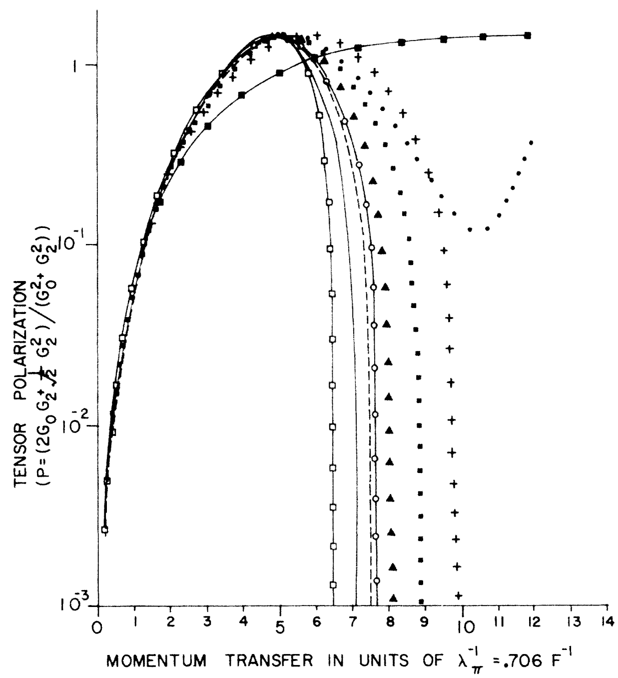

FIG. 3. Tensor polarization P in elastic electron-deuteron scattering versus the momentum transfer q on a semilog scale. For a legend of the curves, see Fig. 1.

retical curves (except for the primitive Hulthén soft core).[5] The small difference might be due to baryonic resonances states.[6] At large q^2 there are somewhat larger differences, but experiments are inaccurate and the theory uncertain because of unknown meson currents and relativistic corrections. For example, see Ref. 4, inspite of the ingenious devices used for that analysis.

Thus we must separate G_0 from G_2 by polarization.[7] Vector polarization is approximately zero,[8] so we turn to tensor polarization, given by (1),

$$P = \frac{3}{\sqrt{2}} \frac{(d\sigma/d\Omega)_0 - (d\sigma/d\Omega)_1}{(d\sigma/d\Omega)_u}$$
$$= \frac{2G_0G_2 + G^2/\sqrt{2}}{{G_0}^2 + {G_2}^2}, \qquad (3)$$

where the subscripts 1 and 0 on the cross sections indicate the polarization of the deuteron, and the subscript u means "unpolarized." The projection of the deuteron spin (eigenvalues 0 and ± 1) should be taken along q. We have then $(d\sigma/d\Omega)_1 = (d\sigma/d\Omega)_{-1}$, so that $(d\sigma/d\Omega)_u = \frac{2}{3}(d\sigma/d\Omega)_1 + \frac{1}{3}(d\sigma/d\Omega)_0$. The tensor polarization is independent of G_{ES} and depends only on G_2/G_0.

G_2 depends on the D-state wave function; G_0 does not. Hence P can determine the percentage of D state which is still uncertain within a factor of 2.[1] We will, however, exploit P differently. Since G_0 and G_2 change signs at different q's, we can have a negative G_0G_2. Furthermore, G_2 is positive for all practical q's, but G_0 flips sign. The exact place where P flips sign is therefore very sensitive to u at small distances because mostly the first lobe of the Bessel function contributes.

P is shown in Figs. 3 and 4, versus q on semilog and linear scales, for the various wave functions[9-19] plotted in Figs. 1 and 2. We see that up to about $q=5$, P is insensitive to the particular wave function since the first lobe of the Bessel function is broad and includes large-distance contributions where wave functions agree. As q increases, the lobe narrows so that it includes the wave function only within 1.5 fm, where wave functions greatly differ. We see that between $q=6$ and 10 different wave functions give P's differing by orders of magnitude and even sign. In the region $q=6$–8 just the sign of P can eliminate many wave functions, and its order of magnitude excludes all but two or three. At very large q the lobe is very narrow and includes nothing of any wave function, thus giving always a vanishing P.

Since G_2 is proportional to the square root of the D-wave probability, P will vary by 1.4 between the limits of that probability. This is negligible compared to the shape dependence, as

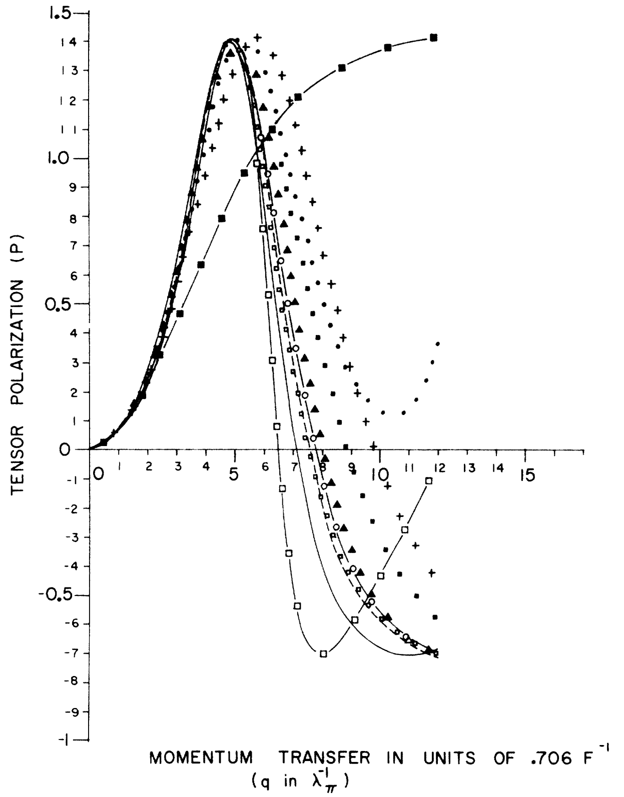

FIG. 4. Same as for Fig. 3, but on a linear scale.

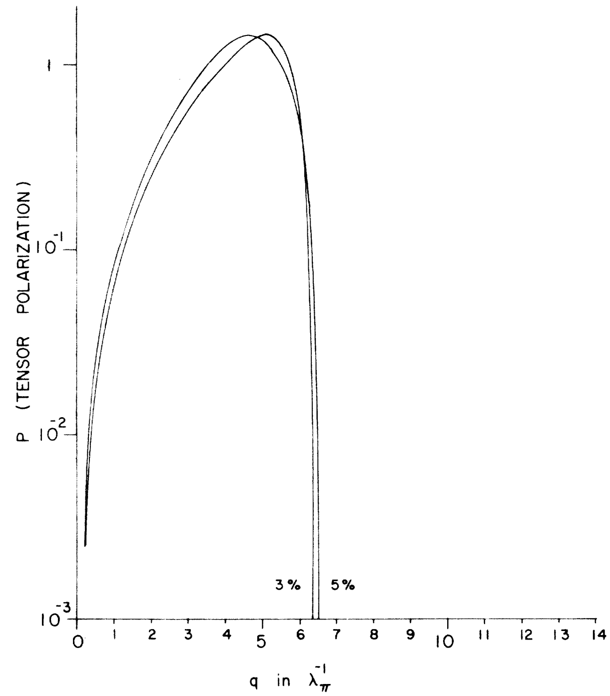

FIG. 5. Tensor polarization P in elastic electron-deuteron scattering for the Hulthén-Sugawara deuteron wave function, with two different D-state probabilities of 3% and 5%, respectively.

illustrated in Fig. 5, if we are at $q = 6-10$. At small q, however, the shape dependence is much less than the dependence on D-state probability.

The interesting region, $q^2 \approx 20-40$ fm^{-2}, can be reached with present electron accelerators. Unpolarized cross sections up to $q^2 = 34$ fm^{-2} have been measured,[4,5,20] and numerical estimates show that at higher energies and smaller angles giving the same q, cross sections are still large enough for measuring. For $\theta \leq 10°$ magnetic scattering is negligible, so the cross section is limited only by how small a θ can be produced. At $\theta = 5°6'$ we reach the desired q region, from 10 to 18.1 GeV, while at $\theta = 10°$ the range is 5.1 to 6.8 GeV.

At near-forward angles detector resolutions must be good to eliminate inelastic events, and so some authors[4] prefer to analyze the recoil deuteron momentum in coincidence. One can either measure the deuteron polarization by a second scattering, or use a polarized deuteron target. In the latter case one can polarize perpendicularly to the plane of scattering, thus getting $(d\sigma/d\Omega)_0$, which, together with $(d\sigma/d\Omega)_u$, gives $(d\sigma/d\Omega)_1$.

We conclude, therefore, that a careful measurement of P in the region of $q = 6-10$ would yield a rather definitive determination of the deuteron wave function down to about 0.2 fm. Such a determination would represent a major breakthrough in our understanding for strong interaction physics as well as for astrophysical calculations.

We are indebted to J. S. Levinger for some informative communications. He, with his coworkers, has obtained, independently of us, results[21] somewhat similar to ours, with an emphasis on the smaller values of q.

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Maximum Mass of a Neutron Star*

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On the basis of Einstein's theory of relativity, the principle of causality, and Le Chatelier's principle, it is here established that the maximum mass of the equilibrium configuration of a neutron star cannot be larger than $3.2M_\odot$. The extremal principle given here applies as well when the equation of state of matter is unknown in a limited range of densities. The absolute maximum mass of a neutron star provides a decisive method of observationally distinguishing neutron stars from black holes.

The estimate of the range of the critical mass for a neutron star varies from 0.32 to $1.5M_\odot$. The greatest uncertainty comes from the equation of state at nuclear densities and above. In fact the knowledge of physical properties of neutron-star material at densities smaller than 10^{13} g/cm^3, essential to describe the properties of the crust of neutron stars[1] and perhaps the change in period of the pulsars,[2] is of no relevance for the determination of the maximum mass of a neutron star. The reason is that on increase of the central density the star becomes more and more compact and its crust becomes only a few tens of meters thick, or even less, depending on the models.[3] At nuclear densities and above, the equation of state is very poorly known because of the presence of strong interactions[4] between nucleons and threshold effects in the creation of resonances[5,6] because of unavailability of phase space.

In recent times it has become clear that the most powerful tool in determining the difference between neutron stars and black holes relies on the possible difference in mass of the two objects.[7] No possibility exists of differentiating them on the basis of electrodynamic properties.[8] Moreover, the recent discovery of x-ray sources in binary systems gives the possibility of determining the mass of a collapsed object with great accuracy.[9] We therefore have the clear need of establishing on solid ground the maximum mass of a neutron star. Instead of trying to analyze the details of nuclear interactions we follow here a different approach. We take that most extreme equation of state that produces the maximum critical mass compatible solely with these three conditions: (1) standard general-relativity equation of hydrostatic equilibrium, (2) Le Chatelier's principle, and (3) the principle of causality. While no suggestion is made that the resultant equation of state accurately represents the actual physical behavior of matter, it does illustrate a point of principle by yielding a maximum mass for the critical mass. It is not altogether new to approach the equation of state from the side of hydrostatic theory rather than from the side of the structure of matter. Gerlach[10] has shown that from a set of measurements on a sequence of stars at the end point of thermonuclear evolution one can, in principle, work back to deduce