is not generally applicable to the hopping situations which are envisaged in noncrystalline solids. Furthermore, it is suggested that the generally observed nonactivated temperature dependence of the dc conductivity is, at least in part, a manifestation of the multiphonon nature of acoustic-phonon-assisted hopping between well-localized states. A detailed account of the herein-summarized consideration is currently being prepared.

The author wishes to acknowledge the aid of E. Young in obtaining the curves of Fig. 3.

*Work supported by the U.S. Atomic Energy Commission.

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Magnitude of the Deformation of the Proton-Nucleus Spin-Orbit Coupling

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(Received 25 October 1973)

A simple explanation is presented for the observed tendency of the spin-orbit part of the optical potential for a proton scattered by a nonspherical target nucleus to be more deformed than the central part. The explanation is based on the fact that the range of the central two-body force is longer than that of the two-body spin-orbit force.

The macroscopic description of inelastic scattering is based on the extended optical model, which differs from the conventional optical model in that the spherically symmetric optical potential is replaced by a deformed potential. It is now generally accepted that in addition to the real central potential, the absorptive central and spin-orbit potentials are also affected by the nuclear deformation. With respect to the spin-orbit coupling, two deformed forms have been proposed: the so-called Oak Ridge[1] and full Thomas[2] forms. The latter has been shown to be in better agreement with experiment.[2]

A general feature following from the early investigations, for both full Thomas and Oak Ridge forms, was a slight but consistent perference for a value of the experimental spin-orbit deformation parameter $\beta^{s.o.}$ larger than that for the central potential, β^{cent}. It is well known[3] that this discrepancy can partly be resolved by following Blair's suggestion[4] that the deformation distances $\alpha(\theta\varphi)$ over which the various parts of the optical potential are displaced in the deformation process are taken equal, which implies equal products βR, R standing for the radius parameter of the specific potential term considered. The relative smallness of the spin-orbit radius parameter $R^{s.o.}$, given by elastic scattering fits, then yields an increased value for $\beta^{s.o.}$. It turns out, however, that this effect is not sufficient to explain the discrepancy, the tendency being that even the product βR for the spin-orbit coupling is anomalously large. A similar tendency has been observed in later work.[5-7] Cases for which there is no clear indication of a discrepancy within the experimental accuracy have also been reported,[8] and even cases in which the discrepancy is in the opposite direction.[9] It is, however, remarkable that deviations take place almost without exception in the direction of larger spin-or-

bit deformation. A recent rather extensive investigation confirming the preference for larger spin-orbit deformation has been published by Tait and Edwards.[10]

Raynal has attempted an explanation of the discrepancy on the basis of microscopic structure of the nuclear states involved.[9] The regularity, however, with which deviations from equality of the deformations take place almost without exception in the direction of larger spin-orbit deformation, suggests the possibility of a more simple explanation. In this note we propose an explanation within the framework of the macroscopic description. The resulting ratio $(\beta R)^{\text{s.o.}}/(\beta R)^{\text{cent}}$ is larger than 1 and varies slowly as a function of mass number. The fluctuations of the ratio from nucleus to nucleus apparent from experimental data might then be interpreted as a superposed nuclear-structure effect, resulting even in a ratio smaller than 1 for a small number of nuclei.

Let us try to ascribe the discrepancy to the longer range of the N-N central force relative to that of the N-N spin-orbit force. On this basis Greenlees, Pyle, and Tang[11,12] were able to explain the difference in radius parameters between central and spin-orbit optical potentials in the case of spherical nuclei. Here we suggest that the same effect may be responsible for the difference in angular variation of the potentials for nonspherical nuclei. If one thinks of the macroscopic potential felt by a projectile as the average sum of N-N interactions with the target nucleons inside its range of interaction, one would expect the central N-N force to scan a larger part of the density distribution in the nuclear surface region. Therefore an average value of the deformation distance over such a portion of the surface of the nuclear mass distribution may express itself in the central proton-nucleus potential,[13] so that $(\beta R)^{\text{cent}} < (\beta R)^{\text{mass}}$. On the other hand, the finite-range effect may be small enough for the optical spin-orbit potential to maintain its Thomas form[14] in terms of the gradient of the nonspherical mass distribution, which implies $(\beta R)^{\text{s.o.}} = (\beta R)^{\text{mass}}$. In combination with the decrease of $(\beta R)^{\text{cent}}$ this may serve as an explanation for the anomalous deformation ratio. We are thus led to the consistent picture that the geometry of the optical spin-orbit potential is more directly related to the geometry of the mass distribution than is the central potential, both with respect to radial as well as to angular variation.

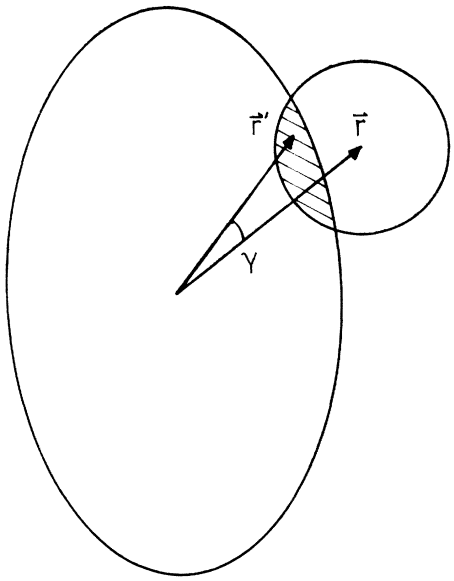

FIG. 1. Proton interaction sphere overlapping deformed nuclear mass distribution.

To estimate the averaging effect, we consider the simple relation[12]

$$U(\vec{r}) = \int \rho(\vec{r}')\, v(|\vec{r} - \vec{r}'|)\, d^3r' \tag{1}$$

between the real proton-nucleus potential U, the nuclear density ρ, and the N-N central interaction v, and study the relation between equidensity and equipotential surfaces. In a region of greater curvature of the nuclear surface the proton "interaction sphere" must be displaced inward to obtain the same total amount of attractive energy (see shaded area in Fig. 1), so that the equipotential surfaces will be less deformed than the equidensity surfaces.[15] This consideration suggests that we write a Taylor expansion to second order in $\vec{r}' - \vec{r}$ for the nuclear density within the interaction sphere. Since the range of the central N-N force is not small compared to the diffuseness parameter of the mass distribution, it is more realistic to carry out a second-order Taylor expansion in tangential directions only.

Writing the spherical mass distribution as $\rho_0(r, R)$, where the radius parameter R is indicated explicitly, the deformed mass distribution is given to first order in α by (see Fig. 1)

$$\begin{aligned} &\rho(\vec{r}') = \rho_0(r', R + \alpha(\theta', \varphi')) = \rho_0(r', R) + \alpha(\theta', \varphi')(\partial/\partial R)\rho_0(r', R), \\ &\alpha(\theta', \varphi') = [1 + i\vec{\gamma}\cdot\vec{l} - \tfrac{1}{2}(\vec{\gamma}\cdot\vec{l})^2 + \ldots]\,\alpha(\theta, \varphi), \end{aligned} \tag{2}$$

where γ is the angle between $\vec{r}$ and $\vec{r}'$ ($\vec{\gamma} \parallel \vec{r} \times \vec{r}'$) and the angular-momentum operator $\vec{l}$ operates on $\alpha(\theta, \varphi)$. Substituting in the integral (1), the first-order γ term vanishes, while in the second-order term we operate on each of the spherical harmonics contained in α:

$$\alpha(\theta, \varphi) = \sum_{\lambda,\mu} \xi_{\lambda\mu}^{\text{mass}} Y_{\lambda\mu}^*(\theta, \varphi). \tag{3}$$

The cross term in the components of $\vec{\gamma}$ perpendicular to $\vec{r}$ vanishes for symmetry reasons, while the component of $\vec{l} = \vec{r} \times \nabla / i$ parallel to $\vec{r}$ vanishes. To second order in γ we thus obtain

$$U(\vec{r}) = U_0(r, R) + \sum_{\lambda,\mu} Y_{\lambda\mu}^*(\theta, \varphi) \xi_{\lambda\mu}^{\text{mass}} (\partial / \partial R) \int d^3r' \rho_0(r', R) v(|\vec{r} - \vec{r}'|)[1 - \tfrac{1}{4}\lambda(\lambda+1)\gamma^2]. \tag{4}$$

Without the γ^2 term this is equal to the usual deformed potential with its multipole deformation distance $\xi_{\lambda\mu}^{\text{cent}}$ equal to that for the mass distribution. The γ^2 term introduces a correction due to the angular variation of the nuclear density and can be roughly estimated by replacing γ^2 by its average value over the interaction sphere. Noting that the $\vec{r}'$ integral in Eq. (4) depends on R only if $\vec{r}$ is in the nuclear surface region, we approximate: $\langle\gamma^2\rangle \simeq \frac{2}{5}(r_0/R^{\text{mass}})^2$, where r_0^2 is the mean square radius of v. Finally we obtain

$$(\beta R)^{\text{cent}} = (\beta R)^{\text{mass}}[1 - \tfrac{1}{6}\lambda(\lambda+1)(r_0/R^{\text{mass}})^2]. \tag{5}$$

Following Greenlees, Makofske, and Pyle,[16] we take $r_0^2 = 4.27$ fm^2. Furthermore, $R^{\text{mass}} = 1.2A^{1/3}$. For $\lambda = 2$ this leads to a 10% additional decrease of β^{cent} relative to $\beta^{\text{s.o.}}$ for the heaviest nuclei and a 30% one for the lightest nuclei for which experimental data on $\beta^{\text{s.o.}}/\beta^{\text{cent}}$ are available.

Note that this effect is linear in the deformation, in contrast to the correction calculated recently by Hendrie[17] using the model of a contact interaction between sharp-edged colliding particles. This model may be considered to express in a simplified way the idea that the equipotential surfaces will tend to have constant *normal* distance to the nuclear surface, rather than constant *radial* distance, as is implied by the usual formulas when $\xi_{\lambda\mu}^{\text{cent}} = \xi_{\lambda\mu}^{\text{mass}}$. In our treatment based on the more realistic Eq. (1) the corresponding correction should be contained automatically as a contribution of second order in α. However, it leads to more complicated radial form factors and cannot be taken into account by a correction to βR. A similar complication arises in Hendrie's model if the projectile interacts with the target nucleus also at smaller distances of its center of mass to the nuclear surface. The second-order correction to β then varies and even changes sign when the projectile center of mass is inside the nuclear surface.

If the deformation distances for mass and charge distributions can be identified, the above-mentioned decrease of $(\beta R)^{\text{cent}}$ may show up in a comparison between $(\beta R)^{\text{cent}}$ and $(\beta R)^{\text{charge}}$, although the estimate (5) is probably too rough for a detailed comparison with experimental data to be meaningful. The relative values of nuclear and Coulomb β values tabulated by Hinterberger *et al.*,[18] after introducing the Blair correction, do not show any systematic deviation of the type expected. On the other hand, the values tabulated by Hendrie[17] seem to lend some support to the validity of the averaging effect in the case of α particles, if we leave out Hendrie's second-order correction in view of the above-mentioned objection, but take into account the first-order Blair correction. The β_2 values indicated as "first order" show a systematic 5–10% discrepancy with the Coulomb-excitation results. For α particles and for the target nuclei considered this is of the correct order of magnitude and sign to be identified with the correction implied by Eq. (5). For the higher multipoles the less satisfactory agreement may be due to the fact that the spatial oscillation of the nuclear surface inside the interaction sphere becomes too rapid to allow the neglect of higher powers in γ. In addition, however, it should be noted that the empirical basis for the identification of deformation distances for mass and charge distributions is rather weak as yet. Further accurate experimental data on β^{cent}, $\beta^{\text{s.o.}}$, and β^{charge} for strongly collective transitions are needed before one can draw more definite conclusions on the angular properties of potentials and on mass and charge distributions, comparable to the conclusions by Greenlees and co-workers[11,12,16] on the radial variations.

The author is grateful to Dr. W. C. Hermans for helpful discussions.

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Difference between Polarization and Analyzing Power in the Reaction $^3H(p,n)^3He$

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We have examined the difference between polarization and analyzing power for the reaction $^3H(p,n)^3He$. We find that this difference is due to the presence of $^3P_2 \leftrightarrow {}^3F_2$ transitions which are enhanced in the vicinity of the lowest 2^- state of ^4He.

In a recent Letter, Haight *et al.* observed a surprisingly large and systematic difference between their analyzing power (A) data for the reaction $^3H(p,n)^3He$ using polarized protons and published polarization data (P) for $^3H(p,n)^3He$ for polarized neutrons in the energy range from 1.5 to 4 MeV. Their observation is of interest because the assumptions of charge symmetry [which implies $P = \bar{P}$, where $\bar{P}$ denotes the polarization for the reciprocal reaction $^3He(n,p)^3H$ with polarized protons] and time-reversal invariance (which implies $A = \bar{P}$) together require P and A to be equal for this reaction. An approximate equality between P and A is to be expected if all charge-dependent effects are small. In the present Letter, we show how the difference between P and A provides a strong constraint on a partial-wave analysis of the reaction $^3H(p,n)^3He$, and that a simple analysis of the difference observed in the energy range 1.5–4 MeV leads to the conclusion that f waves are important in the nucleon channels at these low energies. We discuss implications of this result for the structure of ^4He.

Subsequent measurements[2,3] of both P and A have revealed that these quantities are equal to within experimental uncertainties in the energy