Parametric Up-Conversion of Langmuir Waves into Transverse Electromagnetic Waves

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A mechanism is given for the generation of a transverse electromagnetic wave by a finite-amplitude Langmuir wave of lower frequency. The relationship of this effect to the oscillating two-stream instability is indicated and the conditions for its observation experimentally are discussed.

The parametric down-conversion of a finite-amplitude electromagnetic wave into a Langmuir wave and an ion-acoustic wave is well known.[1-7] Similarly, a finite-amplitude Langmuir wave can decay into a transverse electromagnetic wave and an ion-acoustic wave. A necessary condition for this latter process is, of course, that the frequency of the Langmuir wave be greater than the frequency of the transverse wave. However, there is another possibility which does not seem to have been considered. This is the conversion of a Langmuir wave into a transverse electromagnetic wave of higher frequency! This frequency-up-conversion process can be described as the inverse oscillating two-stream instability. In order to describe the effect I use the equations describing the nonlinear interaction between coherent Langmuir, transverse, and ion-acoustic waves, derived elsewhere[8]:

$$\partial A_L^+/\partial t + \tfrac{1}{2}\nu_e A_L^+ = -ic_{sT}\{(A_s^+)^* A_T^+ \exp[-i(\delta-\omega_s)t] + (A_s^-)^* A_T^+ \exp[-i(\delta+\omega_s)t]\}, \quad (1)$$

$$\partial A_L^-/\partial t + \tfrac{1}{2}\nu_e A_L^- = -ic_{sT}\{(A_s^+)^* A_T^- \exp[i(\delta+\omega_s)t] + (A_s^-)^* A_T^- \exp[i(\delta-\omega_s)t]\}, \quad (2)$$

$$\partial A_T^+/\partial t + \tfrac{1}{2}\nu_e(\omega_{pe}^2/\omega_T^2) A_T^+ = -ic_{Ls}\{A_s^+ A_L^+ \exp[i(\delta-\omega_s)t] + A_s^- A_L^+ \exp[i(\delta+\omega_s)t]\}, \quad (3)$$

$$\partial A_T^-/\partial t + \tfrac{1}{2}\nu_e(\omega_{pe}^2/\omega_T^2) A_T^- = -ic_{Ls}\{A_s^+ A_L^- \exp[-i(\delta+\omega_s)t] + A_s^- A_L^- \exp[-i(\delta-\omega_s)t]\}, \quad (4)$$

$$\partial A_s^+/\partial t + \tfrac{1}{2}\nu_i A_s^+ = -ic_{LT}\{(A_L^+)^* A_T^+ \exp[-i(\delta-\omega_s)t] - (A_L^-)^* A_T^- \exp[i(\delta+\omega_s)t]\}, \quad (5)$$

$$\partial A_s^-/\partial t + \tfrac{1}{2}\nu_i A_s^- = +ic_{LT}\{(A_L^+)^* A_T^+ \exp[-i(\delta+\omega_s)t] - (A_L^-)^* A_T^- \exp[i(\delta-\omega_s)t]\}, \quad (6)$$

where A_L, A_T, and A_s are the slowly varying amplitudes of the Langmuir, transverse, and ion-acoustic waves. The frequencies of these waves (which are always taken as positive) are denoted by ω_L, ω_T, and ω_s, respectively, and the wave vectors are $\vec{k}_L$, $\vec{k}_T$, and $\vec{k}_s$.[9] $A_L^\pm$ denotes a Langmuir wave propagating in the $\pm\vec{k}_L$ direction. The remaining amplitudes have similar meanings. $|A_L^+|^2$ gives the total energy in that mode and so on for the other amplitudes. ν_e is the electron-ion collision frequency, ν_i simulates the damping of the ion-acoustic wave, and $\delta \equiv \omega_T - \omega_L$. The coupling coefficients are given[8] by

$$c_{sT} = \frac{\eta_L}{8\eta_T\eta_s} k_L \frac{\omega_{pe}^2\omega_{pi}^2}{\omega_s^2\omega_T^2},$$

$$c_{Ls} = \frac{\eta_T}{8\eta_s\eta_L} \omega_T \frac{k_s^2 c_s^2 k_L \gamma_e v_{Te}^2}{n_0^2\omega_L\omega_s^2},$$

$$c_{LT} = \frac{\eta_s}{8\eta_T\eta_L} k_L \frac{\omega_{pe}^2\omega_s}{\omega_T^2\omega_L},$$

where

$$\eta_L = \frac{1}{2\sqrt{2}} \frac{\gamma_e k_L v_{Te}^2}{\omega_L\omega_{pe}}, \quad \eta_T = \frac{1}{2\sqrt{2}} \frac{n_0 e}{\omega_T \epsilon_0^{1/2}},$$

$$\eta_s = \frac{1}{2\sqrt{2}} \left(\frac{m_i}{n_0}\right)^{1/2} \frac{|k_s| c_s^2}{\omega_s},$$

with γ_e the ratio of specific heats for the electron fluid, m_i the ion mass, n_0 the equilibrium density of the plasma, v_{Te} the electron thermal velocity, c_s the ion sound speed, e the proton charge, and ϵ_0 the dielectric constant of free space (mks units).

The polarization of the transverse wave is

$$\vec{E} = (0, E_y^T, 0), \quad \vec{B} = (B_x^T, 0, 0), \quad \vec{k}_T = (0, 0, k_T),$$

and $|k_T| \ll |k_L|$ so that $\vec{k}_s \approx -\vec{k}_L$, and I assume the Langmuir and ion-acoustic waves propagate in the y direction (or very nearly). If the frequencies of the waves were all large compared with the rate of change of the slowly varying amplitudes and with the mismatch between wave triplets, then these six equations would decouple into two sets of three. However, since the ion-acoustic frequency is very much less than the frequencies of the Langmuir and transverse waves the allowance for a mismatch which can be comparable to the ion-acoustic frequency results in all six waves being coupled.

If one takes the pump wave to be a transverse electromagnetic wave and assumes that $|A_T^{\pm}| \gg |A_L^{\pm}|$ and $|A_T^{\pm}| \gg |A_s^{\pm}|$, then one must solve Eqs. (1), (2), (5), and (6). When $|A_T^+| = |A_T^-|$, these equations have been shown[8] to give rise to the dispersion relation for the decay and oscillating two-stream instabilities previously derived by Nishikawa.[5,6]

Consider the case when the pump is a finite-amplitude Langmuir wave. We now assume that $|A_L^{\pm}| \gg |A_T^{\pm}|$ and $|A_L^{\pm}| \gg |A_s^{\pm}|$ and must therefore solve Eqs. (3)–(6) where $A_L^{\pm}$ is assumed to be constant. Under the transformation

$$\alpha_T^+ = A_T^+, \quad \alpha_T^- = A_T^- e^{i(2\delta)t}, \quad \alpha_s^+ = A_s^+ \exp[i(\delta - \omega_s)t], \quad \alpha_s^- = A_s^- \exp[i(\delta + \omega_s)t],$$

Eqs. (3)–(6) become

$$\partial \alpha_T^+/\partial t + \gamma_T \alpha_T^+ = -ic_{Ls}[A_L^+ \alpha_s^+ + A_L^+ \alpha_s^-], \tag{7}$$

$$\partial \alpha_T^-/\partial t - i(2\delta)\alpha_T^- + \gamma_T \alpha_T^- = -ic_{Ls}[A_L^- \alpha_s^+ + A_L^- \alpha_s^-], \tag{8}$$

$$\partial \alpha_s^+/\partial t - i(\delta - \omega_s)\alpha_s^+ + \gamma_s \alpha_s^+ = -ic_{LT}[(A_L^+)^* \alpha_T^+ - (A_L^-)^* \alpha_T^-], \tag{9}$$

$$\partial \alpha_s^-/\partial t - i(\delta + \omega_s)\alpha_s^- + \gamma_s \alpha_s^- = ic_{LT}[(A_L^+)^* \alpha_T^+ - (A_L^-)^* \alpha_T^-], \tag{10}$$

where $\gamma_T = \frac{1}{2}\nu_e \omega_{pe}^2/\omega_T^2 \approx \frac{1}{2}\nu_e$ and γ_s is the damping constant for the ion-acoustic wave. Looking for a solution of the form $\exp(-i\omega t)$ one obtains the following dispersion relation:

$$(\Omega - \delta + i\gamma_T)(\Omega + \delta + i\gamma_T)(\Omega - \omega_s + i\gamma_s)(\Omega + \omega_s + i\gamma_s) + 2c_{Ls}c_{LT}\omega_s[|A_L^-|^2(\Omega - \delta + i\gamma_T) - |A_L^+|^2(\Omega + \delta + i\gamma_T)] = 0, \tag{11}$$

where $\Omega = \omega + \delta$. The simplest case to analyze is when the Langmuir wave pump is a standing wave, i.e.,

$$A_L^+ = A_L^- = A_L.$$

Neglecting γ_s^2 in comparison with ω_s^2, Eq. (11) then takes the same form as the equation describing the oscillating two-stream and decay instabilities,[5,6]

$$(\Omega - \delta + i\gamma_T)(\Omega + \delta + i\gamma_T)(\Omega^2 - \omega_s^2 + 2i\gamma_s\Omega) - K\omega_s\delta = 0, \tag{12}$$

where $K \equiv 4c_{Ls}c_{LT}|A_L|^2$. For $\delta < 0$, i.e., $\omega_L > \omega_T$, one has growing waves. These solutions correspond to the usual decay instability of a longitudinal wave into a transverse wave and an ion-acoustic wave. Following Nishikawa[5] one can obtain the minimum threshold for this case:

$$K_m = 4\gamma_s\gamma_T(1 - \gamma_s^2/4\omega_s^2), \quad \omega_s \gg \gamma_T, \tag{13}$$

$$K_m = \sqrt{3}\,\tfrac{16}{9}\gamma_s\gamma_T^2/\omega_s, \quad \omega_s \ll \gamma_T. \tag{14}$$

By analogy with the oscillating two-stream solution we now have the additional possibility that the Langmuir pump wave can excite transverse waves of *higher* frequency than the pump frequency. Since $\delta \equiv \omega_T - \omega_L$, this case corresponds to $\delta > 0$, and we find the threshold condition from Eq. (12):

$$K = (\delta^2 + \gamma_T^2)\omega_s/\delta. \tag{15}$$

The minimum value of this expression occurs for $\delta = \gamma_T$ when

$$K_m = 2\gamma_T\omega_s. \tag{16}$$

Using the expressions for c_{Ls} and c_{LT} and the fact that[8]

$$|A_L| = 2\frac{n_0 e}{m_e}\frac{\eta_L}{\gamma_e k_L v_{Te}^2}E^l,$$

we can express the threshold condition in terms of the longitudinal electric field E^l:

$$\epsilon_0|E^l|^2/n_0\kappa T_e = 4\gamma_e\nu_e/\omega_{pe}. \tag{17}$$

For the standing-wave pump considered above, the ion wave is excited at zero frequency and the transverse wave is frequency shifted to the Langmuir-wave value. The situation is different for a traveling-wave pump, however.

The dispersion relation for the traveling-wave pump is obtained from Eq. (11) by putting A_L^-

$=0$. The equation then becomes

$$(\Omega - \delta + i\gamma_T)(\Omega^2 - \omega_s^2 + 2i\gamma_s\Omega) - \tfrac{1}{2}K\omega_s = 0. \quad (18)$$

Note that the dispersion relation is now cubic (rather than quartic). This is because the A_T^- mode is no longer excited when $A_L^- = 0$. Equation (18) does not have any purely growing solutions. However, for $\mathrm{Re}\Omega < 0$ instability can still occur for both $\delta < 0$ (decay instabilities) and $\delta > 0$ (corresponding to the purely growing solution of the standing-wave case). When $\gamma_T \ll \gamma_s$, the instability thresholds are given by

$$K = 4\gamma_s\gamma_T|\delta|/\omega_s, \quad \delta < 0, \quad (19)$$

$$K = 2\delta\omega_s, \quad \delta > 0, \quad (20)$$

where $|\delta| > (2\gamma_T/\gamma_s)^{1/2}\omega_s$. The threshold for the decay instability $\delta < 0$ in this case is much lower than the up-conversion instability. Both instabilities give rise to propagating ion waves.

Next consider the case $\gamma_T \sim \omega_s$. The instability thresholds are now

$$K = 4\gamma_s\gamma_T, \quad \delta < 0, \quad (21)$$

$$K = 4\gamma_s\gamma_T, \quad \delta > 0, \quad (22)$$

The decay and up-conversion instability thresholds are equal and independent of δ!

Finally, solving Eq. (18) well above threshold where damping terms can be neglected, I find for $K = \omega_s^2$, $\Omega = \omega_s(-1.08 + 0.4i)$, $\delta = -\omega_s$, and $\Omega = \omega_s(-0.55 + 0.156i)$, $\delta = 0.1\omega_s$. These solutions result in the A_s^+ and A_s^- waves becoming identical. For the decay instability the ion wave propagates with velocity $1.08c_s$ and for the up-conversion instability the ion wave propagates with the velocity $0.55c_s$. In both cases the frequency of the transverse wave excited is shifted so that the frequency-matching condition (energy conservation) is satisfied.

The effect considered in this Letter should be more easily observed than its analog—the oscillating two-stream instability—for the following reasons. Firstly, since an electromagnetic wave is generated it could be detected outside the plasma (the electromagnetic wave propagates approximately perpendicularly to the Langmuir pump with its electric field vector in the direction of the pump wave). Secondly, for the case of a traveling-wave pump the low-frequency acoustic waves have finite frequencies and could be detected within the plasma. In this case the frequency of the electromagnetic wave is different from that of the Langmuir pump. Thirdly, it should be easier to excite a Langmuir pump wave having a lower frequency than the transverse mode than vice versa.

For a standing-wave pump the up-conversion process has a lower threshold than the decay process when $\omega_s^2 < \gamma_s\gamma_T$. For a traveling-wave pump the thresolds are equal when $\gamma_T \sim \omega_s$. Since the up-conversion process has a greater range of the dispersion curve available to it this effect should occur preferentially under these conditions.

The mechanism for the generation of electromagnetic radiation considered in this Letter may be important for any situation where a suprathermal level of Langmuir waves is built up, e.g., in laser fusion, electrostatic shocks, and turbulently heated plasmas.

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