form $A = H(a/M)(\sigma - \sigma_0)^\alpha$, and again the equation $g(a/M, H(a/M)(\sigma-\sigma_0)^\alpha, \sigma) = 0$ has a continuous solution for nearby a/M. Thus no trajectory can have an end point on the positive imaginary axis (except possibly at $a/M = 1$) and the eigenfrequency of any unstable mode must have passed through $\sigma = 0$.

Finally, we observe that Carter's theorem excludes such modes. That is, Carter proves that there are no stationary axisymmetric perturbations of Kerr. But if $h_{ij}(\sigma)e^{-i\sigma t}$ are a family of solutions to the time-dependent field equations, we can show that $h_{ij}(\sigma=0)$ is a solution to the time-independent field equations as well. We conclude that there are no unstable axisymmetric modes of the Kerr geometry exterior to a black hole.

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Nonrenormalizability of the Quantized Einstein-Maxwell System*

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The one-loop divergences of coupled general relativity and electrodynamics cannot be absorbed by renormalization.

Although quantization of general relativity has been discussed by many authors, it is only quite recently that the first complete calculation, at the one-loop level, of divergences due to gravitons has been supplied by 't Hooft and Veltman.[1] These authors used dimensional regularization and a general algorithm,[2] based on the background-field method,[3] to obtain the one-loop counterterms. In the case of pure gravitation, the divergent terms could be absorbed by a field renormalization; however, this was no longer possible upon coupling to a quantized scalar field.

While the scalar example is discouraging, one might hope that more realistic matter sources would keep one-loop renormalizability. We report here that this is *not* the case for the Einstein-Maxwell system. We also give some results for (1) Brans-Dicke theory (nonrenormalizable except for the singular case $\omega = -\frac{3}{2}$ which is equivalent to Einstein theory), (2) fermion loops, and (3) pure gravitation with a cosmological term (formally renormalizable).

In the coupled Einstein-Maxwell Lagrangian

$$L \equiv L_E + L_M = -(-\bar{g})^{1/2}[\kappa^{-2}R(\bar{g}) + \tfrac{1}{4}\bar{F}_{\mu\nu}\bar{F}_{\rho\sigma}\bar{g}^{\mu\rho}\bar{g}^{\nu\sigma}], \quad (1)$$

the fields ($\bar{g}_{\mu\nu}$, $\bar{F}_{\mu\nu} \equiv \partial_\mu \bar{A}_\nu - \partial_\nu \bar{A}_\mu$) are written as sums of background fields ($g_{\mu\nu}$, $F_{\mu\nu}$) and quantum fields ($\kappa h_{\mu\nu}$, $f_{\mu\nu}$). The Lagrangian being invariant under gravitational and electromagnetic gauge transformations of the quantum fields $h_{\mu\nu}$ and $f_{\mu\nu}$, we add to L the gauge-breaking terms

$$L_B \equiv L_{EB} + L_{MB} = -\tfrac{1}{2}\sqrt{-g}[(D^\nu h_{\mu\nu} - \tfrac{1}{2}D_\mu h_\alpha{}^\alpha)^2 + (D^\mu A_\mu)^2], \quad (2)$$

where all tensor operations, including covariant differentiation D, are with respect to the background metric $g_{\mu\nu}$. The above choice corresponds to the usual de Donder (harmonic) and Lorentz gauges, which in turn give rise to a vector and a scalar ghost with Lagrangian

$$L_G = \sqrt{-g}[\eta^{*\alpha}(g_{\alpha\beta}D_\gamma D^\gamma - R_{\alpha\beta})\eta^\beta + \varphi^* D_\gamma D^\gamma \varphi]. \quad (3)$$

Note that the usually uncoupled electrodynamical ghost contributes here as a result of the metric dependence of L_{MB}. After the Lagrangian $L+L_B+L_G$ is expanded in the quantum fields about the background, the algorithm determines the one-loop counter Lagrangian from the coefficients of the terms bilinear in the quantum fields.

Before giving explicit results, we note that dimensional regularization has the virtues of avoiding the usual momentum cutoff in favor of a logarithmically divergent dimensionless parameter $(1/\epsilon)$ and of eliminating tadpoles. It follows that in our system, where there is only one dimensional constant, κ, there will be no divergences proportional either to L itself or to a cosmological term $\kappa^{-4}\sqrt{-g}$. This means that there is no coupling-constant renormalization.

We now list the possible gauge-invariant ingredients from which the counter Lagrangian ΔL is to be constructed, namely, $R_{\nu\alpha\beta}{}^{\mu}$, $F_{\mu\nu}$, and D_α. Both F and D must necessarily appear to an even power (by the continuity of photon lines for F and because the other building blocks have an even number of indices for D). Dimension counting requires ΔL to be bilinear in $R_{\nu\alpha\beta}{}^{\mu}$, $\kappa^2 F_{\mu\nu}F_{\alpha\beta}$, and $D_\mu D_\nu$. Since we are not interested in total divergences, the only explicit D dependence is of the form $(D_\alpha F_{\beta\gamma})^2$, $(D_\alpha F_{\beta\gamma})(D_\gamma F_{\beta\alpha})$, or $(D^\mu F_{\mu\nu})^2$. The former two can be eliminated in favor of the latter by using cyclic identities on $D_\alpha F_{\beta\gamma}$ and commutator identities linking $[D_\mu, D_\nu]$ to the curvature. The *a priori* possible scalars are then the following (absorbing κ in F):

$$\Delta L = (\sqrt{-g}/\epsilon)\{c_1 R_{\mu\nu}{}^2 + c_2 R^2 + c_3 (R_{\beta\gamma\delta}{}^{\alpha})^2 + c_4[\mathrm{Tr}F^4 - \tfrac{1}{4}(F^2)^2] + c_5 (F^2)^2 + c_6 R^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta} + c_7 R^{\mu\nu}(F_{\mu\gamma}F_\nu{}^\gamma - \tfrac{1}{4}g_{\mu\nu}F^2) + c_8 R F^2 + c_9 (D^\mu F_{\mu\nu})^2\}. \quad (4)$$

The c_3 term can be eliminated in favor of (c_1, c_2) because of the identity[4,5]

$$R_{\alpha\beta\mu\nu}{}^2 = (4R_{\mu\nu}{}^2 - R^2) + \text{a divergence} \quad (5)$$

which holds in four-dimensional Riemann space.

Explicit calculation shows that because of miraculous (?) cancelations, $c_5 = c_6 = c_8 = 0$, which means that $F_{\mu\nu}$ only appears in the combination $T_{\mu\nu} \equiv F_{\mu\alpha}F_\nu{}^\alpha - \frac{1}{4}g_{\mu\nu}F^2$ and $(D^\mu F_{\mu\nu})^2$, and the full curvature $R_{\nu\alpha\beta}{}^\mu$ is absent. The remaining coefficients have the values $c_1 = 9/20$, $c_2 = -29/10$, $c_4 = 13/24$, and $c_7 = c_9 = 1/6$. Since we are only interested in the renormalizability question, we may insert the field equation obeyed by the unperturbed background fields, namely,

$$R_{\mu\nu} = -\tfrac{1}{2}T_{\mu\nu}, \quad D^\mu F_{\mu\nu} = 0; \quad (6)$$

if ΔL does not then vanish, the divergences are not renormalizable.[2] There is effectively only one term left since modulo the field equations $R_{\mu\nu}{}^2$, $T_{\mu\nu}{}^2$, and $R_{\mu\nu}T^{\mu\nu}$ are equivalent, and $R=0$. The final result is the nonvanishing one-loop Einstein-Maxwell counter Lagrangian

$$\Delta L = (\sqrt{-g}/\epsilon)(137/60)R_{\mu\nu}{}^2 \quad (7)$$

and the theory is *not* one-loop renormalizable.

Since the coupling in question presumably does exist in nature as a fundamental one, we are faced with the alternatives that (a) perturbative renormalizability is not a relevant criterion, (b) general relativity must be replaced by a "better" theory for quantizing, or (c) photons are not "fundamental" fields. Possibility (a) is not very fruitful at our present level of understanding. With respect to possibility (b), one hope might lie in a Brans-Dicke-type theory, since by replacing κ^{-1} by a scalar field φ one might avoid the traditional nonrenormalizability of theories with dimensional coupling constants.[6] However, direct examination of this class of theories shows that they can always be cast, by suitable transformations $g \to g\varphi^n$, $\varphi \to f(\varphi)$, into the unrenormalizable scalar-graviton system. The only exception is the singular case $\omega = -\frac{3}{2}$, which is equivalent to Einstein theory. These remarks do not rule out, perhaps, a deeper use of the Brans-Dicke models in which the Higgs phenomenon for φ might be suitably exploited. With respect to possibility (c), one should consider fermion couplings, since presumably fermions at least are necessary as fundamental fields. We have computed the divergent part of the fermion loop contribution to the graviton propagator, which for $m=0$ gives a term of the form[7]

$$\Delta L_{FL} = (\sqrt{-g}/\epsilon)\tfrac{1}{40}(R_{\mu\nu}{}^2 - \tfrac{1}{3}R^2) \quad (8)$$

and the complete ΔL is under investigation. Finally, we consider pure gravitation with a cosmological term, $\lambda\kappa^{-2}\sqrt{-g}$, which involves a new dimensional constant λ. A formal one-loop calculation leads to a ΔL which may be written as proportional equivalently to $\sqrt{-g}$ or R, and hence absorbed by renormalization of λ/κ^2. This is, however, not a very physical quantum theory, since in it the h field corresponds to a tensor-scalar ghost admixture.

In a general vein, we remark that coupling several matter fields in the conventional way is unlikely to lead to miraculous cancelations, since all $R_{\mu\nu}^2$ terms obtained so far have the same sign. Also, if one starts with an original gravitational Lagrangian with $R_{\mu\nu}^2$ terms to absorb the unrenormalizable ΔL, these terms will in turn generate higher powers of R at the one-loop level (in addition to implying a very unpleasant higher-order equation for the quantum fields $h_{\mu\nu}$), so that a Lagrangian which is a finite polynomial in the curvature is unlikely to help.[8]

As for two-loop calculations, they present enormous technical complications. It would be most important if one could at least discover whether pure gravity involves terms $\Delta L \sim \mathrm{Tr}[(R_{\nu\alpha\beta}{}^{\mu})^3]$ there, since these are presumably a sign of nonrenormalizability.

Detailed results will be published elsewhere.

Note added.—The coupling of fermions to gravitation has now been analyzed; the one-loop divergences include a term proportional to $(\bar{\psi}\gamma_5\gamma_\alpha\psi)^4$. Hence this system is also nonrenormalizable.

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Observation of Structure in $\bar{p}p$ and $\bar{p}d$ Total Cross Sections below 1.1 GeV/c*

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Total cross sections of $\bar{p}p$ and $\bar{p}d$ have been measured between 360 and 1050 MeV/c, with high statistical precision. Structures are observed in both cross sections at about the same momenta. For $\bar{p}p$, the central mass is 1932 ± 2 MeV/c^2, and a fit to the data with a simple Breit-Wigner resonance plus background gives $\Gamma = 9^{+4}_{-3}$ MeV/c^2. The data suggest that the structures are in the isospin-1 state.

There have been many claims[1-11] of narrow meson resonances with masses in the range 1900 to 2000 MeV/c^2; the observed widths varied from ~10 to 200 MeV/c^2. However, many other similar experiments have not observed such resonances and others have directly challenged the basis of such claims.[12-16] Searches for meson resonances in this mass region have been made in missing-mass experiments, multipion mass spectra, and $\bar{p}p$-formation experiments. A review of the subject is given by Diebold,[17] where the conclusion is that no narrow resonances in this mass region have been confirmed.

Total-cross-section measurements have in the past been very successful in searching for new resonances, because even though they are not intrinsically sensitive, they can be carried out to a very high statistical accuracy and with small point-to-point systematic errors. In an attempt to clarify the problem of the possible existence of meson resonances in the mass region around 1950 MeV/c^2, we have measured $\bar{p}p$ and $\bar{p}d$ total cross sections using antiprotons of momenta 360 to 1050 MeV/c (center-of-mass