t matrices without arbitrary parameters and without explicitly using a potential; bound states are easily handled; some amount of inelasticity can be included, allowing one to use experimental phase shifts well above the inelastic threshold; spin coupling due to the tensor force can be incorporated; and the inclusion of partial or complete relativistic effects is possible. The determination of the half-shell t matrix directly from relativistic meson theory by the above established formalism is in progress and the results will be reported later elsewhere.

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Evidence for Coexistence of Spherical and Deformed Shapes in ^{72}Se

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We report evidence for an excited $K^\pi = 0^+$ rotational band in ^{72}Se and its mixing with vibrational states. The mean life of the 937-keV $0^{+\prime}$ state was measured to be 22.8 ± 1.4 nsec, yielding $B(E2) = 36 \pm 7$ single-particle units for the 75-keV $0^{+\prime} \rightarrow 2^+$ transition. From in-beam, γ-ray spectroscopy following the reaction ^{58}Ni(^{16}O, $2p$), a strong γ-ray cascade was observed from an yrast band of levels 0^+, 2^+, ... to spin 12^+ in ^{72}Se.

The lowest few energy levels in even-even nuclei with $A = 70-80$ usually follow the pattern expected for a simple phonon vibrational model characteristic of spherical nuclei. However, notable exceptions to this model are observed[1,2] in ^{70,72}Ge where the first excited $0^{+\prime}$-state energies are just above and just below the first 2^+ states. Various suggestions have been made to explain these two very low-energy $0^{+\prime}$ states. Lure, Peker, and Prokof'ev[3] proposed that the $0^{+\prime}$ state in ^{72}Ge might be the head of a quasi-β band. Stewart and Castel[4] have considered the possibility of nuclear shape coexistence with a mixing of harmonic vibrator and rotational levels, and they could reproduce the energies of the 2^+, $0^{+\prime}$, $2^{+\prime}$, and 4^+ levels and some, though not all, of the branching ratios of the $0^{+\prime}$, 2^+, and $2^{+\prime}$ states in ^{70,72}Ge. The data in the Ge isotopes are limited to only a few low-spin states, and thus the interpretation of these levels is still open to question. Recently, the $0^{+\prime}$ state in ^{72}Se was observed[5,6] only 75 keV above the first 2^+ 862-keV state. In addition, in ^{72}Se the 4^+-to-2^+ transition is lower than the 2^+-to-0^+ one, in contrast to $^{74-78}$Se. Thus something unusual seems to be occuring in ^{72}Se.

Here we report measurements of absolute and relative transition rates of the low-spin states and of a proposed yrast band of even-spin levels to spin 12 in ^{72}Se. We suggest that the $0^{+\prime}$ state and higher-spin states strongly excited through nuclear-induced reactions are members of a $K^\pi = 0^+$ rotational band associated with a deformed shape which coexists with the vibrational states associated with a spherical ground state. Our ^{72}Se data show a striking similarity to the very recent evidence[7] that the yrast states in $^{186}_{80}$Hg go from a near-spherical to a deformed shape about spin 6. In ^{72}Se the transition to a deformed shape in the yrast band occurs at even lower spin and in contrast to ^{186}Hg, where the lowest-spin states in the deformed well are not seen, two 2^+ states and the 0^+ head of the band built on the deformed shape are seen too.

First, the lifetime of the 937-keV $0^{+\prime}$ in ^{72}Se was measured. The ^{72}Se levels were populated by ^{72}Br ($T_{1/2} = 1.31$ min)[5] produced in the reaction ^{58}Ni(^{16}O, pn) at 46 MeV from the Oak Ridge National Laboratory Tandem Van de Graaff accelerator. The 937-keV $0^{+\prime}$ state is fed by a 1062-keV transition and primarily decays by a 75-keV transition to the first 2^+, 862-keV level.[5] The combined lifetime of the $0^{+\prime}$ and 2^+ levels was measured by observing the delayed coincidences between the 1062- and 862-keV transitions since lead x rays masked the 75-keV transition. With a lifetime of ~10 psec expected for the 862-keV state and 7-nsec system resolution, any observable lifetime is associated with the $0^{+\prime}$ level. Information on (γ-ray energy)$\times$time in a 256×64 array were stored in a 16×10^3 analyzer gated by coincidences between the linear signals from the Ge(Li) detector (window 500-1400 keV) and the 862-keV photopeak from the NaI(Tl) detector (details to be published[8]).

The exponential decay part of the time spectra of the 1062–862 keV cascade was analyzed to yield a mean life of 22.8 ± 1.4 nsec for the $0^{+\prime}$ state. From the time distribution of the $E0$ electrons of the 937-keV transition with respect to a cyclotron beam burst, a mean life of 27.8 ± 0.6 nsec was measured.[9] The ratio of the $E0$ to $E2$ decays of the 937-keV level was measured to be 0.37 ± 0.23 by comparing the 1062- and 1136-keV transitions (the latter feeds directly the 862-keV level[5]) in the singles spectra with the coincidence spectra where the prompt plus delayed spectra were added to obtain the $E2$ branching. Correcting for $E2$ internal conversion and the $0^{+\prime} \rightarrow 0^+$ $E0$ strength we find $B(E2; 0' \rightarrow 2) = 0.32 \pm 0.06 e^2$ b^2 or 36 ± 7 single-particle units (30 ± 6 for 27.8 nsec) for a single-particle $B(E2) = 2.95 \times 10^{-5} A^{4/3} \times e^2$ b^2 and find $\rho = 0.176^{+0.048}_{-0.070}$.

Next, a 48-h in-beam γ-γ coincidence experiment in ^{72}Se produced by the reaction ^{58}Ni(^{16}O, $2p$) was carried out with two Ge(Li) detectors. The coincidence data establish the following levels above the $2^{+\prime}$ 1320-keV level: 1637, 4^+; 2467, 6^+; 3425, 8^+; 4502, (10^+); and 5702 keV, (12^+). Angular-distribution work (preliminarily reported earlier[10]) yielded spins of 6 and 8 for states at 2467 and 3425 keV. The tentative 10^+ and 12^+ assignments are based on the strong cascade character of the transitions out of these levels. For example, in the ($12^+ \rightarrow 10^+$) 1200-keV gate spectrum, only the $2\rightarrow0$, $4\rightarrow2$, $6\rightarrow4$, $8\rightarrow6$, and $10\rightarrow8$ transitions are seen with equal intensities. The coincidence data confirm that most if not all of the γ-ray intensity of a 379.9-keV transition in the ^{72}Br decay[5] belongs to the $2^{+\prime}\rightarrow 0^{+\prime}$ transition.

The striking features of the ^{72}Se levels and their decays are as follows: first, the low energy of the $0^{+\prime}$ state and its strong $B(E2)$ to the 2^+ state; second, the strong $2^{+\prime}\rightarrow 0^{+\prime}$ transition with a $B(E2)$ comparable to that of the $2^{+\prime}\rightarrow 2^+$ transition; and third, the low energy of the $4\rightarrow2$ transition compared to all the other transitions in the yrast band and then a regular increase in transition energy with increase in spin above $I = 4$.

The strong $B(E2{:}0^{+\prime}\rightarrow 2^+)$ suggests that the $0^{+\prime}$ and 2^+ levels are members of a collective family, yet this close proximity is inconsistent with the vibrational model. A $2^{+\prime}\rightarrow 0^{+\prime}$ transition is strictly forbidden in the pure vibrational model so its presence with a $B(E2)$ comparable to that of the $2^{+\prime}\rightarrow 2^+$ transition indicates a sharp variance with a phonon model. In the most careful study[11] of zero-phonon transitions, $B(E2{:}4^+\rightarrow 2^+)/B(E2{:}4^+ \rightarrow 2^{+\prime})$ is 360 in ^{134}Ba. The unusual character of the yrast band in ^{72}Se is seen in a plot of $2\mathscr{g}/\hbar^2$ versus $(\hbar\omega)^2$ (Fig. 1), with $\mathscr{g}$ the moment of inertia and $\hbar\omega$ essentially $\frac{1}{2}$ the transition energy. (Sorensen[12] discusses the various definitions of

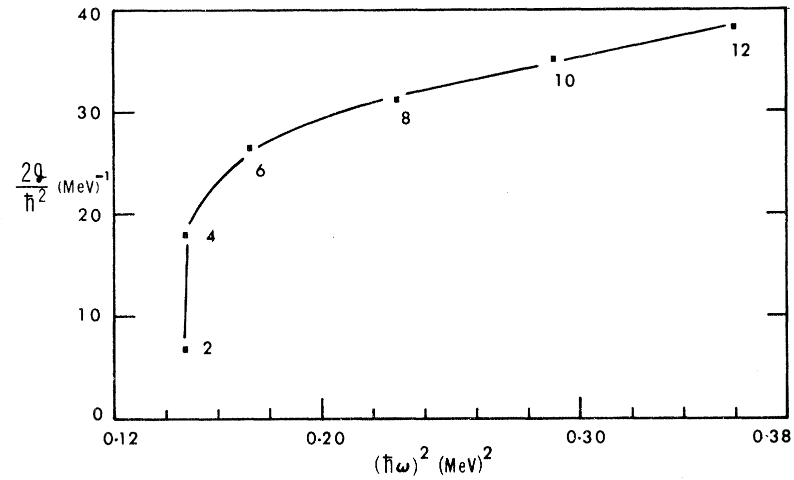

FIG. 1. A plot of $2\mathscr{I}/\hbar^2$ versus $(\hbar\omega)^2$ [for $\hbar\omega$ defined by Eq. (14d), Ref. 11] for ^{72}Se. The lowest point is for the $2 \to 0$ transition.

$\hbar\omega$.) Sudden changes in $\mathscr{I}$ with increase in nuclear spin indicate sudden changes in the structure of the nucleus. The curve in Fig. 1 is based on the ground and first 2^+ states in ^{72}Se as the lowest members of the band. The curve which starts out nearly vertical and bends sharply above spin 4 is markedly different from that of any other reported yrast bands (e.g., Refs. 12–14), except for ^{186}Hg (Ref. 7). The 2^+ and 4^+ states have energy spacings that are reasonably characteristic of a pure spherical vibrator, as indicated by the nearly vertical rise in $\mathscr{I}$ with increasing spin. The spin-6 to -12 states follow reasonably well the rotational-energy formula $E_I = AI(I+1) + BI^2(I+1)^2$.

The enhanced $B(E2{:}0' \to 2)$, the branching ratio of the $2^{+\prime}$ state, and the band of states to spin 12^+ in ^{72}Se can be understood in terms of the coexistence of deformed and spherical states. Such could occur with a near-spherical ground state if there is a second minimum in the potential relatively low in energy with large deformation. (As a point of general interest, in quite a different situation than in ^{72}Se, namely the problem of the backbending of $\mathscr{I}$ in rotational bands in nuclei with deformed ground states, Thieberger[15] has speculated that there may be a connection between the anomalous behavior of $\mathscr{I}$ and the second minima in the potential-energy surfaces seen in the actinides.) We assume that the $0^{+\prime}$ state is a deformed state which is the lowest member of a $K^\pi = 0^+$ rotational band, and further assume that in lowest order the 2^+ member of this band and the 2^+ one-phonon level are close together so there is strong mixing of the rotational and vibrational wave functions and large shifts of these 2^+ levels. The $0^+, 4^+, 6^+, \ldots$ members of the rotational band are assumed to have small mixing with phonon states because phonon states of the same I^π are not close in energy. Then the $0^{+\prime} \to 2^+$ and $2^{+\prime} \to 0^{+\prime}$ transitions are transitions between rotational states.

The 0^+, 4^+, 6^+, 8^+, 10^+, and 12^+ members of the rotational band were assumed to be the levels at 937, 1637, 2467, 3425, 4502, and 5702 keV, respectively (Table I). A least-squares fit of the rotational energy formula to the $0^{+\prime}$, 4^+, 6^+, and 8^+ energies (relative to $0^{+\prime}$) yielded $A = 38.1$ and $B = -0.047$ from which the $2^{+\prime}$, 10^+, and 12^+ energies in Table I were predicted. The fit is strikingly good all the way to spin 12. It is surprising that these data fit so well this relatively simple rotational formula which in general yields

TABLE I. Experimental energy levels (keV) in ^{72}Se and calculated levels for a rotational band built on the 937-keV $0^{+\prime}$ state. A fit of the 0^+, 4^+, 6^+, and 8^+ experimental energies by the rotational energy equation yielded $A = 38.1$ and $B = -0.047$. From these values the 2^+, 10^+, and 12^+ energies were predicted as given.

I^π	E_{exp}	E_{calc}
0^+	937	928
2^+	$\cdots$	1155
4^+	1637	1671
6^+	2467	2444
8^+	3425	3426
10^+	4502	4549
12^+	5702	5727

poor fits at high spin. The near linearity of the high-spin data in Fig. 1 show that the variable moment-of-inertia model works well, too. Since we assume that the two experimental 2^+ states result from the interaction of this rotational state and a one-phonon state, trace invariance of the Hamiltonian matrix can be used to locate the vibrational state. The result is 1024 keV, in reasonably close proximity to the 2^+ rotational state predicted at 1155 keV. The two-phonon triplet occurs about 2050 keV in zero order so its 0^+ and 2^+ members are quite far from the other 0^+ and 2^+ states to justify not mixing these and the lower 0^+ and 2^+ states. There is less justification for not mixing the 4^+ states, but they are much further apart (about 380 keV for the calculated 4^+ levels) than the lower 2^+ states (130 keV). Several states[5] seen near 2.0 MeV could be members of the two-phonon triplet.

Also, relative $B(E2)$'s and branching ratios were calculated. With strong mixing of the 2^+ states assumed, it is not unreasonable to take the mixing to be 50%-50%. Assume further that the microscopic structure of the ground state differs from the intrinsic state of the rotational band by at least two quasiparticles so that the rotational and vibrational states are not connected by a one-body operator such as the quadrupole operator. The $0^{+\prime}\rightarrow 2^+$ rotational transition has the strongest $B(E2)$ predicted ($\frac{1}{2}$ the square of the intrinsic quadrupole moment), which is supported by our large $B(E2)$. The $B(E2)$ ratio, $(2^{+\prime}\rightarrow 2^+)/(2^{+\prime}\rightarrow 0^+)$, is predicted to be 0.7 and experimentally[5] is found to be 1.5 ± 0.3 for $B(M1)=0$ and the $2^{+\prime}\rightarrow 2^+$ transition may contain some $M1$ admixture which could significantly lower the experimental ratio.

The deformation parameter $|\beta|$ calculated in a model-independent way[16] from

$$\beta_{\mathrm{rms}}^2 \approx (4\pi/3eZR_0^2)^2 B(E2{:}0\rightarrow 2),$$

is quite large ($|\beta|=0.31\pm0.06$) for the $0^{+\prime}$ state. The model dependence with the 862-keV 2^+ state an admixture of spherical and deformed shapes would yield a larger β value.

Thus with a simple approach of nuclear coexistence, we can explain semiquantitatively the decay properties of the low-spin states and fit the energies of the rotational band to spin 12 remarkably well. The very low crossing of the rotational band with the "phonon states" at spin 2 and the wide energy spacings of the vibrational states is most fortunate in that one can see the effects of this crossing relatively easily.

There is a striking similarity in the behavior of the yrast level energy spacings in ^{72}Se and ^{186}Hg (Ref. 7) where the $4\rightarrow 2$ and $6\rightarrow 4$ spacings, respectively, drop below the lower ones and then the higher ones increase with spin. It was just such a sharp bend in $\mathcal{I}$ as in Fig. 1 followed by a linearity of the plot of $\mathcal{I}(\omega^2)$ for the spins above 6 in ^{186}Hg that was interpreted[7] as evidence for a change in nuclear shape from near spherical to deformed. In each case the data indicate that the lowest-spin members are associated with near-spherical states and the high-spin states with deformed shapes with the shift occuring at lower spin in ^{72}Se. Our data clearly show that in ^{72}Se this change is consistent with a crossing of a deformed band with the 2^+ spherical state, where we see the lower members of the deformed band. It would be most interesting to search for the lower members of the deformed band in ^{186}Hg to show the complete similarity of these cases.

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Axisymmetric Stability of Kerr Black Holes*

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A variational expression is used to prove that all axisymmetric modes of the Kerr sequence of rotating black holes are stable.

Recent theorems of Hawking[1] and Carter[2] indicate that the Kerr family of rotating black holes is likely to be the unique end point of any gravitational collapse in which an event horizon forms. The stability of these configurations is therefore a question of some astrophysical interest, at least to the extent that such collapse is itself a common occurrence. Numerical calculations by Press and Teukolsky[3] indicate that the entire physical part of the Kerr sequence (angular momentum $< M^2G/c$) is in fact stable, but as yet, no analytic proof has been found. In this Letter we report a proof that all axisymmetric modes of the Kerr sequence are stable. Full details will be published elsewhere.

There are three steps to the proof. We show first that the linearized field equations form a self-adjoint system for unstable axisymmetric modes, whose eigenfrequencies are therefore purely imaginary. Then, by examining some analytic properties of Teukolsky's[4] equations, we find that the eigenfrequency of each such mode varies continuously along the Kerr sequence. We infer that instability in an axisymmetric mode can only set in when its frequency vanishes, an eventuality which we exclude by invoking Carter's theorem.[2]

The Kerr metric has the form

$$ds^2 = -(\omega^0)^2 + (\omega^1)^2 + (\omega^2)^2 + (\omega^3)^2,$$

where, in Boyer-Lindquist[5] coordinates,

$$\omega^0 = (\Delta\rho^2/D)^{1/2}\,dt,$$

$$\omega^1 = \sin\theta\left(\frac{D}{\rho^2}\right)^{1/2}\left(d\varphi - \frac{2Mar}{D}\,dt\right),$$

$$\omega^2 = (\rho^2/\Delta)^{1/2}\,dr,$$

$$\omega^3 = \rho\,d\theta,$$

and where

$$\rho^2 = r^2 + a^2\cos^2\theta,$$

$$\Delta = r^2 - 2Mr + a^2,$$

$$D = (r^2+a^2)^2 - a^2\Delta\sin^2\theta.$$

The Kerr parameters M and a are the mass and specific angular momentum of the solution they index, which has an event horizon at $r = r_+ \equiv M + (M^2 - a^2)^{1/2}$. The one-forms ω^i are a pseudo-orthonormal basis of locally nonrotating observers. All tensor indices below will refer to that basis; Latin indices run from 0 to 3 and Greek indices from 2 to 3.

Chandrasekhar and Friedman[6] derived a variational principle for the equations governing axisymmetric perturbations of any axisymmetric stationary solution of Einstein's equations. When no matter is present, their action can be written in the gauge-independent form

$$I = \int d\tau [h_{ij}{}^* \delta G^{ij} + \delta G_{1\alpha}{}^* \delta G^{1\alpha}], \qquad (1)$$

where h_{ij} is the perturbation in the metric, and δG^{ij} is the corresponding first-order perturbation in the Einstein tensor. The quadratic functional I has two important properties: First, it is Hermitian apart from integrations by parts over the spatial variables; and second, in a gauge where $h_{0\alpha} = 0$, only second time derivatives appear.

Suppose now that h_{ij} is a solution to the perturbed field equations, so that $I = 0$; further, choose a gauge in which $h_{0\alpha} = 0$, and assume a time dependence of the form $e^{-i\sigma t}$ for some complex eigenfrequency σ. Then, solving the equa-