used in the evaluation of the kernels are the same as in Ref. 2. Although the two-phonon states come from a broad band, we shall for computational convenience put $\omega_\Xi$ in Eq. (12) equal to 28.5 meV corresponding to the 230-$cm^{-1}$ peak of the Raman spectrum.[9] Inserting a value of $\mathfrak{D}_\Xi^{(11)} = 2.6\times10^3$ eV, equivalent to $d_\Xi^{(11)} = 26$ eV, we obtain $T_c = 0.3$ K at concentration $n = 10^{20}/cm^{-3}$ in agreement with the experimental value.[2] The $T_c$ calculated with this $d_\Xi^{(11)}$ as a function of $n$ fits the data of Ref. 2 well. The value of $d_\Xi^{(11)}$ deduced incorporates the contribution from at least three combinations of transverse optic phonon branches. In order to evaluate whether the magnitude of the second-order deformation potentials taken above is reasonable it is useful to compare it with a value for pure GaP with $\mathfrak{D}_\Xi = 965$ eV[13] for two TO phonons from a single TO branch. Moreover, this value, deduced from resonant second-order Raman scattering,[13] is expected to be smaller[14] than its component $\mathfrak{D}_\Xi^{(11)}$.

In summary we have proposed the two-phonon coupling mechanism for superconductivity in a single-valley degenerate semiconductor and applied it to $SrTiO_3$. The second-order Raman spectrum of $SrTiO_3$ lends strong support to the proposed mechanism.

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[15]The main point is that the occurrence of strong nonresonant second-order Raman scattering means that the $H_2$ contribution is strong when the $V_s^{(11)}$ contribution is small due to the nonresonance condition. This implies that the $V_s^{(11)}$ contribution will be strong in the resonance condition arising in the superconducting pairing and not canceled by the screened $H_2$ appropriate for degenerate semiconductors.

# Vortex-Line Pinning by Thickness Modulation of Superconducting Films

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We present experimental evidence for the interaction of vortex lines with a controlled pinning configuration induced by periodic thickness modulation of superconducting films. The critical current density as a function of a transverse magnetic field shows characteristic peaks at well-defined field values. These features give evidence for matching of the two-dimensional vortex-line lattice with the one-dimensional periodic pinning structure. A model is proposed which accounts for the observed temperature dependence of the matching effects.

It is well known that the transport properties of type-II superconductors depend strongly on the interaction of vortex lines with pinning centers.[1] In several cases, however, the interpretation of the experimental results becomes very uncertain because of the complexity, variety, and random distribution of the pinning structure involved. For this reason, great interest has recently been devoted to experiments dealing with well-defined pinning configurations.[2,3]

In this Letter we present preliminary experiments on superconducting films having controlled pinning induced by a periodic one-dimensional modulation of their thickness $d$. This idea has already been exploited by Morrison and Rose[4] in their experiments on controlled surface pinning in superconducting-alloy foils. In this connection, the pioneering work of Niessen *et al.*[5] on guided vortex motion in rolled specimens should also be mentioned.

The mechanism leading to additional pinning in our "modulated" films can easily be understood if one remembers that the vortex-line energy is proportional to its length.[6] For this reason, film regions where the thickness reaches a minimum give rise to a periodic one-dimensional pinning array. As a consequence one expects that, for current flow parallel to the grooves, there will be an enhancement $\Delta j_c$ of the critical current density compared with that of a similar completely "flat" film. Moreover, for well-defined values of a transverse magnetic field $H$, characteristic maxima should appear in the $\Delta j_c$-versus-$H$ curves, corresponding to matching of the vortex distance $a$ to multiples of the modulation period $\lambda_g$, i.e., for $a = n\lambda_g$, where $n$ is an integer.

In our experiments $\vec{H}$ is normal to the film plane and therefore $H \simeq B$. The relation between the magnetic induction $B$ and $a$ is then given by $B \simeq H = \varphi_0/a^2$ (here we neglect the small differences between triangular and square lattice structures). For the matching fields $H = \varphi_0/(n\lambda_g)^2$, the lowest-energy configurations of the vortex lattice are achieved by flux lines localized at thickness minima. For these fields one expects the pinning force to reach a (relative) maximum value, since each flux line of the vortex lattice simultaneously experiences the pinning effect of the periodic array.

Our experiments were performed on granular Al films.[7] These layers behave like dirty, extreme type-II superconductors. The thickness modulation was obtained by producing a grating-like structure on the film surface. After evaporation the Al films were covered with a photosensitive layer. A grating was then reproduced on the photolayer using the interference pattern of two incident He-Cd–laser plane waves. Suitable development of the photolayer and subsequent etching of the underlying Al film give the desired modulation amplitude $\Delta d$. Characteristic properties of our films are listed in Table I. In this connection we notice that the ratios of the second, third, and fourth harmonics to the fundamental one, deduced from a Fourier analysis of the film profile, are approximately 0.2, 0.05, and 0.015, respectively.

The enhancement $\Delta j_c$ of the critical-current density as a function of $H$ is shown in Fig. 1. These curves were obtained by subtracting from each other the $j_c$-versus-$H$ characteristics for current flow parallel to the grooves of a modulated film and a flat reference film, both having otherwise the same background pinning, as demonstrated by the absence of any additional critical current (i.e., $\Delta j_c \simeq 0$) for current flow perpendicular to the grooves. For this reason, we think that $\Delta j_c$ also characterizes our periodic pinning mechanism quantitatively.

For $T$ close to $T_c$, several interesting features are clearly apparent in Fig. 1. It is very easy, using the arguments given above and the value of $\gamma_g$ given in the table, to identify the maximum located at $H_{11}$ as the "fundamental peak," corresponding to one-to-one matching ($a = \lambda_g$, $n = 1$) of

TABLE I. Properties of the Al films. The mean free path $l$ was deduced from $\rho l = 0.49 \times 10^{-11}$ $\Omega$ cm$^2$ (Ref. 8) and $\xi_0$ from the bulk Al value ($\xi_{0\text{bulk}} = 16\,000$ Å, Ref. 11) scaled according to our $T_c$ (Ref. 7). $\lambda_L(0)$ is the London penetration depth [$\lambda_L(0) = 157$ Å, Ref. 11]. $\langle d \rangle$ represents the averaged film thickness. $\Delta d$ was determined with an inductive profile-scanning monitor (Talystep).

| Sample No. | $T_c$ (K) | $\rho$(4.2 K) ($\mu\Omega$ cm) | $\langle d \rangle$ (Å) | $\Delta d$ (Å) | $\lambda_g$ ($\mu$m) | $(\xi_0 l)^{1/2}$ (Å) | $\lambda_L(0)(\xi_0/l)^{1/2}$ (Å) |
|---|---|---|---|---|---|---|---|
| Al 2*a* | 2.0 | 35.5 | 5200 | ~200 | 1.9 | 365 | 4140 |
| Al 2*b* | 2.0 | 34.5 | 5350 | ~200 | 1.9 | 370 | 4080 |

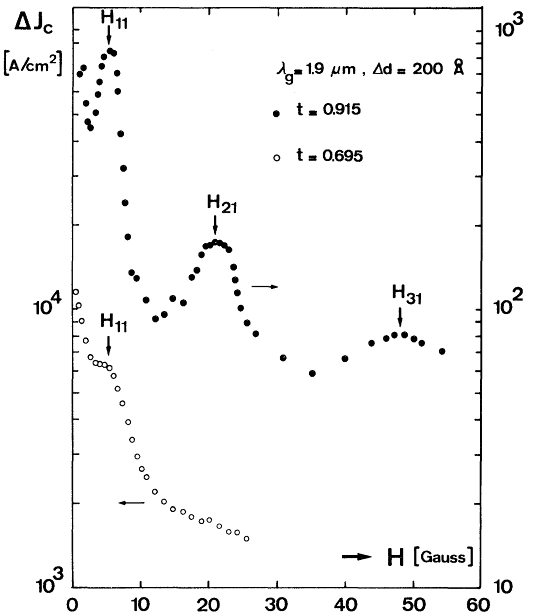


FIG. 1. Change in critical current density $\Delta j_c$ due to thickness modulation of granular Al films (sample 2*a*) as a function of applied magnetic field $H$. $t = T/T_c$. The experimental values of $H_{11}$, $H_{21}$, and $H_{31}$ are, respectively, 5.3, 21, and 48 G.

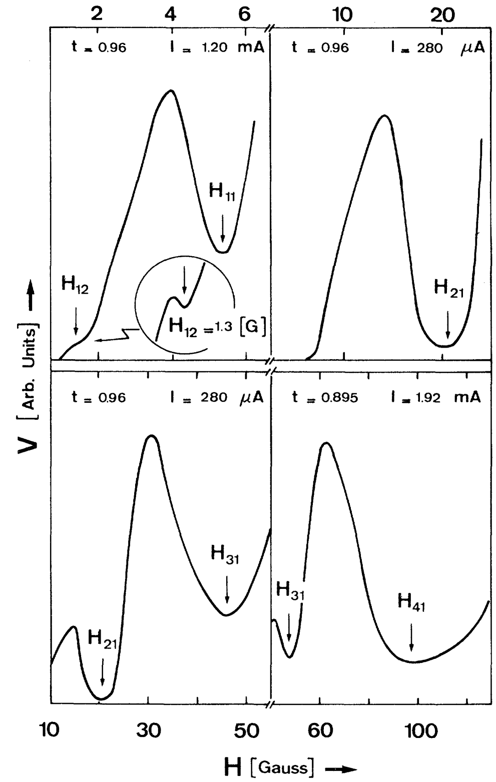


FIG. 2. Voltage-versus-$H$ curves showing resistance minima where $\Delta j_c$ reaches a maximum value. Sample Al 2*b*. $t = T/T_c$.

the vortex lattice with the periodic pinning structure: The calculated value of $H_{11}$ is 5.7 G (for a square lattice), in very good agreement with the experimental result $H_{11} = 5.3$ G. On the other hand, the sensitivity of a $\Delta j_c$-versus-$H$ plot like that of Fig. 1 is still not sufficiently high to reveal the presence of peaks we may attribute to vortex-line configurations corresponding to $a = n\lambda_g$ with $n = 2, 3, \ldots$. There is, however, a measuring technique which is particularly sensitive to the form of our $j_c(H)$ curves. If one considers the flux-flow $I$-$V$ characteristics of the modulated films, one immediately recognizes that for a suitable current level voltage minima should be detected at those field values for which $j_c$ reaches a maximum. Such $V$-versus-$H$ curves are shown in Fig. 2 where we discover a resistance minimum at $H_{12} = 1.3$ G corresponding to a vortex-line configuration satisfying the matching condition $a = 2\lambda_g$ ($n = 2$).

There are further, at first sight unexpected, peaks at $H_{21}$, $H_{31}$, and $H_{41}$ in Figs. 1 and 2. These cannot be interpreted on the basis of the above considerations which assume a pure harmonic modulation of the film profile. Experimentally, we find that $H_{m1}$ is closely related to $H_{11}$ by a relation of the form $H_{m1} = m^2 H_{11}$, where $m$ is an integer. This strongly suggests that the $\Delta j_c$ maxima at higher fields ($m > 1$) correspond to the higher harmonics in the Fourier expansion of the thickness modulation for which the matching condition becomes $a = \lambda_g / m$, in agreement with our experimental results. We may thus conclude that characteristic features corresponding to matching of the vortex-line lattice to the periodic film structure are found when $H$ satisfies the equation

$$H_{mn} = (\varphi_0 / \lambda_g^2)(m/n)^2, \quad (1)$$

where $m$ refers to a harmonic of the thickness modulation and $n$ defines the occupation of the periodic pinning array by the flux lines.

We now briefly discuss the temperature dependence of the matching effect. As can be seen from Fig. 1, the $\Delta j_c$ peaks tend to disappear when the temperature is lowered. At the lowest temperature of our experiment only the fundamental peak is partially resolved. Its position is temperature independent as it must be for a genuine effect. The presence of structure in the $\Delta j_c$-versus-$H$ curves is strongly related to the temperature dependence of the parameters characterizing the mixed state: the coherence length $\xi(T)$ and the penetration depth $\lambda(T)$. One can easily verify, using the values given in the table, that $\xi(T)$ does not play an important role in our experiments since $\xi(T)$ is always much smaller than $\lambda_g$ except for $1-t \lesssim 10^{-4}$, a temperature region difficult to explore.

The condition $\lambda(T) > d$ is nearly satisfied in the temperature range of our investigations so that the calculations of Pearl[9] and Fetter and Hohenberg[10] for the mixed state of superconducting films apply to our case. According to these models the strength of the interaction between vortices is controlled by an effective penetration depth $\Lambda(T) = 2\lambda^2(T)/d$. Near $T_c$ we find $\Lambda(T) > \lambda_g$, and at low temperatures, $\Lambda(T) < \lambda_g$. Thus near $T_c$ there is the required long-range interaction of the vortices *over several pinning chains* of the one-dimensional array for producing the matching effects predicted by Eq. (1). As a matter of fact the structure[10] of the vortex lattice near $T_c$ and in the field region of interest is essentially determined by flux-line interactions which dominate the small energy perturbations introduced by the periodic thickness modulation and lead to a nearly uniform field penetration. Consequently, the above considerations, which assume a uniform-flux line distribution, are valid in this temperature range and we are led to the matching condition Eq. (1).

The picture drastically changes at low temperatures where the periodic film structure rather than the interaction between the vortices is the basic mechanism determining the flux-line arrangement. Since $\Lambda(T) < \lambda_g$, vortex nucleation takes place preferentially along the periodic pinning chains, giving rise to nearly noninteracting parallel vortex rows. The distance $b$ between flux lines within a chain is given by $H = \varphi_0/\lambda_g b$. This anisotropic picture of the vortex configuration is appropriate at low fields, probably as long as $b > \Lambda(T)$, but is certainly not valid at high fields $[b < \Lambda(T)]$, where we again expect an isotropic behavior of the vortex lattice ($H = \varphi_0/a^2$). In any case one easily sees that for $T \ll T_c$ no matching effects can be detected in the interesting low-field region of our experiments. The anisotropic model does indeed predict a monotonic behavior of the $\Delta j_c$-versus-$H$ curves, $\Delta j_c$ having its maximum value at the vortex penetration field.

We conclude by emphasizing that the results of this experiment are a consequence of the flux quantization in type-II superconducting films.

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