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Pressure-Balance Limitations in Z Pinches with Diffusion Heating*

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Conditions of global energy and pressure balance are applied to a cylindrically symmetric plasma column having a B_θ confining field and a B_z stabilizing field, as exist in stabilized z pinches and tokamaks. In the process of producing stable configurations the plasma pressure due to Ohmic heating during magnetic field diffusion may become too large for the confining field to contain.

In fusion research there is revived interest in z pinches having magnetic field configurations with diffuse profiles since stable configurations exist which confine high-β plasmas according to magnetohydrodynamic (MHD) theory.[1-3] Experimentally the poloidal field diffuses rapidly and most of the profiles now being considered have longitudinal currents nearly uniform inside the pinched-plasma column. In many experiments the plasma β_θ's are high. The production of stable configurations would be greatly simplified by operating at lower values which by MHD theory would have a greater margin of stability. Attempts, however, to lower the plasma energy density have not been completely successful.

These considerations suggest that there are limitations on β_θ due to the processes involved in setting up the plasma and field profiles. In particular, the plasma is Ohmically heated during the diffusion of the poloidal field. For an incompressible, constant-conductivity, plane conductor excited by a step-function current the result[4] that the local Joule heating is greater than the field energy density led early workers on the diffuse pinch to the prediction[5] that $\beta_\theta > \frac{1}{3}$ without losses. These considerations led to the expectation that unless special measures were taken the Joule heating would raise the β above the low values then believed necessary for stability. A more detailed cylindrical calculation[6] on the incompressible model gives heating corresponding to $\beta_\theta > 2$, at the time the field has diffused so that it is uniform within 10% across the conductor, for all values of the current rise time. This result indicates the possibility that in the formation of a z-pinch plasma, heating by diffusion prevents magnetic containment by exceeding pressure-balance conditions as well as those for stability. This paper treats this problem using general energy- and pressure-balance conditions

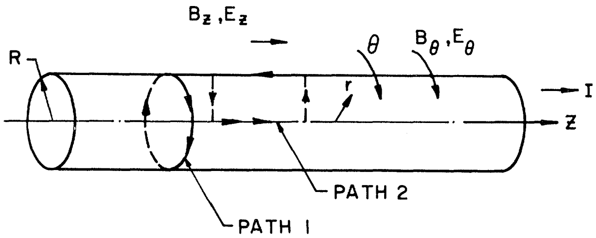

FIG. 1. Geometry for derivation of energy- and pressure-balance conditions.

for a mixed-field plasma column.

Consider a cylindrically symmetric plasma and microscopic field configuration with $\vec{B} = \hat{\theta} B_\theta(r,t) + \hat{z} B_z(r,t)$ (see Fig. 1). Using a cylindrical coordinate system (r,θ,z) we treat a volume V of unit length in the z direction and bounded by an outer radius R. Energy balance requires that the energy present in V in the form of plasma energy W_p and field energy W_f be given by the energy initially present, $W(0)$, plus the difference between the energy input W_{in} and the cumulative losses W_L both measured from $t=0$:

$$W(0) + W_{\text{in}} - W_L = W_p + W_z + W_\theta. \tag{1}$$

The field energy W_f has been expressed as the sum of the longitudinal and azimuthal field energies W_z and W_θ. The loss term W_L includes all energy which is not present as plasma kinetic energy or as field energy inside V. The total energy W_{in} that enters V between $t=0$ and $t=T$ is assumed to be supplied by the externally applied fields and is given by the time and surface integrals of the Poynting vector $\vec{P} = \vec{E} \times \vec{B}/\mu$. (We use rationalized mksa units; μ is the permeability of the vacuum.) We express W_{in} in terms of the flux functions φ_z and φ_θ by applying the Faraday induction law to the integration paths 1 and 2 in Fig. 1. Using the relation $B_\theta(R) = \mu I/2\pi R$, where I is the total longitudinal current, we obtain

$$\begin{aligned} W_{\text{in}} &= \int_0^T E_z(0,t) I(t)\, dt + \varphi_\theta I\big|_0^T \\ &\quad - \int_0^T \varphi_\theta \dot{I}\, dt + \mu^{-1} \int_0^T \dot{\varphi}_z B_z(R,t)\, dt \\ &\overset{\text{def}}{=} W_E + \varphi_\theta I\big|_0^T - W_{\theta p} + W_{z\,\text{in}}. \end{aligned} \tag{2}$$

We call W_E the flux annhilation integral since it will be nonzero only when the time integral of the longitudinal applied voltage, φ_{input}, differs from φ_θ. This leads us to define the flux annhilation factor which normally exceeds unity after the electric field reaches the axis:

$$f = \varphi_{\text{input}}/\varphi_\theta \equiv \int_0^T E_z(R,t)\, dt/\varphi_\theta(T). \tag{3}$$

The time integral of the electric field on axis as deduced from the Faraday induction law is $(f-1)\varphi_\theta$. The integral W_E may be written

$$W_E = \bar{I} \int_0^T E_z(0,t)\, dt \equiv (f-1)\xi I(T) \varphi_\theta(T), \tag{4}$$

where $\bar{I} = \xi I(T)$ $(0 < \xi < 1)$ is the value of current given by the mean value theorem.

Assuming that at some time T the plasma energy thermalizes and reaches static pressure balance, we use the MHD equilibrium equation,

$$(\partial/\partial r)(p + B_z^2/2\mu) + (2\mu)^{-1} r^{-2} \partial (r B_\theta)^2/\partial r = 0. \tag{5}$$

A global pressure-balance condition is obtained by multiplying Eq. (5) by πr^2 and integrating over the radius.[7] An integration by parts yields

$$\tfrac{2}{3} W_p = W_0 - W_z + W_R, \tag{6}$$

where W_p is the plasma thermal energy, W_0 is $\mu I^2/8\pi$, and W_R is $\pi R^2[p(R) + B_z^2(R)/2\mu]$. If we use β_θ defined as[1,5]

$$\beta_\theta = \int_0^R p\, 2\pi r\, dr/W_0, \tag{7}$$

Eq. (6) can be written

$$\beta_\theta = 1 + (W_R - W_z)/W_0. \tag{8}$$

For a simple z pinch in equilibrium with zero plasma pressure at the wall, $W_R = W_z = 0$ and β_θ is unity. For this case Eq. (7) reduces to the Bennett relation[8] and we may regard Eq. (8) as a generalized Bennett relation for a mixed-field plasma column.

Letting $I(0) = 0$, eliminating W_p between Eqs. (1) and (6), and using Eqs. (2) and (4), we obtain

$$\begin{aligned} W(0) + [(f-1)\xi + 1]\varphi_\theta I - W_{\theta p} + W_{z\,\text{in}} \\ = \tfrac{3}{2}(W_0 + W_R) - \tfrac{1}{2} W_z + W_\theta. \end{aligned} \tag{9}$$

All quantities except $W(0)$ are to be evaluated at a time T when equilibrium is established.

The experiments on the Los Alamos toroidal z pinch ZT-1 and elsewhere indicate that the poloidal magnetic field rapidly diffuses giving a nearly uniform current in the compressed plasma column. An idealized model of such a diffuse, uniform current pinch of plasma radius r_p has a B_θ field which varies linearly with radius inside the plasma and inversely outside. When the expressions for W_θ and $\varphi_\theta I$ and $W(0) = W_p(0) + W_f(0)$ are inserted into Eq. (9), there results

$$\begin{aligned} W_p(0) &+ 2[2(f-1)\xi + 1] W_0 \ln K + 2(f-1)\xi W_0 \\ &- W_{\theta p} + W_{z\,\text{in}} + \tfrac{3}{2}(W_z - W_R) \\ &- [W_z - W_f(0)] - W_L = 0, \end{aligned} \tag{10}$$

where $K = r_p/R$ with R taken as the wall radius.

We apply Eq. (10) to a stabilized z pinch energized with a step function $I(t)$ which reaches an equilibrium state with compressed B_z and the diffuse B_θ field discussed above. At the time of onset of the longitudinal current ($t=0$) we assume that the only field present is an initial uniform B_z supplied by a crowbarred conductor so that $\dot{\varphi}_z = 0$ for $t > 0$. Under these conditions $W_f(0) = W_z(0)$ and $W_{z\text{in}}$ is zero. For a step function $I(t)$ the term $W_{\theta p}$ is zero. Since $W_p(0)$, K, ξ, and $f-1$ are all positive, the first three terms in Eq. (10) cannot be negative. For the compressed field case, if $p(R)$ is negligible, we have $W_z \geq W_z(0) \geq W_R$ so that the grouping of terms $\frac{3}{2}(W_z - W_R)$ is positive and dominates the negative grouping $-[W_z - W_z(0)]$. Thus the loss term must be nonzero in order to satisfy Eq. (10). The condition is only marginally met with no losses for the nontypical special case: (1) There is no initial plasma energy at the time of current onset $[W_p(0)=0]$, (2) the B_θ flux is conserved ($f=1$), and (3) there is no plasma or B_z-field compression ($K=1$, $W_z = W_z(0) = W_R$).

Thus, *if a cylindrically symmetric stabilized pinch is driven with a current which rises to its final value very rapidly compared to the compression and flux diffusion times, it cannot achieve static equilibrium after the B_θ field diffuses to a uniform current distribution unless losses or plasma pressure at the wall are present.* It follows from continuity that this conclusion will be true also if a small amount of reversed longitudinal field outside the compressed plasma column is added for stability. It may be possible, however, to achieve magnetic containment without the loss requirement by increasing W_R using substantial amounts of reversed longitudinal field outside the pinch applied before the poloidal field diffusion is complete. If this is done by external field programming, any additional energy input ($W_{z\text{in}}$) must be included and, if positive, acts in the opposite fashion to W_R so that care must be taken not to defeat the purpose. This reduction of the loss required to achieve magnetic containment amounts to arranging the configuration so that the longitudinal field supports a portion of the excess pressure arising from poloidal-field diffusion, in addition to its usual role of supplying stability. To achieve confinement the whole operation must be done in such a fashion that a stable configuration results.

For a slow current rise one expects less heating for the final diffused state since the $W_{\theta p}$ term in Eq. (10) will give a negative contribution rather than zero as for the step-function case. A measure of the closeness of a given finite current rise to the step function is given by the smallness of the ratio $D = W_{\theta p}(T)/\varphi_\theta(T)I(T)$.

We inquire as to whether the ratio D for z pinches is high enough that equilibrium can be obtained with negligible losses and plasma pressure at the wall. We solve Eq. (10) for D, and since $W_p(0)$ is positive it can be omitted giving a lower limit on D for the diffuse B_θ profile. Making use of Eq. (8) we obtain the following necessary condition for energy and pressure balance without losses:

$$D \geq \frac{\frac{1}{2}[2(f-1)\xi + 1]\ln K + \frac{1}{2}(f-1)\xi + \frac{1}{8}(1-\beta_\theta) + [W_{z\text{in}} + W_z(0) - W_R]/4W_0}{\ln K + \frac{1}{2}}. \tag{11}$$

For common current wave forms, ξ is greater than $\frac{1}{2}$ since $E_z(0,t)$ is low at the beginning of the current rise and becomes larger with time as the field diffusion progresses, thus weighting the larger current values. [For the rise time of a sinusoidal current and $E_z(0,t) \propto t^n$, ξ ranges from $2/\pi$ for $n=0$ to unity for $n \to \infty$.] For stabilized z-pinch operation with compression ratios of about 2, with small amounts of external reversed field, and $p(R)=0$, the above condition is typically $D \gtrsim 0.5$. If this condition on D is not satisfied in fast-compression experiments, radiation and ionization losses are likely to be insufficient and equilibrium can be obtained only after wall contact. The conditions required to establish stable equilibria are difficult to predict since the experimental flux penetration rates are anomalous.

It is of interest to apply condition (11) to a tokamak. Approximating this case with $K=1$, $B_\theta = 1$, $W_{z\text{in}} = 0$, and $W_z(0) = W_R$, we obtain $D \geq \xi(f-1)$. An upper limit on D for the incompressible case can be obtained by assuming that the B_θ field diffuses instantaneously to a uniform current distribution during the current rise. The direct evaluation of $W_{\theta p}$ then gives $W_{\theta p} = W_0$ and $I = 2W_0$, so that the upper limit on D for the incompressible case is $\frac{1}{2}$. Since $\xi > \frac{1}{2}$ the condition on D will be violated if $f > 2$. Since (1) tokamaks typically operate with f factors larger than 2 for the initial setup phase, (2) the flux does not penetrate instantaneously, and (3) ξ will most likely be considerably greater than $\frac{1}{2}$, the above argument predicts that an energy loss is also needed for a tokamak to achieve a final state of uniform current for confinement by the poloidal field, i.e., $\beta_\theta = 1$. This suggests that the poorly understood early phase of the tokamak discharge before it falls into a steady regime is related to the excess heating caused by the poloidal field diffusion. The discharge can lose any such excess energy (above that needed for ionization) by instability, recycling, or other means before it settles down to its equilibrium phase.

The sometimes-suggested method of reducing β_θ by reducing the plasma density is far from being a simple conclusion. The plasma density nowhere enters explicitly in the above considerations. Its effect appears indirectly in determining the flux-penetration rates which determine the energy-input integrals. The density can become important however, by affecting the compression rate, and also the diffusion rate if it is anomalous.

The arguments given are based on the general assumptions of energy and pressure balance, cylindrical symmetry, and plasma thermalization. The results apply (toroidal corrections aside) to a wide class of containment devices.

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Collision-Induced Light Scattering Observed at the Frequency Region of Vibrational Raman Bands

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We show that collision-induced light scattering at the frequency range of vibrational Raman bands of perturbed molecules can be observed and analyzed and that for gases the scattering process is similar to the one of depolarized Rayleigh scattering. In particular the I_{VH} component of the totally symmetric ν_1 vibration of SF_6 and CF_4 has been studied. The depolarized light scattering at the ν_1 Raman band of liquid SF_6 is also discussed.

Collision-induced light scattering arises from molecular interaction of atoms and molecules at higher gas densities or in the liquid state. This type of scattering has been studied in the frequency region of the Rayleigh wings, firstly in rare gases[1,2] and then in optically isotropic molecules in the gaseous and liquid state. The light scattering by a collision pair of identical atoms or molecules may be attributed[3] to an additional anisotropy of the polarizability for this pair:

$$\beta = 6\alpha_0^2 r^{-3} + \beta_{sr} \tag{1}$$

where $3\alpha_0$ is the trace of the polarizability of an unperturbed atom (molecule) and r is the intermolecular distance. The first term represents the dipole-induced dipole contribution which may be described either by the local field fluctuations[1] or by double light scattering.[4] The second term β_{sr} takes short-range contribution into account, in particular electron overlap[5] and frame distortion[6] during the collisions between molecules. For the approximation $\beta_{sr} = 0$ and ideal gas the Rayleigh depolarization ratio η_s is given for this first approach by

$$\eta_s \approx \tfrac{6}{5}\alpha_0^2 \langle r^{-6}\rangle, \tag{2}$$

where the bracket is an average over the collision pairs; this factor is proportional to the num-